Which compensation for whom?

Pascal Gastineau* Emmanuelle Taugourdeau†

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Abstract

This paper examines a situation where a decision-maker determines the appropriate compensation for a given ecological damage. We consider that the policy-maker can use either or both money and natural units to meet three goals: i) no aggregate welfare loss, ii) minimization of the cost associated with the compensation, iii) minimal environmental compensation requirement.

We provide a simple two-period model to analyze the problem faced by the decision-maker. The findings suggest that - in some cases - providing both monetary and natural compensation can be the best option. Some comparative statics and welfare implications are analyzed.

Keywords: Natural resources, Welfare, Ecological equivalency methods

JEL codes: H43, Q51, Q57

*IFSTTAR, Transport and Environment Laboratory, EconomiX, 25 avenue François Mitterrand, 69675 Bron Cedex. Email: pascal.gastineau@ifsttar.fr

†CNRS, Paris School of Economics, CES-ENS Cachan, 63 avenue du Président Wilson, 94235 Cachan Cedex. Email: taugour@univ-paris1.fr.

1 Introduction

This paper aims to analyse the choice of a policy-maker in charge of determining the scaling of compensation for environmental damage. As compensation, the policy-maker may choose between granting a uniform amount of money to each individual and/or restoring a natural resource similar to the damaged one. Given the properties of the injured population (number of agents and heterogeneity in wealth or preferences), the policy-maker pursues a trade-off between two conflicting objectives: equity and efficiency. Here, equity refers to the idea that each agent does not support similarly the damage, also for the compensation. As a result, the pattern of compensation may either restablish equity (no change in individual and social welfare) or maintain a certain level of inequity arised by the damage, since agents support any welfare losses whereas others benefit from welfare gain. We oppose this equity purpose to an efficiency one, here defined in terms of costs: an efficient compensation will be the one which ensures no social welfare change together with a minimum level of costs.

Decision-makers are aware of the need to prevent and to remedy for environmental damage. This growing environmental awareness was notably embodied in various statutes such as the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA) and the Oil Pollution Act of 1990 (OPA) in the U.S. and the Directive 2004/35/EC on Environmental Liability with regard to the prevention and remedying of environmental damage in the European Union. These texts highlight the role that authorities have to play in order to establish a common framework that any polluter may comply with it.

In addition, there is a sharp debate on the best way to offset the damages on natural resources and services. Generally, two types of compensation are distinguished: natural compensation and monetary compensation. The first one consists in providing a natural restoration or implementing other actions that provide benefit to the restoration. The second one consists in an amount of money paid to the prejudiced people. Within the last years, ecological compensation for the loss of some environmental assets (whether the ecological damage is planned or accidental) gains popularity. Besides, the resource-to-resource (R-R) or service-to-service (S-S) equivalence approaches are considered as a first option by the European Directive. Furthermore, the european directive precludes the use of direct monetary payments to victims.

Non-monetary methods such as equivalency analyses (EA) aim to implement actions that provide natural resources and/or services of the same

type, quality and quantity as those damaged (i.e. 'in-kind' compensation[1]) (Dunford et al., 2004, Zafonte and Hampton, 2007). These techniques determine the necessary compensation to offset past, current and future damages without directly valuing them in economic terms by equalizing the amount of loss and gain of resources and services over time. To do so, they use a selection of *proxies* (metrics) representing the most important ecosystem services (English et al., 2009).[2] The presupposed advantages of S-S and R-R methods (i.e. no net loss principle) stand in contrast with drawbacks associated with well-known monetary valuation techniques. However, none of the methods are perfect and reliability of the equivalency methods to measure the environmental damage and/or scale and determine the appropriate compensation is still discussed. On the ecological side, while stressing the usefulness of the equivalency methods, Dunford et al. (2004) also emphasize their weaknesses: a high degree of uncertainty concerning estimates of compensatory restoration and their difficulty to consider complex impacts and phenomenon. Many attempts are made to improve ecological equivalency methods by focusing on a specific issue[3]: uncertainty (Moilanen et al., 2009), temporal dynamics (Bendor, 2009) or spatial analysis (Bruggeman et al., 2005; Bruggeman and Jones, 2008). On the economics side, Zafonte and Hampton (2007) suggest that, under certain conditions, resource equivalency analysis (REA, i.e. R-R) provide an acceptable approximation of compensating wealth. By contrast, many authors argue that ecological equivalence specified in biophysical equivalents could fail to provide a satisfactory compensation in a welfare perspective (Flores and Thacher, 2002; EPA, 2009). Flores and Thacher (2002) also stress the potential economic inefficiencies that could occurred when not including money component in the analysis and recommend a case by case determination of the adequate compensation that would better consider distributional issues associated with compensatory projects.

