Resilience of natural-resource-dependent economies

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Abstract: We study how human preferences affect the resilience of economies that depend on more than one type of natural resources. In particular, we analyze whether the degree of substitutability of natural resources in consumer needs may give rise to multiple steady states and path dependence even when resources are managed optimally. This is a major shift in the interpretation and analysis of resilience, from viewing (limited) resilience as an objective property of the economy-environment system to acknowledging its partially subjective, preference-based character. We find that for a given set of initial conditions, society is less willing to buffer exogenous shocks if resource goods are complements in consumption than if they are substitutes. Hence, the stronger the complementarity between the various types of resource goods, the less resilient the economy.

Keywords: complementarity, discount rate, multiple steady states, path dependence, preferences, resilience, substitutability, tipping point, natural resources

JEL-Classification: C62, O13, Q01, Q20

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1 Introduction

Resilience is typically viewed as a property of natural systems and is defined as the extent to which they can buffer exogenous shocks (Scheffer et al. 2001). Natural systems, whether they are ecosystems, species populations or climate systems, tend to be robust to small perturbations. But if shocks are large enough that a system crosses a threshold or tipping point, its dynamics may be such that it collapses to a bad state. Examples of natural systems characterized by limited resilience include populations with a minimum population size below which extinction is inevitable (e.g., Gould 1972; Berck 1979; van Kooten and Bulte 2000), ecological systems with complex interactions between the various components of the system such as shallow lakes and semi-arid rangelands (e.g., Mäler et al. 2003; Anderies et al. 2002), and the Earth's climate system, where events like melting of the Greenland ice sheet or of the permafrost in the Northern Hemisphere may cause the Earth's climate to change dramatically (e.g., Ridley et al. 2010; Gough and Leung 2002).

The extent to which a natural system is resilient against exogenous shocks is not just a function of the underlying ecological processes; it also crucially depends on the way in which the system is managed. An example in point is the stock of cod in the North-East Arctic which collapsed in the late 1980s due to very high harvesting pressure combined with a sudden shortage of its main prey species, capelin (Hersoug et al. 2000). Poorly managed natural resource systems, including so-called open-access resources, are generally less resilient to shocks than optimally managed systems. This does not mean, however, that optimally managed systems never collapse. Optimal policy making in the face of potential future negative disturbance requires comparing, in terms of intertemporal social preferences, the benefits and costs of ex-ante precaution and of ex-post restoration of the system – if restoration is physically possible at all. Hence, resilience is not just an intrinsic feature of natural resources; institutions and preferences are likely to play an important role, too (Horan et al. 2011).

In this paper, we explore the impact of human preferences on the resilience of optimally managed economies that depend on more than one type of renewable natural resource. We find two characteristics of preferences to be of key importance. The first is – not surprisingly – the discount rate used by the social planner. For a given amount of time needed for natural resources to return to their good state in the wake of a negative shock, system restoration is less likely to be optimal the higher the social discount rate – even if abstaining from intervening results in the demise of one or more resources. The second key characteristic of preferences is more surprising: it is the extent to which the various types of natural resources are substitutes or complements in the consumers' utility function.[1] While intuition would suggest that society's willingness to protect a natural resource from collapse would be larger the more it depends on its output, i.e., when natural resources are complements rather than substitutes in consumption, we find the exact opposite. The reason is that if restoring the resource requires a moratorium on its exploitation, it is less costly to do so if there are good substitutes available so that postponing exploiting the resource does not reduce consumer welfare by much. While we pay most attention to the impact of the discount rate and the degree of complementarity in resource consumption on the resilience of the resource-dependent economy, we analyze the impact of other factors, too, including the rate of resource regeneration and the opportunity costs of harvesting.

Over the past decade, many papers have tried to provide explanations for the collapse of historic societies as diverse as those of Easter Island, the Anasazi, and the Maya (Diamond 2005), as better insights into the fate of previous civilizations may help the

[1] In two previous papers (Baumgärtner et al. 2011; Derissen et al. 2011) we present a few numerical examples where multiple steady states exist in a related model when natural resources are complements in consumption, and when resources are exploited under pure open access. In this paper, we analytically characterize the dynamic properties of the system under optimal forward-looking resource management.

current one to better cope with today's major environmental crises such as climate change or biodiversity loss (Arrow et al. 1995, Taylor 2009). For example, Brander and Taylor (1998) and Taylor (2009) develop models to explain the disappearance of the civilization on Easter Island, suggesting that its demise may have been due to a nonlinear interaction between population growth and the dynamics of natural resources, especially forest resources, resulting in feast-and-famine cycles. While lack of property rights and myopia are most likely the underlying causes of the collapse of the Easter Island civilization, our paper suggests that depletion of the forest resource is not necessarily suboptimal. To feed the population, fish need to be caught, and hence trees need to be logged continuously to produce boats. Timber consumption and catching fish are thus complements, and the instantaneous costs of reduced fishing activity to restore forest stocks may have been too large compared to the long-run benefits of recovered timber resources.

Modern society is admittedly much more complex than that of Easter Island, but our model still provides important insights for the challenges we face today. For example, because the availability of good substitutes is smaller the higher the level of physical aggregation (e.g., protein intake from wild deer can easily be replaced by farmed beef, but no good substitutes are available for the Earth's climate system), dealing with the large-scale environmental challenges posed today may even be more difficult than previously thought.

The paper is organized as follows. In Section 2, we present a stylized model of an economy that depends on the use of two types of renewable natural resources. In Section 3, we derive the conditions for dynamically optimal resource use. In Section 4, we use this information to analyze the steady states and path-dependence of the resource-dependent economy. In particular, we study how the number of optimal steady states and their stability properties depend on the degree of complementarity of the two resources in consumption and on the social discount rate, providing both analytical results and numerical examples. We use the results to determine the consequences for optimal resilience management of a resource-dependent economy in Section 5, taking into account the possibility that the stocks may be hit by a negative shock. We conclude by discussing the implications of our model for the management of global natural resources in Section 6.

2 Model of a natural-resource-dependent economy

The representative agent derives utility from the consumption of a (composite) manufactured good, $y(t)$, and from the consumed quantities of two different natural-resource goods, $h_1(t)$ and $h_2(t)$. The agent's instantaneous utility function is specified as

$$u(y(t), h_1(t), h_2(t)) = y(t) + \gamma \ln \left[\sum_{j=1,2} h_j(t)^{1-\kappa} \right]^{\frac{1}{1-\kappa}} , \tag{1}$$

where $\gamma > 0$ is the weight the agent attaches to consumption of natural resources, and $1/\kappa$ is the elasticity of substitution between the two natural-resource goods.

The quasi-linear utility function (1) captures some stylized facts about preferences on natural resources and manufactured goods. Considering that natural resources satisfy basic needs, marginal utility, according to specification (1), goes to infinity (zero) as the consumption of resources goes to zero (infinity). Marginal utility of the manufactured good, in contrast, is constant. Hence, this good is not essential: marginal utility does not go to infinity as consumption goes to zero. Next, utility function (1) is flexible in that it allows the two natural-resource goods to be complements in consumption or substitutes, depending on the value of κ. The two natural-resource goods are perfect substitutes in consumption if $\kappa \to 0$ and perfect complements if $\kappa \to \infty$, with $\kappa = 1$ as the special case where the sub-utility from consumption of resources is the Cobb-Douglas function. Hence, κ measures the degree of complementarity of the two natural-resource goods in consumption: the higher κ, the stronger the complementarity between the two. Finally, note that in case $\kappa > 1$, depletion of one or more resources (implying that $h_1(t)$ and/or $h_2(t)$ equals zero) results in instantaneous utility being equal to minus infinity. This suggests that it is never optimal to deplete either resource – but see Section 4.

The representative agent inelastically supplies one unit of labor on a competitive labor market. Labor is allocated between the three activities of producing the manufactured good, y, and harvesting of the two natural resource stocks to produce resource goods, h_1 and h_2. We assume that all goods-markets always clear, so that quantities consumed of each of the three commodities equal the quantities supplied. Regarding the production of the manufactured good, we assume that the quantity produced is a linear function of just one input, labor, with constant marginal productivity equal to $\omega > 0$. Using $e_1(t)$ and $e_2(t)$ to denote effort allocated to respectively harvesting of resource goods 1 and 2 at time t, the aggregate output of the manufactured good produced thus equals

$$y(t) = \omega \left(1 - \sum_{j=1,2} e_j(t)\right) . \tag{2}$$

Normalizing the sales price of the manufactured good to unity, linearity of the manufacturing production function (2) implies that the general equilibrium wage rate equals ω – as long as manufacturing is still taking place. To ensure that this is the case, we assume that $\omega > \gamma$ (see Appendix A.1), that is, the marginal product of manufacturing should be larger than the weight the representative agent attaches to the consumption of natural resource goods.

Regarding the natural resource sectors, we assume that resource goods are produced according to the standard Schaefer production function (Gordon 1954, Schaefer 1957):

$$h_j(t) = q_j \, x_j(t) \, e_j(t), \qquad j = 1, 2 \ , \tag{3}$$

where $x_j(t)$ is the size of the resource stock j at time t, and q_j is a technology parameter reflecting what share of the resource stock j can be harvested per unit of effort. Note that the marginal product of labor, $e_j(t)$, allocated to harvesting resource j is larger the larger the size of the resource stock, $x_j(t)$.

Regarding the dynamics of the resource stocks, current harvesting $h_j(t)$ reduces the remaining stock $x_j(t)$, but there is also natural regeneration. The change in the stock size of resource j at time t, $\dot{x}_j(t) \equiv dx_j(t)/dt$, equals the net natural growth of the resource stock, $f_j(x_j(t))$, minus the quantity harvested, $h_j(t)$:

$$\dot{x}_j(t) = f_j(x_j(t)) - h_j(t), \quad j = 1, 2 \ , \tag{4}$$

where we assume that both resources regenerate according to the standard logistic growth function (Verhulst 1838):

$$f_j(x_j(t)) = r_j \, x_j(t) \left(1 - \frac{x_j(t)}{K_j}\right), \quad j = 1, 2 \ . \tag{5}$$

Here r_j denotes the intrinsic (or maximum) growth rate of resource j, and K_j its carrying capacity (or the maximum stock the ecosystem can sustain). Substituting (5) into (4), our model implies that net natural growth of resource j is a function of just the size of its stock and of the rate at which it is being harvested – there is no physical interaction between the two resource stocks. We thus assume that the two stocks are geographically separated, and that the only interaction between the two is via consumer preferences (see also Halsema and Withagen 2008, Quaas and Requate 2011).

We assume that a social planner maximizes the representative household's present value of utility

$$\int_0^\infty u(y(t), h_1(t), h_2(t)) \, e^{-\delta t} \, dt \ , \tag{6}$$

where $\delta > 0$ is the social discount rate. This discount rate is assumed to reflect the representative household's impatience to consume, but possibly also the limited permanence of society's institutions. With a positive probability, the current institutions may cease to exist at any given point in time by forces beyond the planner's control.