In this paper, we go further in the analysis of compensation by showing that natural and monetary compensation are not antinomic and may be implemented simultaneously. Due to heterogeneous individual preferences (or income), compensation can result in some losers and winners relative to their initial (pre-injury or pre-project) utility. Therefore, careful attention must be given to the characteristics and the size of the population affected

[1]This option is preferred to 'out-of-kind' compensation in which the adverse impacts to one resource (or habitat) are mitigated through the creation, restoration, or enhancement of another resource (or habitat).

[2]When equivalency approaches can not be used, valuation scaling approaches (value-to-cost and value-to-value) are recommended.

[3]See Quétier and Lavorel (2011) for a synthesis.

by an environmental damage when determining the compensation to be implemented. Thus, we study how the decision-maker can combine both of them in order to provide the adequate compensation at minimal cost.

In line with Cole (2012), this paper allows us to investigate equity and cost efficiency issues associated with an enforced ecological compensation. We depart from Cole by considering equity issues for the prejudiced population instead of considering the society on its whole. Moreover, contrary to Cole (2012) who compares both compensation schemes, we allow for a mixed compensation in which both of the compensatory methods may be implemented at the same time.

To reach our goal, we propose a simple model of an economy with two goods, a composite good and a natural resource. In this model, we determine which type of compensation the decision-maker may enforce the polluter to implement given the magnitude of the damage, the number and the characteristics of the prejudiced agents, and the cost associated with each compensation scheme. Our paper refers to marginal damages in the sense that they do not alter the agents' preferences. For instance, these damages could be either an accidental release of hazardous-substance into the environment (soil or river) or temporary damages to verges and footpaths due to road building process. To determine the optimal compensation scheme, the decision-maker pursues three goals:

- no welfare loss for the public impacted by the environmental damage;
- minimization of the cost of the compensation scheme, in line with recommendation of "reasonable cost" of the European directive 2004/35/EC;
- natural compensation cannot be less than a given quantity.

In doing so, the objective of the present paper is in line with the objective of the european Directive 2004/35/EC, namely "to establish a common framework for the prevention and remedying of environmental damage at a reasonable cost to society". We consider an heterogeneous affected population and examine the appropriate compensation for the case of a log linear utility function. We show that the eligible compensation mechanism (which meets the three conditions) varies with the magnitude of the environmental impact, the design of heterogeneity and number of agents that need compensation. We also show that enforcing a minimal non-monetary compensation not only implies ecological effects but also impacts the equity and cost efficiency issues associated with the compensation. More precisely, when the constraint is binding, an ecological constraint can reduce inequity at the expense of a rise in inefficiency.

The article is organised as follows. Section 2 presents the model. Optimal compensation schemes are derived in Section 3 according to two types of population heterogeneity: heterogeneity in preference for goods and heterogeneity in wealth. The last section concludes and suggests future directions for additional works.

2 The Model

We consider a two-period economy composed by n heterogeneous agents in which the agent i 's lifetime utility is given by:

$$U_i = \delta^t u_{it}\left(X_{it}, q_t\right) \quad \text{with } i = 1, \ldots, n \text{ and } t = 1, 2$$

where u_{it} is the agent i 's utility in period t, δ is the time-preference rate, X_{it} measures the agent i's private consumption and q_t the level of the environmental or resource service measured in physical units at time t.
The "lifetime" indirect utility of agent i is given by:

$$V_i = v_i\left(W_i, q_1, q_2\right) \tag{1}$$

where W_i stands for the agent i's intertemporal income which is exogeneously given.

We assume that the natural resource is damaged in the first period and compensated in the second period according to a compensating rule decided by a policy-maker. The compensation is twofold: a monetary compensation identical for each agent whatever his type, and an environmental compensation.