To maximize (6), the social planner chooses the quantity harvested of each of the two resources, $h_1(t)$ and $h_2(t)$, in every period as well as the amount of the manufactured good produced, $y(t)$, taking into account constraints (1)–(5).[2]

3 Conditions for optimal resource use

The conditions for dynamically optimal resource use are derived in Appendix A.1 as the necessary first-order conditions for the social planner's maximization problem. In the following, we use π_j to denote the shadow price of consuming resource j. This shadow price is equal to the direct marginal costs of harvesting, i.e., the cost of effort needed to harvest an extra unit of resource j, plus the opportunity costs of reducing the current resource stock with one unit, which are given by the shadow price of the stock of resource j, μ_j.

Using (4) and the conditions for optimal resource use derived in Appendix A.1, we obtain the following system of differential equations ($i, j = 1, 2, j \neq i$):

$$\dot{x}_j = f_j(x_j) - h_j = f_j(x_j) - \gamma \, \frac{\pi_i^{-\frac{1}{\kappa}}}{\sum\limits_{j=1,2} \pi_j^{1-\frac{1}{\kappa}}} \; , \tag{7}$$

$$\dot{\pi}_j = \left[\delta - f_j'(x_j)\right] \left[\pi_j - \frac{\omega}{q_j \, x_j}\right] - \frac{\omega \, f_j(x_j)}{q_j \, x_j^2} \; , \tag{8}$$

that governs the optimal dynamics of the resource-dependent economy together with the initial conditions, $x_j(0) = x_{j0}$, and the transversality conditions, $e^{-\delta \, t} \, \mu_j \, x_j \xrightarrow{t \to \infty} 0$, for both resources $j = 1, 2$. The interaction between the two resources is captured by the harvesting term in Equation (7). Equation (8), in contrast, depends only on the stock and shadow price of the resource $j = 1, 2$ itself.

The resilience of the resource-dependent economy is determined by the number of optimal steady states, and their stability properties. A steady state is characterized by $\dot{x}_1 = \dot{x}_2 = 0$ and $\dot{\pi}_1 = \dot{\pi}_2 = 0$. Using these conditions in (7) and (8) we obtain (for

[2]From here on, we omit the time indicators, unless it may cause confusion to do so.

$i, j = 1, 2$ and $j \neq i$):

$$\pi_j = \pi_i \left[\frac{\gamma}{\pi_i \, f_i(x_i)} - 1 \right]^{\frac{\kappa}{\kappa - 1}} , \tag{9}$$

$$\pi_j = \frac{\omega}{q_j \, x_j} \left[1 + \frac{f_j(x_j)}{x_j \, \left[\delta - f_j'(x_j) \right]} \right] = \frac{\omega}{q_j \, x_j} \frac{\delta + r_j \, \frac{x_j}{K_j}}{\delta - r_j + 2 \, r_j \, \frac{x_j}{K_j}} . \tag{10}$$

Note that in Equation (10) the shadow price of consuming resource j is just a function of the size of stock j, while the interaction between the two resources is captured in Equation (9). Using (5), we can rewrite (10) as

$$x_j(\pi_j) = \frac{K_j}{4} \left[\sqrt{\frac{8 \, \delta \, \omega}{r_j \, q_j \, K_j \, \pi_j} + \left[\frac{\delta - r_j}{r_j} - \frac{\omega}{K_j \, q_j \, \pi_j} \right]^2} - \frac{\delta - r_j}{r_j} + \frac{\omega}{K_j \, q_j \, \pi_j} \right] , \tag{11}$$

while combining (9) and (10) yields:

$$\pi_j(x_i) = \frac{\omega}{q_i \, x_i} \frac{\delta + r_i \, \frac{x_i}{K_i}}{\delta - r_i + 2 \, r_i \, \frac{x_i}{K_i}} \left[\frac{\gamma \, q_i}{\omega \, r_i} \frac{\delta - r_i + 2 \, r_i \, \frac{x_i}{K_i}}{\left[\delta + r_i \, \frac{x_i}{K_i} \right] \, \left[1 - \frac{x_i}{K_i} \right]} - 1 \right]^{\frac{\kappa}{\kappa - 1}} . \tag{12}$$

Together, Conditions (11) and (12) give the optimal steady-state stock x_j of resource j as a function of the steady-state stock x_i of resource i, the $x_j(x_i)$-isocline.

4 Steady states and path-dependence of optimal resource management

To be able to derive clear-cut analytical results, we assume in the following that the two natural resources are governed by the same dynamic processes. That is, the parameters of the biological growth functions are assumed to be the same, and so are the parameters of the harvesting functions: $r_1 = r_2 = r$, $K_1 = K_2 = 1$, and $q_1 = q_2 = q$. Furthermore, because $(x_1, x_2) = (0, 0)$ is an absorbing state (see Equation 5), we assume throughout the analysis that at least one of the resource stocks is initially strictly positive.

If the two resource stocks are driven by the same dynamic processes, there always exists a unique interior symmetric steady state, as stated in the following lemma.

Lemma 1. *For symmetric resources, and if and only if* $\max\{2\,r\,\omega - \gamma\,q, r\gamma\,q - \delta(\gamma\,q - 2\,r\,\omega)\} > 0$*, there is one (and only one) symmetric steady state* $(x_1^S, x_2^S) = (x^S, x^S)$*, where*

$$x^S = \frac{1}{2\,r}\left[r - \delta - \frac{\gamma\,q}{\omega} + \sqrt{(\delta + r)^2 + \left(\frac{\gamma\,q}{\omega}\right)^2}\right] > 0\ . \tag{13}$$

Proof: see Appendix A.2. ■

The resource stocks in the symmetric interior steady state do not depend on the degree of complementarity, κ, as both resources are used in equal quantities. Of course, the sizes of the resource stocks do depend on the social discount rate, δ: the higher the discount rate, the smaller the optimal resource stocks. If the condition $2\,r\,\omega > \gamma\,q$ is met, however, the steady state stocks are positive even for $\delta \to \infty$. Higher weights on resource consumption (γ), higher harvesting productivities (q) and lower productivity levels in manufacturing (ω) make instantaneous resource harvesting more attractive – especially if society does not care much about the future (i.e., if $\delta \to \infty$). In that case, stock depletion can still be avoided, but only if the intrinsic growth rate of the resource (r) is sufficiently high.

We are interested in how the number of steady states depends on society's preferences, including the degree of complementarity of resources in consumption. The following proposition yields a first result.

Proposition 1. *The symmetric steady state is locally stable independent of the degree of complementarity* κ *if either* $\gamma\,q < r\,\omega$ *or* $\delta < r^2\,\omega/\left(2\,(\gamma\,q - r\,\omega)\right) \equiv \delta_{MSY}$*.*

Proof: see Appendix A.3. ■

Combined with Lemma 1, Proposition 1 states that the steady-state level is independent of κ if $\min\{r\,\omega - \gamma\,q, \delta_{\mathrm{MSY}} - \delta\} > 0$. The reason is that if this condition is met, the steady-state resource stock x^{S} – as defined in (13) – is larger than the maximum-sustainable-yield stock, x_{MSY}.[3] If $\gamma\,q < r\,\omega$, demand for the resources (captured by

[3]The maximum-sustainable-yield stock level is defined as the level at which the amount that can

γ) and productivity of resource extraction (captured by q) are so low relative to resource productivity (captured by r) and opportunity costs of harvesting (captured by ω) that even under full myopia ($\delta \to \infty$, or equivalently: open access to resources) the symmetric steady-state stocks are larger than the stocks that would generate the maximum sustainable yield (see Appendix A.2). If $\gamma\, q > r\, \omega$, one has $x^{\mathrm{S}} = x_{\mathrm{MSY}}$ if $\delta = r^2\, \omega / \left(2\, (\gamma\, q - r\, \omega)\right) \equiv \delta_{\mathrm{MSY}}$, and hence the symmetric steady-state levels are smaller than x_{MSY} if and only if $\min\{r\, \omega - \gamma\, q, \delta_{\mathrm{MSY}} - \delta\} > 0$.

We are interested in how preferences affect both the number of steady states and also their stability. Proposition 1 states that if $\max\{r\, \omega - \gamma\, q, \delta_{\mathrm{MSY}} - \delta\} > 0$, the symmetric steady-state stocks are stable independent of κ. Having established this result, we now move on to analyzing the case in which neither of these conditions is met. This means that for the rest of the paper we explore all cases for which the following condition holds:

Condition 1: $r\, \omega < \gamma\, q < 2\, r\, \omega$ and $\delta > \delta_{\mathrm{MSY}}$.

For all cases in which condition 1 holds, the steady state stocks are in between 0 and the maximum sustainable yield stock, and preferences now crucially affect both the number of steady states and their stability. Therefore, we now turn to analyzing the situations in which resources are either substitutes (Section 4.1) or complements (Section 4.2) for all cases in which condition 1 is met.

4.1 Optimal dynamics when resources are substitutes

Let us now analyze the case where Condition 1 holds and where the degree of complementarity of the two resources in consumption is such that they are substitutes in consumption (i.e., $\kappa < 1$).

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be harvested without reducing the stock, is maximal. For any x the sustainable harvesting level is $h_j \leq f(x_j)$, and this amount is largest at the stock level where net resource regeneration is largest. Solving $f\,'(x_j) = 0$ and using the equation of motion (5), we have $x_{\mathrm{MSY}} = K/2 = 1/2$.

Proposition 2. *When resources are substitutes ($\kappa < 1$) the following holds.*

2a. If the condition

$$\delta < \; r \, \frac{\gamma\, q}{\gamma\, q - \omega\, r} \equiv \delta_0 \tag{14}$$

holds,[4] *there are three steady states: the symmetric steady state $(x_1^S, x_2^S) = (x^S, x^S)$ with $x^S > 0$, and two asymmetric steady states $(x^A, 0)$ and $(0, x^A)$, with*

$$x^A = \frac{1}{2\,r} \left[r - \delta - \frac{2\,\gamma\, q}{\omega} + \sqrt{(\delta + r)^2 + \left(\frac{2\,\gamma\, q}{\omega} \right)^2} \right] > 0 \; . \tag{15}$$

2b. The symmetric steady state (x^S, x^S) is globally[5] *stable.*

Proof: see Appendix A.4. ■

Any optimal steady state, whether or not we assume that resources are identical, is determined by the solution of the fixed-point equation $x_1(x_2(x_1)) = x_1$, where $x_1(x_2)$ and $x_2(x_1)$ are determined by Equations (11) and (12) (for $i, j = 1, 2$ and $j \neq i$). When resources are substitutes the isoclines are upward-sloping over the entire domain (see Appendix A.4), and hence they only intersect once. Proposition 2 states that if $\kappa < 1$, $(x^{\mathrm{S}}, x^{\mathrm{S}})$ is the only optimal steady state, as the asymmetric ones are unstable. That means that $x_j = x^{\mathrm{A}}$, $x_{-j} = 0$ is achieved only if $x_{-j0} = 0$.

These results are illustrated in Figure 1.[6] We will refer to the case where $\delta < \delta_0$ (i.e.

[4] The subscript 0 refers to the result that $x^A = 0$ for all $\delta > \delta_0$ (cf. Proposition 2).

[5] By "global" we mean the entire state space, with the exception of the unstable steady state $(0, 0)$ and potentially the two unstable asymmetric steady states identified by Proposition 2a – which is a subset of measure zero.

[6] To construct the graphs in this and the following figures, we choose intrinsic growth rates of $r = 0.04$ per year. With regard to the technological parameters, we assume $\omega = 0.1$ and $q = 0.1$. The weight of resources in utility is $\gamma = 0.0667 = 0.667\,\omega$, which means that two thirds of the available effort is spent harvesting resources. In Figure 1, we further use $\delta = 0.05$, $\kappa = 0.625$ in panel (a), and $\delta = 0.17$, $\kappa = 0.625$ in panel (b). In Figure 2, we use $\delta = 0.09$, $\kappa = 1.667$ in panel (a); $\delta = 0.09$, $\kappa = 5.0$ in panel (b); $\delta = 0.17$, $\kappa = 1.667$ in panel (c); and $\delta = 0.17$, $\kappa = 2.5$ in panel (d). In Figures 3 and 4, we use $\kappa_A = 5.0$ and $\kappa_B = 1.667$. In Figure 5 we use $\delta = 0.17$.

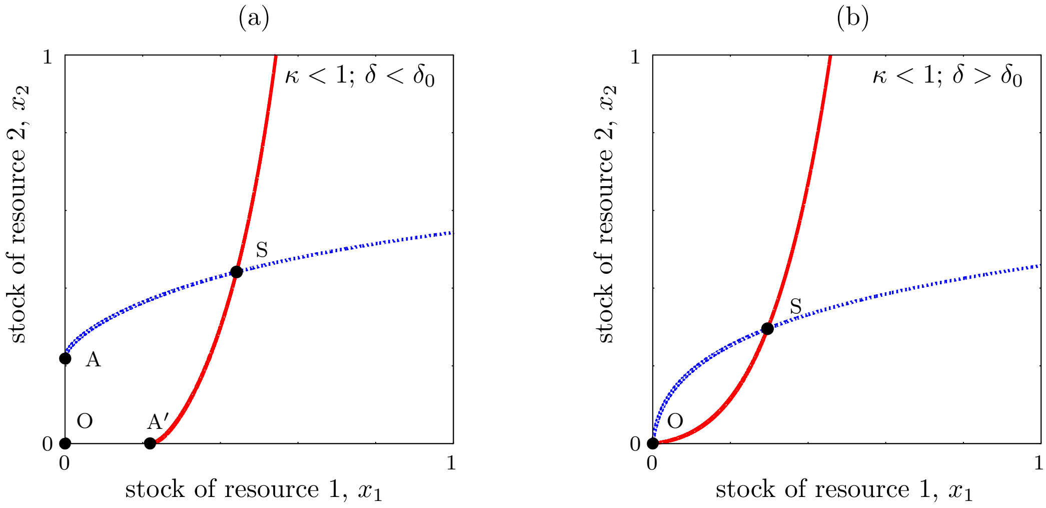

Figure 1: Phase diagrams for (a) low and (b) high social discount rates δ for the case when resources are substitutes, $\kappa < 1$.

when Condition 14 holds) as the case of a *low discount rate* and to $\delta > \delta_0$ as the case of a *high discount rate*. Figure 1(a) shows the case of a low discount rate, where there are two asymmetric steady states $(x^{\mathrm{A}}, 0)$ and $(0, x^{\mathrm{A}})$ with $x^{\mathrm{A}} > 0$ (cf. Proposition 2). For the case of a high discount rate, shown in Figure 1(b), $(x^{\mathrm{S}}, x^{\mathrm{S}})$ is the only non-trivial steady state. To summarize, if $\kappa < 1$, the optimal steady state stocks of both resources are strictly positive; corner solutions are only optimal if and only if one of the two resource stocks is equal to zero initially.

4.2 Optimal dynamics when resources are complements

Let us now check whether this also holds for the case in which the two resource goods are relative complements in consumption, $\kappa > 1$ (taking into account Condition 1). Trivially, $(0, 0)$ is a steady state, and from Lemma 1 we know that there always is an interior symmetric steady state ($(x_1^{\mathrm{S}}, x_2^{\mathrm{S}}) = (x^{\mathrm{S}}, x^{\mathrm{S}})$). However, it is easy to see that in case resources are complements, there are two additional asymmetric steady states as well: $(0, 1)$ and $(1, 0)$. With $\kappa > 1$ we have $x_j \xrightarrow{t \to \infty} 1$ if $x_{-j}(0) = 0$: if one resource has been depleted, harvesting the other one does not provide any utility, and hence the latter stock grows to its carrying capacity (recall we set $K = 1$). Below we also prove that we may also have $x_j \xrightarrow{t\to\infty} 1$ if $x_{-j}(0)$ is positive but close to zero, but only if the discount rate is sufficiently high or the degree of complementarity sufficiently large. If this is the case, (1,0) and (0,1) are locally stable and hence the economy does not necessarily end up in $(x^{\mathrm{S}}, x^{\mathrm{S}})$.

Before we can state this formally in Proposition 3, we first state two Lemmas:

Lemma 2. *When resources are complements ($\kappa > 1$), the symmetric interior steady state is locally stable if $\kappa < \hat{\kappa}(\delta)$ and locally unstable if $\kappa > \hat{\kappa}(\delta)$, where $\hat{\kappa}(\delta)$ with $\hat{\kappa}(\delta) > 1$ is defined by*

$$\hat{\kappa}(\delta) \equiv -\frac{\pi'(x^S)}{\pi(x^S)}\,\frac{f(x^S)}{f'(x^S)} \tag{16}$$

where $\pi(x^S) = \frac{\omega}{q\,x^S}\,\frac{\delta + r\,x^S}{\delta - r + 2\,r\,x^S}$ and $x^S(\delta)$ is given by (13).

Proof: see Appendix A.5. ∎

We will refer to the case $1 < \kappa < \hat{\kappa}(\delta)$ as the resources being *mild complements* and to the case $\kappa > \hat{\kappa}(\delta)$ as the resources being *strong complements.*[7]

Lemma 2 indicates that the degree of complementarity may have an important influence on the stability of steady states, including the symmetric one. We next use this result to derive conditions under which other interior steady states exist and to characterize their respective stability properties.

Lemma 3. *When resources are complements ($\kappa > 1$),*

3a. for a low discount rate ($\delta < \delta_0$) and strong complementarity ($\kappa > \hat{\kappa}(\delta)$) there are two locally stable asymmetric interior steady states (x_1^A, x_2^A) and $(x_1^{A'}, x_2^{A'}) = (x_2^A, x_1^A)$ (with $x_1^A > 0$, $x_2^A > 0$ and $x_1^A \neq x_2^A$).

[7] Note that because $\hat{\kappa}$ is a function of δ, the definitions of mild and strong complementarity depend on the level of the discount rate. A specific degree of complementarity, κ, may be considered 'mild' for some values of δ, and 'strong' for others – depending on whether κ is smaller than $\hat{\kappa}(\delta)$, or not. We will come back to this at the end of this subsection.

3b. for a high discount rate ($\delta > \delta_0$) *and mild complementarity* ($\kappa < \hat{\kappa}(\delta)$) *there are two unstable asymmetric interior steady states* (x_1^A, x_2^A) *and* $(x_1^{A'}, x_2^{A'}) = (x_2^A, x_1^A)$ *with* $x_1^A > 0$, $x_2^A > 0$ *and* $x_1^A \neq x_2^A$.

Proof: see Appendix A.6. ■

Combined with Lemma 1, Lemma 3 indicates that there can be multiple interior steady states. This may come as a surprise, as the resource dynamics are given by logistic growth equations and thus perfectly convex. The objective function, however, depends on the stock sizes of the two resources, as the costs of harvesting resource j depend on x_j, and hence the economy may be characterized by multiple steady states (see also Arrow and Kurz 1970).

Note that δ_0 is lower the higher are γ and q, and the lower is ω. If several interior steady states exist (i.e., if $\kappa > \hat{\kappa}(\delta)$ and $\delta < \delta_0$ or if $1 < \kappa < \hat{\kappa}(\delta)$ and $\delta > \delta_0$), none of them can be globally stable. This gives rise to the following proposition:

Proposition 3. *3a. for mild complements* ($1 < \kappa < \hat{\kappa}(\delta)$) *and a low discount rate* ($\delta < \delta_0$)*, there are three steady states,* (x^S, x^S), $(1, 0)$ *and* $(0, 1)$*, of which* (x^S, x^S) *is locally stable;*

3b. for strong complements ($\kappa > \hat{\kappa}(\delta)$) *and a low discount rate* ($\delta < \delta_0$)*, there are five steady states,* (x^S, x^S), $(1, 0)$, $(0, 1)$, (x_1^A, x_2^A) *and* (x_2^A, x_1^A)*, of which* (x_1^A, x_2^A) *and* (x_2^A, x_1^A) *are locally stable;*

3c. for mild complements ($1 < \kappa < \hat{\kappa}(\delta)$) *and a high discount rate* ($\delta > \delta_0$)*, there are five steady states,* (x^S, x^S), $(1, 0)$, $(0, 1)$, (x_1^A, x_2^A) *and* (x_2^A, x_1^A)*, of which* (x^S, x^S), $(1, 0)$ *and* $(0, 1)$ *are locally stable;*

3d. for strong complements ($\kappa > \hat{\kappa}(\delta)$) *and a high discount rate* ($\delta > \delta_0$)*, there are three steady states,* (x^S, x^S), $(1, 0)$, $(0, 1)$*, of which* $(1, 0)$ *and* $(0, 1)$ *are locally stable.*

Proof: a) and d) follow from Lemma 2; b) and c) follow from Lemma 3. ■

Proposition 3 states that, unlike in the case of resources being substitutes, the symmetric steady state is locally stable if and only if both the discount rate and the degree of complementarity are sufficiently low. This is because the present value of the benefits of building up the relatively small stock are larger the lower is the discount rate, while the costs of doing so are larger the stronger the complementarity between the two resources. Building up a stock requires reducing per-period extraction, and using little of the relatively scarce resource implies that utility during the transition phase is quite low – as the reduced use of the relatively scarce resource can hardly be compensated by a more intensive use of the relatively abundant one. Therefore, the higher the degree of complementarity, the less likely it is that society is willing to invest in building up stocks, and the more so the more impatient it is.

If the two resources are complements and the discount rate is sufficiently small ($\delta < \delta_0$), Proposition 3 states that there are three steady states in the system's interior (case 3b) – unless complementarity is fairly weak ($1 < \kappa < \hat{\kappa}(\delta)$; see case 3a). Proposition 3a is illustrated in Figure 2 (a); whatever the initial steady state stocks, society is sufficiently patient to be willing to invest in building up the relatively small stock – because the degree of complementarity is not very strong so that during the transition phase the decrease in welfare can be kept limited by using more of the relatively abundant stock. In case 3b, illustrated in Figure 2 (b), society is patient enough to ensure the existence of an interior stable steady state, but the high degree of complementarity between the two resources limits the attractiveness to build up both resource stocks. This leads to a path dependency of the economy. For any initial relative stock size $((x_1(0), x_2(0)),$ $x_1(0) \neq x_2(0))$, the steady state stock of the initially scarce resource is relatively small, while the steady state stock of the initially abundant resource is relatively large.