Leaving the utility of an individual unchanged following a natural damage implies:

$$dV_i = \frac{\partial v_i}{\partial W_i} dW_i + \frac{\partial v_i}{\partial q_1} dq_1 + \frac{\partial v_i}{\partial q_2} dq_2 = 0 \tag{2}$$

where $dq_1 < 0$ stands for the natural damage and $dq_2 > 0$, the environmental compensation while dW_i is the monetary compensation.

The individual willingness to accept a monetary compensation for the environmental damage is defined as[4]:

$$WTA_i^W = \left(\frac{\partial v_i}{\partial q_1} / \frac{\partial v_i}{\partial W_i}\right)(-dq_1) \tag{3}$$

[4] WTA_i^W is the value of dW_i obtained by equation (2) stating that $dq_2 = 0$.

It expresses how much money the individual i is willing to accept in exchange for the loss (dq_1). WTA_i^W is identified with the compensating variation.[5]

Using the same reasoning it is possible to express a WTA in terms of environmental unit:

$$WTA_i^q = \left(\frac{\partial v_i}{\partial q_1} / \frac{\partial v_i}{\partial q_2}\right)(-dq_1).$$

Let consider a simple utilitarian social welfare function in a n individuals society:

$$\mathcal{W} = \mathcal{W}\big[v_1(W_1, q_1, q_2), \ldots, v_n(W_n, q_1, q_2)\big] = \sum_{i=1}^{n} V_i \tag{4}$$

When determining the compensation pattern, the decision-maker aims to account for three criteria: minimize the costs involved by the implementation of the whole compensation, leave the social welfare unchanged and comply with a minimal environmental compensation requirement.

The program of the decision-maker writes:

$$\min_{MC, dq_2} \; C(dq_2, MC) \tag{5}$$

subject to

$$d\mathcal{W} = 0 \tag{6}$$

$$dq_2 \geq -(1+r)dq_1 \tag{7}$$

$$MC \geq 0 \tag{8}$$

where $MC = dW_i \; \forall i$ is the monetary compensation, dq_2 the environmental compensation, and C is the cost function associated to the compensation. $\mathcal{W}$ stands for the social welfare. Constraint (6) characterizes the fact that the compensating policy must leave the social welfare unchanged, whereas constraint (7) specifies that the environmental compensation must at least be equal to a given value larger than the initial damage. This value corresponds to the one that would be determined when using Equivalence Approaches (EA) in their simplest formulation, i.e. the "discounted" environmental gain equalize the "discounted" environmental loss. r is the discount rate. Note that no ex-post redistribution of monetary compensation between losers and gainers is possible.

[5] The absence of environmental damage is the reference state for most people. WTA is the better measure to use (Knetsch, 2007).

The Lagrangian associated to this program is given by

$$\mathcal{L} = C\left(dq_2, MC\right) + \lambda_1\left[d\mathcal{W}\right] + \lambda_2\left[dq_2 + (1+r)\,dq_1\right] + \lambda_3\left[MC\right]$$

where λ_1 is the Lagrangian multiplier associated to constraint (6), λ_2 to (7) and λ_3 to (8).

The conditions arising from solving the Lagrangian are:

$$\frac{\partial \mathcal{L}}{\partial MC} = -\frac{\partial C}{\partial MC} + \lambda_1 \frac{\partial d\mathcal{W}}{\partial MC} + \lambda_3 = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial dq_2} = -\frac{\partial C}{\partial dq_2} + \lambda_1 \frac{\partial d\mathcal{W}}{\partial dq_2} + \lambda_2 = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = d\mathcal{W} = \sum_{i=1}^{n} dv_i = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = dq_2 - (1+r)dq_1 \geq 0 \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = MC \geq 0 \tag{13}$$

Four regimes can be distinguished from this program, that determine the pattern of the compensation:

- regime 1: (monetary compensation $[\mathcal{R}_1]$) : $\lambda_2 > 0; \lambda_3 = 0 => dq_2 = -dq_1(1+r); MC > 0$. In this case both compensations are implemented but the level of the environmental compensation being the minimal one defined by the EA constraint, we call this case "monetary compensation". Without the EA constraint, the environmental compensation would be comprised between 0 and $-dq_1(1+r)$. This case leads to the relation

$$\left[\frac{\partial d\mathcal{W}}{\partial MC} \Big/ \frac{\partial d\mathcal{W}}{\partial dq_2}\right] > \left[\frac{\partial C}{\partial MC} \Big/ \frac{\partial C}{\partial dq_2}\right] \tag{14}$$

  The decision-maker generates more welfare per unit spent on monetary compensation than per unit spent on natural compensation. Then he should spend more on monetary compensation in order to compensate at minimal cost.