If the two resources are complements and the discount rate is sufficiently large ($\delta > \delta_0$), Proposition 3 states that there are two additional steady states in the system's interior (case 3c) – unless complementarity is too strong ($\kappa > \hat{\kappa}(\delta)$; see case 3d). Figure 2

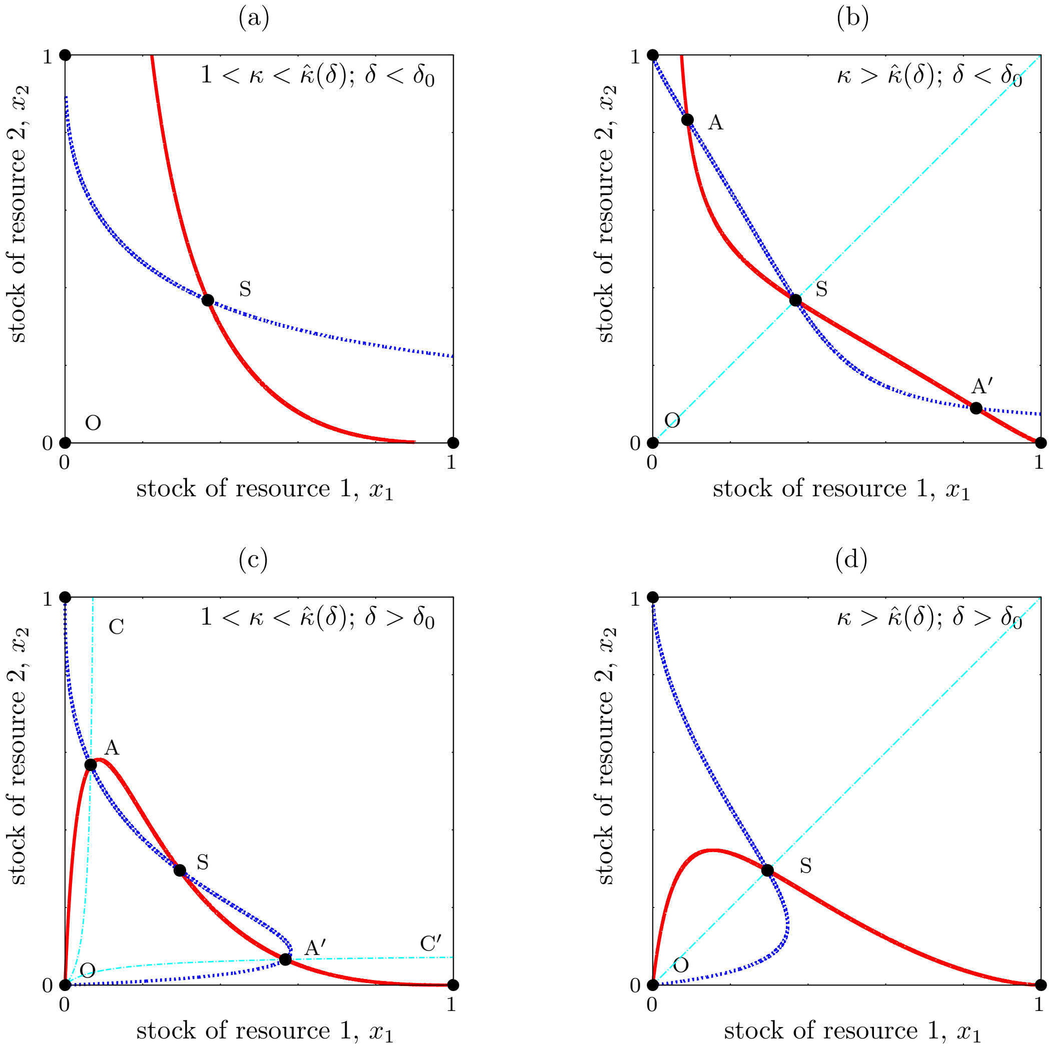

Figure 2: Phase diagrams for different degrees of complementarity $\kappa > 1$ of the resources and different social discount rates δ.

(c) illustrates case 3c with $\delta > \hat{\delta}$ and $1 < \kappa < \hat{\kappa}(\delta)$. The three interior steady states are denoted by A, S, and A′. The two asymmetric steady states A and A′ are saddle-point stable. The dotted lines C and C′ depict the saddle point trajectories that would lead to these steady states. Thus, if $\delta > \delta_0$ and $1 < \kappa < \hat{\kappa}(\delta)$, the initial stocks determine which steady state is optimal – $(x^{\mathrm{S}}, x^{\mathrm{S}})$, (1,0), or (0,1). For all initial states in between C and C′ the optimal paths lead to the symmetric steady state S. For initial states to the west (south) of the saddle path C (C′), it is optimal to continue harvesting both

	low discount rate $\delta < \delta_0$	high discount rate $\delta > \delta_0$
substitutes $\kappa < 1$	$(x^{\mathrm{S}}, x^{\mathrm{S}})$	$(x^{\mathrm{S}}, x^{\mathrm{S}})$
mild complements $1 < \kappa < \hat{\kappa}(\delta)$	regime I $(x^{\mathrm{S}}, x^{\mathrm{S}})$	regime II $(x^{\mathrm{S}}, x^{\mathrm{S}})$, $(1, 0)$, $(0, 1)$
strong complements $\kappa > \hat{\kappa}(\delta)$	regime III $(x_1^{\mathrm{A}}, x_2^{\mathrm{A}})$, $(x_2^{\mathrm{A}}, x_1^{\mathrm{A}})$	regime IV $(0, 1)$, $(1, 0)$

Table 1: Summary of locally stable steady states.

resources at reasonably high rates until the initially scarcest resource is depleted in the limit $t \to \infty$, while the initially most plentiful stock grows logistically until it reaches, in the limit, its carrying capacity $K = 1$. In the limit, the level of well-being would be minus infinity (cf. the utility function (1)), but with a sufficiently impatient society (condition $\delta > \hat{\delta}$ holds) the discount factor would tend to zero faster than the level of well-being would decrease.

Finally, the two alternative interior steady states $(x_1^{\mathrm{A}}, x_2^{\mathrm{A}})$ and $(x_2^{\mathrm{A}}, x_1^{\mathrm{A}})$ vanish if $\delta > \delta_0$ and $\kappa > \hat{\kappa}(\delta)$, and moreover $(x^{\mathrm{S}}, x^{\mathrm{S}})$ loses its (local) stability; see Proposition 3d and Figure 2 (d). Here, the complementarity of resources and the rate of discount are so high that the collapse of the resource-dependent economy is optimal for almost any set of initial conditions. The only exception is the case where the economy initially is on the saddle-path to the symmetric steady state, i.e. where $x_1(0) = x_2(0)$ holds exactly. So, if society starts harvesting in the Garden of Eden (where both $x_1(0)$ and $x_2(0)$ are equal to 1 – their maximum levels), the symmetric steady state may eventually be reached – but only if no shocks occur to either stock.

We summarize all results in Table 1. If resources are substitutes ($\kappa < 1$) and if resource j is relatively abundant ($x_j(0)/x_{-j}(0)$ is relatively large), the costs of building up resource $-j$ are relatively small because the reduced use of resource $-j$ can at least partly be compensated by a more intensive use of resource j. Indeed, we find that the welfare costs in the transition phase are always smaller than the net present value of the benefits of eventually having $x_1 = x_2 = x^{\mathrm{S}}$ – that is, for the relevant case where $\max\{2\,r\,\omega - \gamma\,q, r\gamma\,q - \delta(\gamma\,q - 2\,r\,\omega)\} > 0$; see Lemma 1.[8] If the resources are complements $(\kappa > 1)$, it may still be optimal to build up the relatively scarce resource stock towards its symmetric steady state level x^{S}, but not if the degree of complementarity is too high (that is, if $\kappa > \hat{\kappa}(\delta)$). For high levels of complementarity the economy tends towards asymmetric steady states, which are located in the interior if and only if the benefits of partially building up the relatively scarce resource are sufficiently high – that is, if the discount rate is sufficiently low.

Before we turn to the analyzing the economy's resilience in more detail, it is important to note that the conditions on δ and κ (cf. 14 and 16) are not independent – as already suggested in footnote 7. The following lemma characterizes this relationship in more detail.

Lemma 4. *$\hat{\kappa}(\delta)$ is monotonically decreasing from $\hat{\kappa}(\delta_{MSY}) = +\infty$ to $\lim_{\delta\to\infty} \hat{\kappa}(\delta) = \gamma\,q/(2\,(\gamma\,q - r\,\omega)) \equiv \underline{\kappa} > 1$.*

Proof: see Appendix A.5. ■

The properties of $\hat{\kappa}(\delta)$ are illustrated in Figure 3. Lemma 4 states that $\hat{\kappa}$ is decreasing in δ: the higher the discount rate, the more likely it is that two resources are being labeled 'strong complements', and hence the less likely it is that society is willing to invest in building up the initially relatively scarce one.

Lemma 4 implies an alternative definition for 'mild' and 'strong' complementarity. Using $\hat{\delta}$ to denote the level of the discount rate that solves $\hat{\kappa}(\delta) = \kappa$, resources with

[8] Note that this holds a fortiori if $\min\{r\,\omega - \gamma\,q, \delta_{\mathrm{MSY}} - \delta\} > 0$ also holds, because then the symmetric steady state is interior as well as locally stable for all values of κ anyway – see Proposition 1.

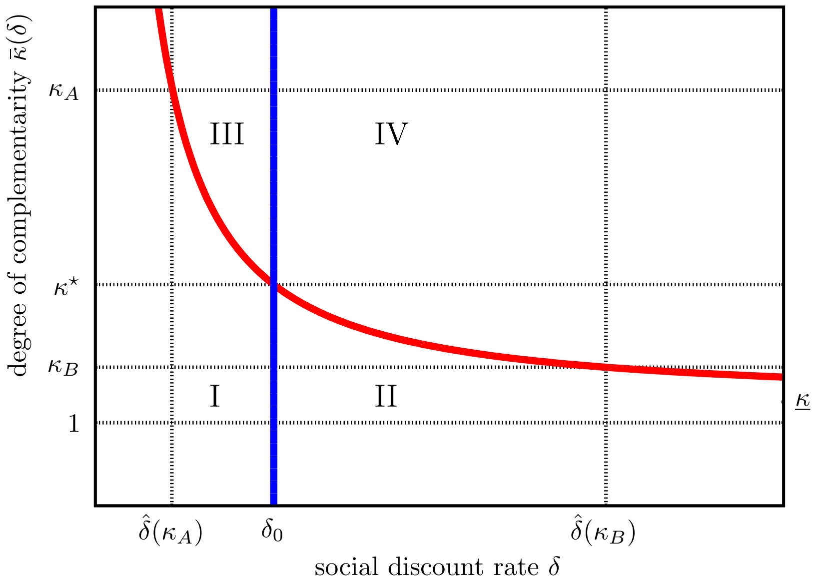

Figure 3: Threshold degree of complementarity as a function of the social discount rate (for $\delta > \delta_{\mathrm{MSY}}$).

a degree of complementarity equal to κ are defined to be mild (strong) complements if $\delta < \hat{\delta}(\kappa)$ ($\delta > \hat{\delta}(\kappa)$). Hence, in reference to Table 1, for $(x^{\mathrm{S}}, x^{\mathrm{S}})$ to be stable for a pair of resources with degree of complementarity equal to κ (that is, for Regimes I or II to apply), the discount rate should be smaller than $\hat{\delta}(\kappa)$ (so that the resources are mild complements; $\kappa < \hat{\kappa}(\delta)$). Furthermore, the corner steady states $(1, 0)$ and $(0, 1)$ are stable for $\delta > \delta_0$ (Regimes II and IV) and unstable for $\delta < \delta_0$ (Regimes I and III). Hence, Regime I applies if $\delta < \min\{\delta_0, \hat{\delta}(\kappa)\}$. Regarding the issue of which of the two conditions ($\delta < \delta_0$, or $\delta < \hat{\delta}$) is more stringent, let us denote with $\kappa^\star$ the value of κ for which the conditions (14) and (16) coincide.[9] This is represented in Figure 3 as follows. For a specific level $\kappa = \kappa_A > \kappa^\star$, $\hat{\delta}(\kappa_A) < \delta_0$, while for $\kappa = \kappa_B < \kappa^\star$, $\hat{\delta}(\kappa_B) > \delta_0$. That is, defining $\kappa^\star = \hat{\kappa}(\delta_0)$, we have $\hat{\delta} > \delta_0$ ($\hat{\delta} < \delta_0$) for all $\kappa < \kappa^\star$ ($\kappa > \kappa^\star$).

[9]Note that this value $\kappa^\star$ exists and is unique, as $\delta_0 > \delta_{\mathrm{MSY}}$ (recall that $\gamma\, q > r\, \omega$; see condition 1).

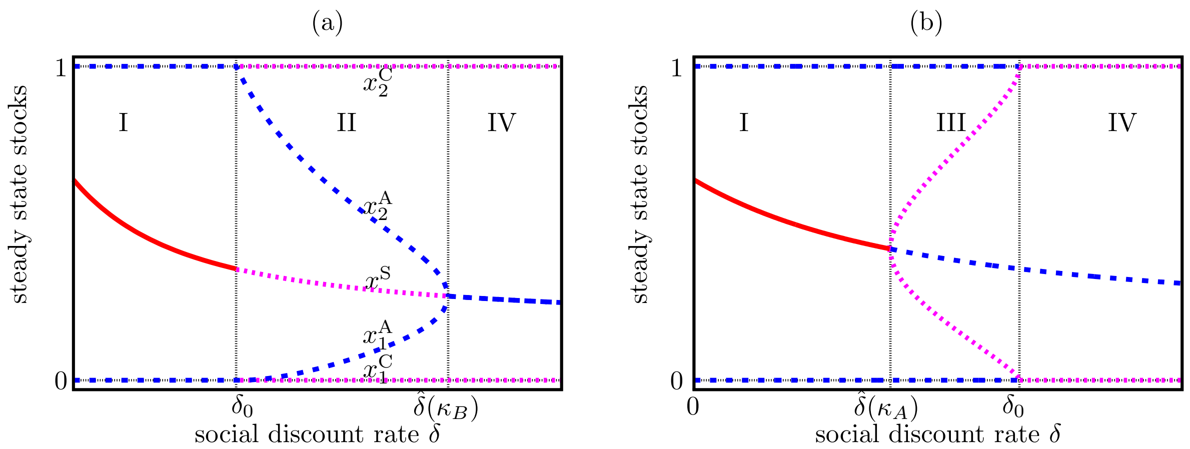

Figure 4: Bifurcation diagrams: steady state stock sizes as function of discount rate for (a) mild and (b) strong complements. Red solid lines depict an (almost) globally stable equilibrium, magenta dotted lines locally stable equilibria and blue dashed lines (almost) unstable equilibria. Roman numbers I–IV refer to the regimes identified in Table 1.

4.3 Bifurcation diagrams

Having established the dynamics of the resource stocks for various levels of δ and $\kappa > 1$, let us now turn to the question how these two key parameters affect the economy's resilience. Suppose that the economy is in steady state and that it is hit by an exogenous shock, does the economy return to its original steady state? In Figure 4 we depict the steady states of the economy as a function of δ (with $\kappa > 1$). In Figure 4(a) κ is sufficiently close to 1 that we have $\hat{\delta} < \hat{\delta}$ (that is, $\kappa < \kappa^\star$), while Figure 4(b) is drawn for a larger κ such that $\hat{\delta} < \delta_0$ (i.e., $\kappa > \kappa^\star$). The bifurcation diagrams summarize the resilience properties of the various cases as identified in Proposition 3, and roman numbers I-IV indicate the regime the economy is in, where the regimes are defined in Table 1; see also Figure 3.

Figure 4(a) shows how the stability properties depend on the discount rate for $\kappa < \kappa^\star$ so that $\delta_0 < \hat{\delta}$. At low levels of δ ($\delta < \min\{\delta_0, \hat{\delta}\}$), the economy is in regime I as identified in Table 1, so there is just one optimal steady state. This steady state is globally stable, and hence the economy will return to it as long as the exogenous shock perturbing the system does not directly exhaust any of the two resources. For larger δ such that $\delta_0 < \delta < \hat{\hat{\delta}}$, consumers are more impatient or society's institutions are of limited permanence ($\delta > \delta_0$), but given the actual degree of complementarity (κ), the discount rate is still sufficiently small ($\delta < \hat{\hat{\delta}}$) that the resources can still be labeled mild complements ($1 < \kappa < \hat{\kappa}(\delta)$). That means that the economy is in Regime II, and the relevant phase diagram is now Figure 2(c). Hence, there are three interior steady states, two of which are unstable (A and A′, depicted as the upper and lower branches in Figure 4)(a), and one locally stable one (S, in the middle between A and A′). If a shock moves the system outside the region bounded by the unstable equilibria A and A′, it is optimal for society to ultimately deplete one of the resources. If that happens, society stops harvesting the other resource – because of the complementarity between the two resources, marginal utility of resource good j is equal to zero if the quantity harvested from resource $-j$ is zero. Intuitively, if a shock negatively affects the size of one resource stock, restoring that stock to its steady state would require reduced harvesting levels for quite a long time. If society is sufficiently impatient or if its institutions are sufficiently brittle, it actually prefers to continue harvesting (and consuming) both stocks until one of the two is depleted. Finally, for even higher levels of δ such that $\delta > \hat{\hat{\delta}}$, the discount rate is sufficiently high that the resources can now be labeled strong complements (see Regime IV) and hence the economy loses its resilience altogether – the basin of attraction for S has shrunk to zero, as it has become an unstable steady state itself. The relevant phase diagram is now Figure 2 (d): the two saddle-point stable branches disappear, and any shock, however small, will result in the ultimate depletion of one of the resources.

Figure 4(b) is drawn for $\kappa > \kappa^\star$ so that $\hat{\hat{\delta}} < \delta_0$. For discount rates close to 0 (so that $\delta < \min\{\hat{\hat{\delta}}, \delta_0\}$), the symmetric steady state is globally optimal, and the economy is in Regime I. For levels of δ such that $\hat{\hat{\delta}} < \delta < \delta_0$, the discount rate itself is still 'low', but it is sufficiently high that the resources can now be labelled 'strong complements'. That means that the economy is in Regime II, and the relevant phase diagram is Figure 2 (b): two asymmetric interior steady states are stable. And if the discount rate is even higher (that is, $\delta > \delta_0 > \hat{\delta}$), the economy is in Regime IV, and only the corner steady states are stable.

5 Optimal resilience management of the resource-dependent economy

In Section 4 we have studied the (multiple) steady states and path-dependence of the optimally managed resource-dependent economy, answering the question whether, for given δ, κ and all other parameters of the economy, society is willing to invest in building up the initially relatively scarce resource, yes or no, for specific levels (and ratios) of $x_1(0)$ and $x_2(0)$. Clearly, this is relevant in itself, but it also provides the necessary input for the analysis of the optimal management of the resource-dependent economy. If a shock to the economy results in a specific ratio of $x_1/x_2 \neq 1$, is it optimal for society to bring the economy back to the state where $x_1 = x_2 = x^{\mathrm{S}}$, or not?

In this section we build on the previous two sections' results to analyze the consequences a one-time random shock $\Delta = (\Delta_1, \Delta_2)$ hitting the resource stocks at time T, such that the stock variables shift from the current state $(x_1(T), x_2(T))$ to another, disturbed state $(x_1(T+dt), x_2(T+dt)) = (x_1(T)-\Delta_1, x_2(T)-\Delta_2)$ an infinitesimal time increment dt later. The random shock Δ is distributed over some bounded support Ω. After such a disturbance, the social planner reoptimizes its harvesting and production plans to maximize (6) given the post-shock stock sizes (as well as Equations 1–5). We will focus on parameter values where $\kappa > 1$ and the economy is in Regime II, as this is the most interesting case with one stable steady state with positive stock sizes (the symmetric steady state $(x^{\mathrm{S}}, x^{\mathrm{S}})$) and two stable steady states where one stock is depleted (the corners $(0, 1)$ and $(1, 0)$).

The social planner's optimization problem at time $t = 0$ before the resource stocks are hit by the random shock at time T is

$$\max_{y(t),h_1(t),h_2(t)} \int_0^T u(y(t), h_1(t), h_2(t))\, e^{-\delta t}\, dt + e^{-\delta T}\, E\left\{V(x_1(T) - \Delta_1, x_2(T) - \Delta_2)\right\} \quad (17)$$

subject to (2)–(5) and using $(x_1(0), x_2(0)) = (x^{\mathrm{S}}, x^{\mathrm{S}})$ as initial stocks. We use E to denote the expectation operator over the random disturbance at time T. Furthermore, the value function $V(x_1, x_2)$ is defined as

$$V(x_1, x_2) = \max_{y(t),h_1(t),h_2(t)} \int_T^\infty u(y(t), h_1(t), h_2(t))\, e^{-\delta\,(t-T)}\, dt \quad (18)$$

subject to (2)–(5) with initial state (x_1, x_2)

The first-order conditions that determine the optimal development of the economy before the shock are identical to those given in Appendix A.1, except for the transversality conditions at T. These transversality conditions require that the shadow price μ_j of resource stock $j = 1, 2$ must equal the expected marginal value of the stock after T, i.e. $\mu_j(T) = E\{V_{x_j}(x_1(T) - \Delta_1, x_2(T) - \Delta_2)\}$.

We numerically study the optimal development of the economy before the shock for two different degrees of complementarity, $\kappa = 1.5$ and $\kappa = 1.8$. Given the other parameter values as specified in footnote 6, we have $1 < \kappa < \hat{\kappa}(\delta)$ and $\delta > \hat{\delta}$ for both values of κ, and hence the economy is in Regime II as identified in Table 1. The corresponding phase diagrams are drawn in Figure 5(a) and (b). For initial states to the west (south) of the saddle path C (C′), it is optimal to continue harvesting both resources at reasonably high rates until the initially scarcest resource is depleted in the limit $t \to \infty$, while the initially most plentiful stock grows logistically until it reaches, in the limit, its carrying capacity $K = 1$. The range of post-shock values of x_1 and x_2 for which (0,1) or (1,0) are optimal, is clearly larger for $\kappa = 1.8$ than for $\kappa = 1.5$. So how does this difference in resilience affect the society's optimal management plans?