- regime 2: (mixed compensation $[\mathcal{R}_2]$): $\lambda_2 = \lambda_3 = 0 \Rightarrow dq_2 > -dq_1(1+r); MC > 0$. There exists a couple of compensation tools (MC^*, dq_2^*) such that:

$$\left[\frac{\partial d\mathcal{W}}{\partial MC} / \frac{\partial d\mathcal{W}}{\partial dq_2}\right] = \left[\frac{\partial C}{\partial MC} / \frac{\partial C}{\partial dq_2}\right] \tag{15}$$

  The ratio of the marginal differences in utility equals the ratio of the marginal costs. In other words there exists a couple (MC^*, dq_2^*) such that the welfare that we get from an additional unit of MC or dq_2 per fund spent is the same.

- regime 3: (environmental compensation $[\mathcal{R}_3]$): $\lambda_2 = 0; \lambda_3 > 0 \Rightarrow dq_2 > -dq_1(1+r); MC = 0$, which implies

$$\left[\frac{\partial d\mathcal{W}}{\partial MC} / \frac{\partial d\mathcal{W}}{\partial dq_2}\right] < \left[\frac{\partial C}{\partial MC} / \frac{\partial C}{\partial dq_2}\right] \tag{16}$$

  This is the opposite case to the regime 1. Decision-maker should spend fund on natural compensation.

- regime 4: (minimal compensation $[\mathcal{R}_4]$) $\lambda_2 > 0; \lambda_3 > 0 \Rightarrow dq_2 = -dq_1(1+r); MC = 0$ For $\lambda_2 > 1$ and $\lambda_3 > 0$ there is no optimal solution. There is a 'second best' option such that there is no monetary compensation and minimal environmental compensation. For $\mathcal{R}_4$, $d\mathcal{W} > 0$. This regime does not fulfill constraint (6).

3 Application

We now specify both the cost and the utility functions. We assume a log linear utility function of the form

$$U_{it} = \alpha_i \ln X_{it} + (1-\alpha_i) \ln q_t$$

where α_i is the agent i's weight for the consumption bundle in utility.

The agent i's lifetime utility function rewrites:

$$U_i = \alpha_i \ln X_{1i} + (1-\alpha_i) \ln q_1 + \delta\alpha_i \ln X_{2i} + \delta(1-\alpha_i) \ln q_2$$

where δ is the time-preference rate.

The arbitrage in private consumption between period 1 and 2 gives:

$$\frac{X_{2i}}{X_{1i}} = \delta_i (1+r)$$

which combined with the intertemporal budget constraint $W_i = X_{i1}(1+r) + X_{i2}$ gives the demand for private goods so that the indirect utility writes

$$V_i = \alpha_i \ln\left(\frac{W_i}{(1+\delta)(1+r)}\right) + (1-\alpha_i)\ln q_1 + \delta\alpha_i \ln\left(\frac{\delta}{(1+\delta)}W_i\right) + \delta(1-\alpha_i)\ln q_2 \tag{17}$$

Finally, we assume that the cost function for compensation is given by:

$$C(dq_2, MC) = nMC + a(dq_2)^b$$

The cost of compensation is decomposed in two parts: a lump sum part (nMC) which characterizes the monetary compensation granted uniformly to all agents, and a second part which is proportional to the restoration and depends on the type of the nature that should be restored ($b > 0$ can be either ≥ 1 or < 1). Note that with the cost function we use, it is straightforward that the program is quasiconvex in MC, whereas it is quasiconvex in dq_2 for $b \geqslant 1$.

3.1 Heterogeneity in preference for goods

In this section, the optimal compensation scheme is derived according heterogeneity in preference for goods. Cost and welfare implications are then explored.

3.1.1 Optimal compensation scheme

We assume that agents are only differentiated by their preference for goods, α_i. The social welfare function writes:

$$\mathcal{W} = \mathcal{W}\big[v_1(W, q_1, q_2), \ldots, v_n(W, q_1, q_2)\big]$$

and considering (4), it can be rewritten as

$$\begin{aligned}\mathcal{W} &= \sum_{i=1}^{n} v_i(W, q_1, q_2) \\ &= n\overline{\alpha}\ln\left(\frac{W}{(1+\delta)(1+r)}\right) + n(1-\overline{\alpha})\ln q_1 + n\delta\overline{\alpha}\ln\left(\frac{\delta}{(1+\delta)}W\right) + n\delta(1-\overline{\alpha})\ln q_2\end{aligned}$$

where $\overline{\alpha} = \frac{1}{n}\sum \alpha_i$ is the mean preference for the private good.

Condition (6) becomes

$$d\mathcal{W} = (1+\delta)\frac{n\overline{\alpha}}{W}MC + \frac{n(1-\overline{\alpha})}{q_1}dq_1 + n\delta\frac{(1-\overline{\alpha})}{q_2}dq_2 = 0 \tag{18}$$

so that

$$MC = W\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{-dq_1}{q_1} - \frac{\delta}{q_2}dq_2\right) \tag{19}$$

or

$$dq_2 = \left(\frac{-dq_1}{q_1} - MC\frac{\overline{\alpha}(1+\delta)}{(1-\overline{\alpha})W}\right)\frac{q_2}{\delta} \tag{20}$$

Given this established relation between both compensations, we are able to distinguish two different regimes according to the value of b: $b \geq 1$ or $b < 1$

Proposition 1 *For $b \geq 1$ four solutions can emerge from the program of the social planer*

1. $dq_2 = -(1+r)\,dq_1$ *and* $MC = -dq_1 W\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{1}{q_1} - \frac{\delta}{q_2}(1+r)\right)$ *iff* $n < \underline{n}$ *and* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 1)*
   where
   $$\underline{n} = ab\frac{(1+\delta)\,\overline{\alpha}}{(1-\overline{\alpha})\,W}\frac{q_2\left(-(1+r)\,dq_1\right)^{b-1}}{\delta}$$

2. $dq_2 = \left(\frac{(1-\overline{\alpha})nW\delta}{\overline{\alpha}(1+\delta)q_2ab}\right)^{\frac{1}{b-1}}$ *and* $MC = \frac{(1-\overline{\alpha})W\left(\frac{-dq_1}{q_1}\right)}{(1+\delta)\overline{\alpha}} - \left(\frac{\delta(1-\overline{\alpha})W}{q_2(1+\delta)\overline{\alpha}}\right)^{\frac{b}{b-1}}\left(\frac{n}{ab}\right)^{\frac{1}{b-1}}$ *iff* $\overline{n} > n > \underline{n}$ *(regime 2)*
   where
   $$\overline{n} = ab\frac{(1+\delta)\,\overline{\alpha}}{(1-\overline{\alpha})\,W}\left(\frac{q_2}{\delta}\right)^b\left(-\frac{dq_1}{q_1}\right)^{b-1}$$

3. $MC = 0$ *and* $dq_2 = -\frac{q_2}{q_1}\frac{1}{\delta}dq_1$ *iff* $n > \overline{n}$ *and* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 3)*

4. $MC = 0$ *and* $dq_2 = -(1+r)dq_1$ *iff* $(1+r) > \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 4)*

Proof. Rewriting the cost function in dq_2 according to (19) gives

$$C\left(dq_2, MC\right) = nW\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{-dq_1}{q_1} - \frac{\delta}{q_2}dq_2\right) + a\left(dq_2\right)^b$$

which is clearly quasi-convex in dq_2 if and only if $b \geq 1$. Minimizing this cost function gives

$$dq_2 = \left(\frac{(1-\overline{\alpha})nW\delta}{\overline{\alpha}(1+\delta)q_2ab}\right)^{\frac{1}{b-1}}$$

and condition (8) gives the value for MC

$$MC = \frac{(1-\overline{\alpha})W\left(\frac{-dq_1}{q_1}\right)}{(1+\delta)\,\overline{\alpha}} - \left(\frac{\delta(1-\overline{\alpha})W}{q_2\,(1+\delta)\,\overline{\alpha}}\right)^{\frac{b}{b-1}}\left(\frac{n}{ab}\right)^{\frac{1}{b-1}}$$