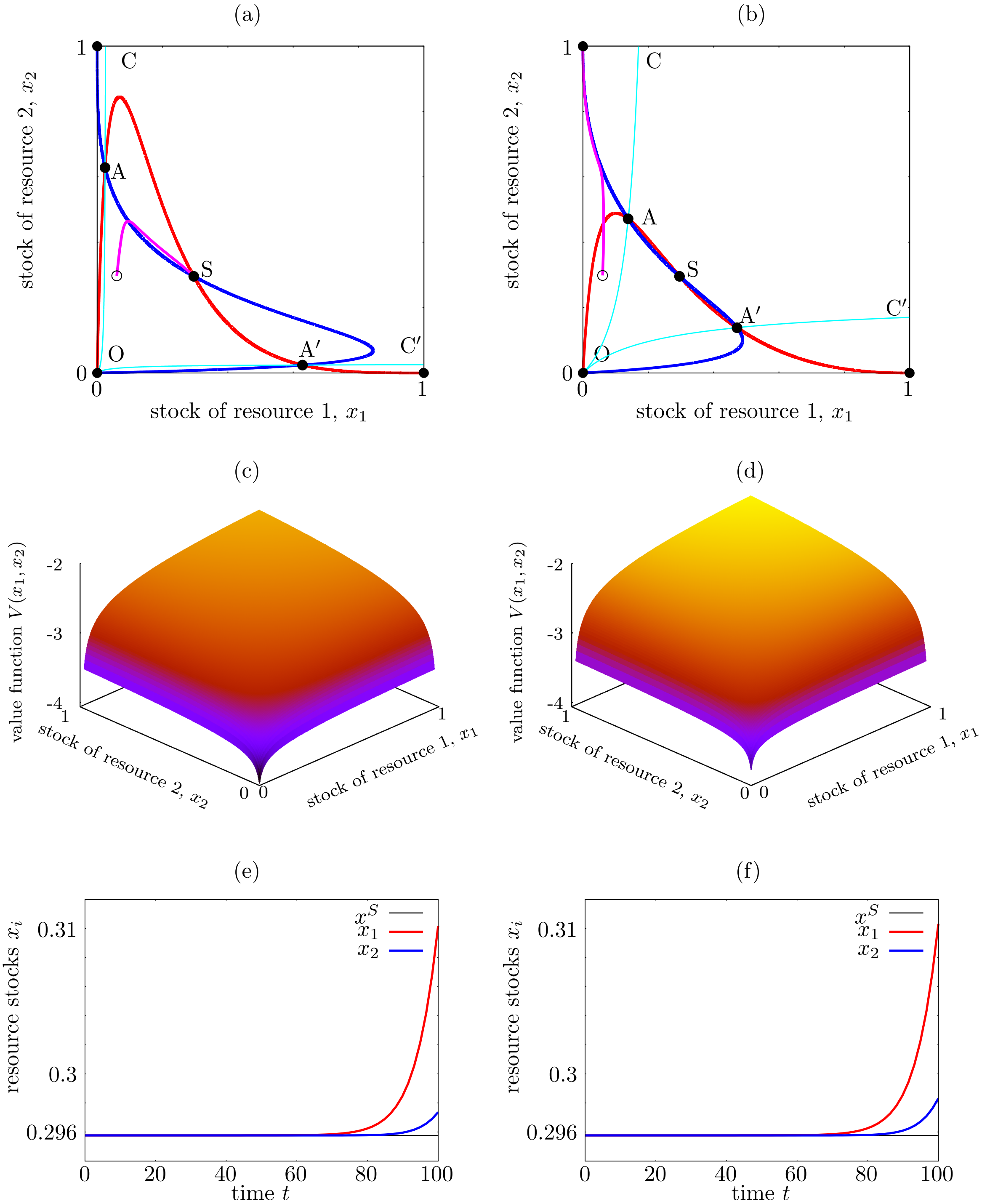

Figure 5: Phase diagrams (top), value functions (middle), and optimal paths of resource stocks before a shock at time $T = 100$ (bottom) for a degree of complementarity of $\kappa = 1.5$ (left) and $\kappa = 1.8$ (right).

We proceed as follows. First, we numerically approximate the value function $V(x_1, x_2)$ using the collocation method for solving the Bellman equation corresponding to the planner's optimization problem (Miranda and Fackler 2002).[10] The resulting value functions are shown in Figure 5(c) and (d). They are smooth over the whole domain. We further assume that the economy initially is in the symmetric steady state $(x^{\mathrm{S}}, x^{\mathrm{S}})$. At time $T = 100$, the stock of resource 1 is reduced by $\Delta_1 = 0.25$ with probability $p = 0.5$, while there is no shock to the stock of resource 2 ($\Delta_2 = 0$ with probability $p = 1$). Next, we derive the optimal time path before the shock by numerically solving the open-loop optimization problem (17), using the previously computed value function $V(x_1, x_2)$ to determine the appropriate transversality conditions.[11] The resulting time paths for the two resource stocks are shown in Figure 5(e) and (f).

Consistent with intuition, the optimal stock size of resource 1 increases over time to insure against the potential shock at T=100. To increase the stock of resource 1, harvest has to be reduced. As a consequence, harvest of the complementary resource 2 decreases as well, and the stock of resource 2 also increases. Still, two additional results are surprising. First, the anticipated effect of the shock starts affecting the optimal management plan only a relatively short period before the negative shock hits (with a 50% probability) at time $T = 100$; for $t < 75$, the optimal steady state is still the symmetric steady state $(x^{\mathrm{S}}, x^{\mathrm{S}})$. Second, the optimal trajectories of x_1 are very similar for the two values of κ. The considerably lower resilience of the economy for $\kappa = 1.8$ has hardly any influence on optimal management before a shock hits the economy.

For illustration, we also compute the optimal path of the economy for an initial state

[10] Programming codes are available from the authors. We use Matlab (version 7.11.0) with the CompEcon toolbox provided by Miranda and Fackler (2002). For the collocation method, we use two-variate Chebychev polynomials of degree 70 in the domain $[0.01, 0.99] \times [0.01, 0.99]$. Approximation residuals are below 10^{-7} over the whole domain.

[11] We use Matlab's built in solver for boundary value problems, `bvp5c`.

$(x_1(T)-\Delta_1, x_2(T))$, using the closed-loop solution to the optimization problem given by the value function. The resulting trajectories are also shown in Figure 5(a) and (b). For the lower degree of complementarity, $\kappa = 1.5$, the symmetric steady state is ultimately approached again after the shock. For the higher degree of complementarity, $\kappa = 1.8$, by contrast, the optimal steady state after the shock is the corner steady state $(0, 1)$.

6 Conclusion

We analyzed how characteristics of consumer preferences affect the (limited) resilience of natural-resource-dependent economies. We focused on two specific characteristics of preferences, the degree of complementarity of resources in the satisfaction of human needs, and the social discount rate. We derived conditions on the degree of complementarity and on the discount rate for which the optimal dynamics of resource use features multiple steady states and path-dependence. We established that if parameters are such that the symmetric steady-state stocks are larger than the maximum-sustainable-yield ones (that is, if Condition 1 is violated), this symmetric steady state is always stable, independent of whether the two resources are substitutes in consumption or complements.

If the optimal stocks are smaller than the maximum-sustainable-yield ones, the degree of complementarity of resources crucially affects the stability of the symmetric steady state. While one may expect that society is more willing to invest in regenerating relatively scarce resources if they are complements than when they are substitutes, we find the exact opposite. If resources are substitutes in consumption, the economy always ends up in the symmetric steady state – independent of the initial stocks. If resources are complements in consumption, however, the economy is characterized by limited resilience; alternative steady states exist that are locally stable, and the symmetric steady state may even become unstable – if the discount rate is sufficiently high.

The intuition behind these results is as follows. Allowing the relatively small stock to regenerate requires reducing the rate at which it is being harvested. Such a reduction reduces instantaneous utility, but more so if the two resources are complements than if they are substitutes – if they are substitutes (complements), the drop in welfare can (can not) be limited by increasing the use of the relatively abundant resource. Hence, the costs of moving the system to its symmetric steady state are larger the higher the degree of complementarity, and if society is sufficiently impatient, it may even be optimal to always let the initially least abundant resource go extinct.

So, we find that society's willingness to invest in regenerating stocks is smaller the stronger the complementarity between the two resources, and hence one would expect that anticipating this, society would be willing to apply more stringent 'safe minimum standards' if it considers resources to be complements rather than substitutes. Surprisingly, we only find very little evidence for this; in anticipation of negative shocks society is willing to invest in larger stocks in order to better buffer the economy against them, but numerical analyses suggest that this willingness is not very sensitive to the degree of complementarity. The higher the degree of complementarity, the smaller the basin of attraction of the symmetric steady state, but the propensity to invest in larger buffers is not much larger.

In real-world economies there are many more resources than just two, and they may be pairwise complements, or substitutes. The higher the level of aggregation of the analysis – that is, at the ecosystem level or even at the level of climate systems – the higher the degree of complementarity tends to be, and our model predicts that even under optimal management, the economy's resilience tends to be smaller the higher the stakes. In our numerical examples we find that by itself this is not really cause for concern because the sheer fact that resource depletion is more likely to be optimal if resources are complements, also implies that it is more costly to prevent resource collapse – because building up buffer stocks is more costly, too. The extent to which the results hold for specific stocks or ecosystems, is an empirical question.

Appendix

A.1 Conditions for optimal resource use

Rewriting (3) as $e_j = h_j / (q_j x_j)$, the current-value Hamiltonian is as follows:

$$H = \frac{\gamma}{1-\kappa} \ln \left[\sum_{j=1,2} h_j^{1-\kappa} \right] + \omega \left(1 - \sum_{j=1,2} \frac{h_j}{q_j \, x_j} \right) + \sum_{j=1,2} \mu_j \, [f_j(x_j) - h_j] \qquad (19)$$

where μ_j is the shadow price of the resource stock j; $j = 1, 2$. The first-order conditions of the social planner's optimization problem are

$$\gamma \, h_j^{-\kappa} \left[\sum_{i=1,2} h_i^{1-\kappa} \right]^{-1} = \frac{\omega}{q_j \, x_j} + \mu_j \equiv \pi_j \qquad j = 1, 2 \qquad (20)$$

$$\frac{\omega \, h_j}{q_j \, x_j^2} = \left[\delta - f_j'(x_j) \right] \, \mu_j - \dot{\mu}_j \qquad j = 1, 2 \qquad (21)$$

together with the transversality conditions $e^{-\delta \, t} \, \mu_j \, x_j \xrightarrow{t \to \infty} 0$ and for given initial sizes of the resource stocks $x_j(0) = x_{j0}$ for $j = 1, 2$. For the following analysis, it is more convenient to use the shadow price of resource consumption, π_j (as defined in Equation 20) than the shadow price of the resource stock, μ_j.

From Equation (20) we obtain harvest as a function of the shadow prices,

$$h_j = \gamma \, \frac{\pi_j^{-\frac{1}{\kappa}}}{\sum\limits_{i=1,2} \pi_i^{1-\frac{1}{\kappa}}}. \qquad (22)$$

Equation (22) shows that a positive consumption of the numeraire commodity is guaranteed because $\sum_{j=1,2} \frac{\omega}{q_j \, x_j} \, h_j \leq \sum_{j=1,2} \pi_j \, h_j = \gamma < \omega$, where the last inequality holds by assumption.

A.2 Proof of Lemma 1

In a symmetric steady state $x_1^{\mathrm{S}} = x_2^{\mathrm{S}} = x^{\mathrm{S}}$, which implies that $\pi_1^{\mathrm{S}} = \pi_2^{\mathrm{S}} = \pi^{\mathrm{S}}$; see (10). From (9) we infer that x^{S} is implicitly determined by $2 \, \pi^{\mathrm{S}} f(x^{\mathrm{S}}) = \gamma$. Using (10) and (5), this condition can be rewritten as

$$\left(2\,r\,x^{\mathrm{S}}+\delta-r+\frac{\gamma\,q}{\omega}\right)^2 = (\delta+r)^2+\left(\frac{\gamma\,q}{\omega}\right)^2$$

Solving for x^{S} yields the unique positive solution given in Equation (13) provided

$$\begin{aligned} & r-\delta-\frac{\gamma\,q}{\omega}+\sqrt{(\delta+r)^2+\left(\frac{\gamma\,q}{\omega}\right)^2}>0 \\ \Leftrightarrow \qquad & \delta\,(\gamma\,q-2\,r\,\omega)<\gamma\,q\,r. \end{aligned}$$

A.3 Proof of Proposition 1

(i) We will use in the following that $\pi'(x^{\mathrm{S}})<0$. To prove this, note first that any real-valued solution to (9) requires that π_j, $j=1,2$ is positive, which means that $\delta>f'(x_j)$ for both resource stocks in steady state. Differentiating (10) with respect to x_j and using (5) yields

$$\pi_j'(x_j) = -\frac{\omega}{q}\,\frac{2\,(r\,x_j)^2+\delta\,(\delta-r+4\,r\,x_j)}{x_j^2\,(\delta-r+2\,r\,x_j)^2}<0$$

as $x_j>(r-\delta)/(2\,r)$. In a similar way, it is easily verified that $\pi_j''(x_j)>0$.

(ii) We now analyze the local stability of the symmetric steady state by considering the Jacobian matrix of the dynamic system (7) and (8). Using $2\,\pi^S\,f(x^{\mathrm{S}})=\gamma$, in symmetric steady state the Jacobian is equal to

$$J^S=\begin{pmatrix} f'(x^{\mathrm{S}}) & 0 & \frac{\kappa+1}{\kappa}\,\frac{f(x^{\mathrm{S}})}{2\,\pi(x^{\mathrm{S}})} & \frac{\kappa-1}{\kappa}\,\frac{f(x^{\mathrm{S}})}{2\,\pi(x^{\mathrm{S}})} \\ 0 & f'(x^{\mathrm{S}}) & \frac{\kappa-1}{\kappa}\,\frac{f(x^{\mathrm{S}})}{2\,\pi(x^{\mathrm{S}})} & \frac{\kappa+1}{\kappa}\,\frac{f(x^{\mathrm{S}})}{2\,\pi(x^{\mathrm{S}})} \\ -\big(\delta-f'(x^{\mathrm{S}})\big)\,\pi'(x^{\mathrm{S}}) & 0 & \delta-f'(x^{\mathrm{S}}) & 0 \\ 0 & -\big(\delta-f'(x^{\mathrm{S}})\big)\,\pi'(x^{\mathrm{S}}) & 0 & \delta-f'(x^{\mathrm{S}}) \end{pmatrix} \tag{23}$$

where x^{S} is stock size and $\pi(x^{\mathrm{S}})$ is the shadow price of harvest in the symmetric steady state, both of which are independent of κ (see Lemma 1). The four eigenvalues of the

Jacobian are

$$\lambda_{1,2} = \frac{1}{2}\left[\delta \pm \sqrt{(\delta - 2\,f'(x^{\mathrm{S}}))^2 - 4\,\frac{\pi'(x^{\mathrm{S}})}{\pi(x^{\mathrm{S}})}\,f(x^{\mathrm{S}})\,\left(\delta - f'(x^{\mathrm{S}})\right)}\right] \tag{24}$$

$$\lambda_{3,4} = \frac{1}{2}\left[\delta \pm \sqrt{(\delta - 2\,f'(x^{\mathrm{S}}))^2 - 4\,\frac{\pi'(x^{\mathrm{S}})}{\kappa\,\pi(x^{\mathrm{S}})}\,f(x^{\mathrm{S}})\,\left(\delta - f'(x^{\mathrm{S}})\right)}\right]. \tag{25}$$

All four eigenvalues are real-valued, as $\pi(x^{\mathrm{S}}) > 0$, $f(x^{\mathrm{S}}) > 0$, $\pi'(x^{\mathrm{S}}) < 0$, and $\delta > f'(x^{\mathrm{S}})$. It follows directly that $\lambda_1 > 0$ and $\lambda_3 > 0$. $(x^{\mathrm{S}}, x^{\mathrm{S}})$ is always stable (independent of κ) if both λ_2 and λ_4 are negative. The last eigenvalue λ_4 is negative if

$$\begin{aligned}
& \delta^2 < \left(\delta - 2\,f'(x^{\mathrm{S}})\right)^2 - 4\,\frac{\pi'(x^{\mathrm{S}})}{\kappa\,\pi(x^{\mathrm{S}})}\,f(x^{\mathrm{S}})\,\left(\delta - f'(x^{\mathrm{S}})\right) \\
\Leftrightarrow \quad & 0 > f'(x^{\mathrm{S}})\left(\kappa + \frac{\pi'(x^{\mathrm{S}})}{\pi(x^{\mathrm{S}})}\,\frac{f(x^{\mathrm{S}})}{f'(x^{\mathrm{S}})}\right)
\end{aligned} \tag{26}$$

and similarly the condition for $\lambda_2 < 0$ is

$$0 > f'(x^{\mathrm{S}})\left(1 + \frac{\pi'(x^{\mathrm{S}})}{\pi(x^{\mathrm{S}})}\,\frac{f(x^{\mathrm{S}})}{f'(x^{\mathrm{S}})}\right). \tag{27}$$

If $x^{\mathrm{S}} > 1/2 \equiv x_{\mathrm{MSY}}$, we have $f'(x^{\mathrm{S}}) < 0$, so that conditions (26) and (27) are always met.

(iii) The stock sizes in the symmetric steady state are larger than the maximum sustainable yield stocks $x_{\mathrm{MSY}} = 1/2$ if and only if

$$\begin{aligned}
& r - \delta - \frac{\gamma\,q}{\omega} + \sqrt{(\delta + r)^2 + \left(\frac{\gamma\,q}{\omega}\right)^2} > r \\
\Leftrightarrow \quad & (\delta + r)^2 + \left(\frac{\gamma\,q}{\omega}\right)^2 >: \left(\delta + \frac{\gamma\,q}{\omega}\right)^2 \\
\Leftrightarrow \quad & \delta\left(1 - \frac{\gamma\,q}{r\,\omega}\right) + \frac{r}{2} >: 0
\end{aligned}$$

which holds if either $\gamma\,q < r\,\omega$ or $\delta < r^2\,\omega/\left(2\,(\gamma\,q - r\,\omega)\right) \equiv \delta_{\mathrm{MSY}}$.

A.4 Proof of Proposition 2

We shall first show that for $\kappa < 1$ we have $x_j'(x_i) > 0$ for all x_i. Total differentiation of (9) leads to

$$\pi_j'(x_j)\,\frac{dx_j}{dx_i} = \pi_i'(x_i)\,\left[\frac{\gamma}{\pi_i\,f(x_i)} - 1\right]^{\frac{\kappa}{\kappa-1}} - \frac{\gamma\,\left[\frac{\gamma}{\pi_i\,f(x_i)} - 1\right]^{\frac{1}{\kappa-1}}}{\frac{\kappa-1}{\kappa}\,f(x_i)}\,\left[\frac{\pi_i'(x_i)}{\pi_i} + \frac{f'(x_i)}{f(x_i)}\right] \tag{28}$$

The first term on the right hand side is negative because $\pi_i'(x_i) < 0$ (cf. Appendix A.3) and because $\gamma > \pi_i\,f(x_i)$; see (9). The last factor (in brackets) of the second term on the right hand side is negative, too:

$$\begin{aligned}\frac{\pi_i'(x_i)}{\pi_i} + \frac{f'(x_i)}{f(x_i)} &= -\frac{2\,[r\,x_i]^2 + \delta\,[\delta - r + 4\,r\,x_i]}{[\delta + r\,x_i]\,[\delta - r + 2\,r\,x_i]} + \frac{1 - 2\,x_i}{x_i\,[1 - x_i]} \\ &= -\frac{\delta\,[\delta + 2\,r\,x_i] + r^2\,[1 - 2\,x_i\,[1 - x_i]]}{[1 - x_i]\,[\delta + r\,x_i]\,[\delta - r + 2\,r\,x_i]} < 0.\end{aligned} \tag{29}$$

We use this to show that for $\kappa < 1$ any steady state must be symmetric. Let $(x_1, x_2) = (x^{\mathrm{S}}, x^{\mathrm{S}})$ be a symmetric steady state. Since $x_j(x_i)$ is monotonically increasing, it may be inverted, such that a steady state is determined by $x_2(x^{\mathrm{S}}) = x_1^{-1}(x^{\mathrm{S}})$. For symmetric resources, we have $x_2(x) = x_1(x)$ for all x. Assume without loss of generality that $x_j'(x^{\mathrm{S}}) > 1$. Then, $x_i'(x^{\mathrm{S}}) = 1/x_j'(x^{\mathrm{S}}) < 1$. Thus, no asymmetric steady state is possible. Furthermore, only one symmetric steady state with $x^{\mathrm{S}} > 0$ exists (Lemma 1).

For $\kappa < 1$, the problem is also well-defined if one of the resource stocks is zero from the very beginning. In this case, the first-order conditions for the optimal the steady-state stock of the resource with positive stock reads

$$\frac{\omega\,h_j}{q\,x_j^2} = \left[\delta - f_j'(x_j)\right]\,\left[\frac{\gamma}{h_j} - \frac{\omega}{q\,x_j}\right] \tag{30}$$

$$\frac{\omega}{\gamma\,q}\,r^2\,(1 - x^{\mathrm{A}})^2 = \left[\delta - r\,(1 - 2\,x^{\mathrm{A}})\right]\,\left[1 - \frac{\omega}{\gamma\,q}\,r\,(1 - x^{\mathrm{A}})\right]. \tag{31}$$

Solving for x^{A} leads to (15). It is straightforward to verify that this steady state is at a positive stock level if condition (14) holds.

A.5 Proofs of Lemmas 2 and 4

By condition 1, we have $f'(x^{\mathrm{S}}) > 0$. With this, the eigenvalue λ_4 of the Jacobian at the symmetric steady state (see Appendix A.3) is negative if $\kappa < \hat{\kappa}(\delta)$ and positive if $\kappa > \hat{\kappa}(\delta)$, as is easily verified using condition (26).

Next, we show that $\hat{\kappa}$ is monotonically decreasing with δ and that $\hat{\kappa}(\delta) > 1$. This shows that the eigenvalue λ_2 of the Jacobian at the symmetric steady state (see Appendix A.3) is always negative. Hence, the symmetric steady state is stable if $\kappa < \hat{\kappa}(\delta)$ and unstable if $\kappa > \hat{\kappa}(\delta)$.

As $f'(x^{\mathrm{S}}) \xrightarrow{\delta \to \delta_{\mathrm{MSY}}} 0$, we have $\hat{\kappa}(\delta) \xrightarrow{\delta \searrow \delta_{\mathrm{MSY}}} +\infty$. By differentiating (16) with respect to δ for $\delta > \delta_{\mathrm{MSY}}$ (condition 1), we obtain

$$\hat{\kappa}'(\delta) = \hat{\kappa}(\delta) \left(\frac{\pi''(x^{\mathrm{S}})}{\pi'(x^{\mathrm{S}})} - \frac{\pi'(x^{\mathrm{S}})}{\pi(x^{\mathrm{S}})} + \frac{f(x^{\mathrm{S}})}{f'(x^{\mathrm{S}})} - \frac{f''(x^{\mathrm{S}})}{f'(x^{\mathrm{S}})} \right) \frac{dx^{\mathrm{S}}}{d\delta}$$

which is negative, as $dx^{\mathrm{S}}/d\delta < 0$ (cf. Lemma 1); $\pi(x^{\mathrm{S}}) > 0$, $\pi'(x^{\mathrm{S}}) < 0$, and $\pi''(x^{\mathrm{S}}) > 0$ (cf. Appendix A.4); and as $f(x^{\mathrm{S}}) > 0$, $f'(x^{\mathrm{S}}) > 0$ (because $\delta > \delta_{\mathrm{MSY}}$) and $f''^{\mathrm{S}}) < 0$.

By condition 1 and according to Lemma 1 the minimal x^{S} is

$$\lim_{\delta \to \infty} x^{\mathrm{S}} = 1 - \frac{\gamma\, q}{2\, r\, \omega} > 0.$$

Under condition 1, we thus obtain

$$\lim_{\delta \to \infty} \hat{\kappa}(\delta) = \frac{\frac{\gamma\, q}{2\, r\, \omega}}{\frac{\gamma\, q}{r\, \omega} - 1} = \frac{\gamma\, q}{2\,(\gamma\, q - r\, \omega)} = \frac{\gamma\, q}{\gamma\, q - (2\, r\, \omega - \gamma\, q)} > 1.$$

A.6 Proof of Lemma 3

We shall consider the $x_j(x_i)$ isoclines, as given by Equations (11) and (12). With $r_1 = r_2 = r$, $K_1 = K_2 = 1$, and $q_1 = q_2 = q$, these two equations become

$$x_j(\pi_j) = \frac{1}{4} \left[\sqrt{\frac{8\, \delta\, \omega}{r\, q\, \pi_j} + \left(\frac{\delta - r}{r} - \frac{\omega}{q\, \pi_j} \right)^2} - \frac{\delta - r}{r} + \frac{\omega}{q\, \pi_j} \right] \tag{32}$$

$$\pi_j(x_i) = \frac{\omega}{q\, x_i} \, \frac{\delta + r\, x_i}{\delta - r + 2\, r\, x_i} \left[\frac{\gamma\, q}{\omega\, r} \, \frac{\delta - r + 2\, r\, x_i}{(\delta + r\, x_i)\,(1 - x_i)} - 1 \right]^{\frac{\kappa}{\kappa - 1}} \tag{33}$$

First, from Equation (33) we have $\pi_j(x_i) \xrightarrow{x_i \to 1} +\infty$ and hence $x_j(x_i) \xrightarrow{x_i \to 1} 0$ from Equation (32).

We shall secondly show that for $\delta > \delta_0$ we have $x_j(x_i) \xrightarrow{x_i \searrow 0} 0$, while for $\delta < \delta_0$ a $\underline{x}_i > 0$ exists such that $x_j(x_i) \xrightarrow{x_i \searrow \underline{x}_i} +\infty$. Consider the expression in brackets in Equation (33). For the case $\delta < \delta_0$ it is zero when

$$x_i = \sqrt{\left(\frac{\delta + r}{2\,r}\right)^2 + \left(\frac{\gamma\,q}{r\,\omega}\right)^2} - \left(\frac{\delta - r}{2\,r} + \frac{\gamma\,q}{r\,\omega}\right) \equiv \underline{x}_i > 0.$$

Hence, $\pi_j(\underline{x}_i) = 0$ and $x_j(x_i) \xrightarrow{x_i \searrow \underline{x}_i} +\infty$ by Equation (32).

For $\delta > \delta_0$, the expression in brackets in Equation (33) is positive even for $x_i = 0$. Thus, $\pi_j(x_i) \xrightarrow{x_i \searrow 0} = +\infty$ and $x_j(x_i) \xrightarrow{x_i \searrow 0} 0$ by Equation (32).

These two arguments together imply that for $\delta > \delta_0$, we have $x_2(x_1(x)) < x$ for x sufficiently close to 1, while for $\delta < \delta_0$, we have $x_2(x_1(x)) > x$ for x sufficiently close to 1.

Third, for $\kappa < \hat{\kappa}(\delta)$ the symmetric steady state is locally stable (Lemma 2) which implies that $x_2(x_1(x^{\mathrm{S}} + \epsilon)) > x^{\mathrm{S}} + \epsilon$ for some small $\epsilon > 0$ (put differently, we have $x_j'(x^{\mathrm{S}}) < -1$). For $\kappa > \hat{\kappa}(\delta)$ the symmetric steady state is locally unstable (Lemma 2) which implies that $x_2(x_1(x^{\mathrm{S}} + \epsilon)) < x^{\mathrm{S}} + \epsilon$ for some small $\epsilon > 0$ (put differently, we have $x_j'(x^{\mathrm{S}}) > -1$).

Now for 3a), we have $x_2(x_1(x^{\mathrm{S}}+\epsilon)) < x^{\mathrm{S}}+\epsilon$ for some small $\epsilon > 0$ and $x_2(x_1(x)) > x$ for some x sufficiently close to 1. Since $x_2(x_2(\cdot))$ is continuous, the equation $x_2(x_1(x^{\mathrm{A}})) = x^{\mathrm{A}}$ must have a solution $x^{\mathrm{A}} > x^{\mathrm{S}}$. As the symmetric steady state is unstable, the asymmetric steady states are stable (Arrow and Kurz 1970).

As for 3b), we have $x_2(x_1(x^{\mathrm{S}}+\epsilon)) > x^{\mathrm{S}}+\epsilon$ for some small $\epsilon > 0$ and $x_2(x_1(x)) < x$ for some x sufficiently close to 1. Since $x_2(x_2(\cdot))$ is continuous, the equation $x_2(x_1(x^{\mathrm{A}})) = x^{\mathrm{A}}$ must have a solution $x^{\mathrm{A}} > x^{\mathrm{S}}$. As the symmetric steady state is stable, the asymmetric steady states are unstable (Arrow and Kurz 1970).

References

Anderies, J.M., M.A. Janssen and B.H. Walker, 2002. Grazing management, resilience, and the dynamics of a fire-driven rangeland system. Ecosystems 5(1), 23–44.

Arrow, K., B. Bolin, R. Costanza, P. Dasgupta, C. Folke, C.S. Holling, B. Jansson, S. Levin, K.-G. Mäler, C. Perrings, D. Pimentel, 1995. Economic growth, carrying capacity, and the environment. Science 268, 520–521.

Arrow, K.J. and M. Kurz, 1970. Public Investment, The Rate of Return, and Optimal Fiscal Policy. Resources for the Future, John Hopkins Press, Baltimore.

Baumgärtner, S., D. Derissen, M.F. Quaas, and S. Strunz, 2011. Consumer preferences determine resilience of ecological-economic systems. Revised version, University of Lüneburg Working Paper Series in Economics, Working Paper No. 109, December 2008.

Berck, P., 1979. Open access and extinction. Econometrica 47(4), 877–882.

Brander, J. and M. Taylor, 1998. The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. American Economic Review 88 (1), 119–138.

Derissen, D., Quaas, M. F. and Baumgärtner, S. 2011. The relationship between resilience and sustainability of ecological-economic systems. Ecological Economics 70(6): 1121–1128.

Diamond, J., 2005. Collapse: How societies choose to fail or succeed. Penguin Group, New York.

Gordon, H., 1954. The economic theory of a common-property resource: The fishery. Journal of Political Economy 62, 124–142.

Gough, W.A. and A. Leung, 2002. Nature and fate of Hudson Bay permafrost. Regional Environmental Change 2(4), 177-184.

Gould, J.R., 1972. Extinction of a fishery by commercial exploitation: a note. Journal of Political Economy 80(5), 1031–1038.

Grainger, A., 1990. The Threatening Desert. Earthscan Publications Ltd.

Halsema, A. and C. Withagen, 2008. Managing multiple fishery pools: property right regimes and market structures. Environment and Development Economics 13, 775–794.

Heal, G., 2009. Climate economics: a meta-review and some suggestions for future research. Review of Environmental Economics and Policy 3(1): 4–21.

Hersoug, B., P. Holm, and S.A. Rånes, 2000. The missing T. Path dependency within an individual vessel quota system – the case of Norwegian cod fisheries. Marine Policy 24(4), 319–330.

Horan, R.D., E.P. Fenichel, K.L.S. Drury and D.M. Lodge, 2011. Managing ecological thresholds in coupled environmental-human systems. Proceedings of the National Academy of Sciences 108(18), 7333-7338.

Mäler, K.G., A. Xepapadeas, and A.J. de Zeeuw, 2003. The economics of shallow lakes. Environmental and Resource Economics 26(4), 603–624.

McAnany, P.A. and N. Yoffee (eds.), 2010. Questioning Collapse. Human Resilience, Ecological Vulnerability, and the Aftermath of Empire. Cambridge University Press, Cambridge.

Miranda, M.J. and P.L. Fackler, 2002. Applied Computational Economics and Finance. MIT Press, Cambridge, MA.

Quaas, M.F. and T Requate, 2011. Sushi or fish fingers? Seafood diversity, collapsing fish stocks and multi-species fishery management. Manuscript, Department of Economics, University of Kiel, Germany.

Ridley, J., J. Gregory, Ph. Huybrechts and J. Lowe, 2010. Thresholds for irreversible decline of the Greenland ice sheet. Climate Dynamics 35(6), 1049-1057.

Schaefer, M., 1957. Some considerations of population dynamics and economics in relation to the management of marine fisheries. Journal of the Fisheries Research Board of Canada 14, 669–681.

Scheffer, M., S. Carpenter, J.A. Foley, C. Folke and B. Walker, 2001. Catastrophic shifts in ecosystems. Nature 413, 591–596.

Taylor, M. S., 2009. Environmental crises: past, present, and future. Canadian Journal of Economics/Revue canadienne d'économique 42 (4), 1240–1275.

Verhulst, P., 1838. Notice sur la loi que la population suit dans son accroissement. Correspondance Mathematique et Physique 10, 113–121.