Conditions (7) and (8) imply

$$dq_2 > -\,(1+r)\,dq_1 \Longleftrightarrow n > ab\frac{(1+\delta)\,\overline{\alpha}}{(1-\overline{\alpha})W}\frac{q_2\,(-\,(1+r)\,dq_1)^{b-1}}{\delta} = \underline{n}$$

$$MC > 0 \Longleftrightarrow n < ab\left(\frac{(1+\delta)\,\overline{\alpha}}{(1-\overline{\alpha})W}\right)\left(\frac{q_2}{\delta}\right)^{b}\left(\frac{-dq_1}{q_1}\right)^{b-1} = \overline{n}$$

both conditions will be fulfilled iff

$$\overline{n} > \underline{n} \Longleftrightarrow (1+r) < \left(\frac{q_2}{q_1}\frac{1}{\delta}\right) \text{ for } b \geq 1$$

- If $n > \overline{n} > \underline{n}$ then $MC = 0$ and dq_2 is derived from (20). It corresponds to condition (14).
- If $n < \underline{n} < \overline{n}$ then dq_2 is implemented at its minimal level i.e. $dq_2 = -\,(1+r)\,dq_1$ and MC is derived from (19). It corresponds to condition (16).
- If $(1+r) > \left(\frac{q_2}{q_1}\frac{1}{\delta}\right)$, which implies $\underline{n} > \overline{n}$, none of condition (7) and (8) are fulfilled so that both compensations are implemented at their minimal level whatever the level of n, i.e. $MC = 0$ and $dq_2 = -(1+r)dq_1$.

■

Proposition 1 highlights four different regimes of compensation scheme.

Regime 4 is the one which corresponds to the seminal EA compensation. Without EA constraint, condition (7) does not hold and regime 4 does not exist anymore. In this regime, the marginal rate of substitution between the environmental good in period 1 and 2 $\left(\frac{\partial v_i}{\partial q_1}/\frac{\partial v_i}{\partial q_2} = \frac{q_2}{q_1}\frac{1}{\delta}\right)$ is low so that a very low level of $dq2$ is required to compensate the loss of one unit of $q1$. In this regime, the minimum compensation $MC = 0$ and $dq_2 = -(1+r)dq_1$ is then higher that the required one. Due to the EA constraint, this scheme of

compensation does not fulfill the welfare condition since it implies a higher level of aggregated welfare than required i.e. $d\sum V_i > 0$.

The three other regimes are most interesting. They occur when the level of the marginal rate of substitution between the environmental good in period 1 and 2 is not too low $\left(\frac{q_2}{q_1}\frac{1}{\delta} > (1+r)\right)$. The compensation schemes (regime 1, 2 or 3) depend on the number of agents injured by the damage. The explanation lies in the cost function and the nature of the goods which characterizes the properties of both types of compensation: the monetary compensation is granted uniformly to all agents affected by the damage so that the cost of such compensation is linearly increasing with the number of compensated agents. Conversely, the amount of natural compensation is fixed whatever the number of agents it benefits to, so that its cost is stable with n. Intuitively, it is more relevant to implement a monetary compensation for a low level of n and a natural compensation for a high level of n.

As Figure 1 shows[6], for a low n ($n < \underline{n}$) [Regime 1], the best compensation scheme is that which implements the minimum required of natural compensation together with the value of the monetary compensation that leaves the social welfare unchanged. The level of MC is fixed whatever the level of $n \in [0, \underline{n}[$. This is due to the EA constraint that implies a higher natural compensation that the level that would be optimal without any constraint. Without any EA constraint, the compensation scheme would lead to an increasing level of dq_2 and a decreasing level of MC. and Regime 1 disappears. $\underline{n}$ is the number of agents which separates two regimes: the first where the EA constraint works (dq_2 would be smaller and MC greater and fixd without the constraint) and the second where dq_2 becomes higher than the value imposed by the EA constraint. For $n < \underline{n}$, the level of MC and dq_2 vary with n. Social planer should provide more monetary compensation and less natural compensation, i.e. $-(1+r)dq_1 > dq_2 > 0$. Moreover, for $n \geq \underline{n}$, the EA constraint plays no role as $dq_2 \geq -(1+r)dq_1$. The value of $\underline{n}$ increases with $\overline{\alpha}$ and $(-dq_1)$ and decreases with W and δ. An agent who values more the future expects a higher level of compensation so that the switch from regime 1 to regime 2 occurs for a lower n. Conversely a lower weight for the environmental good in the utility (high $\overline{\alpha}$) implies a lower need for compensation and the limit between both regimes is moved for a higher n. Finally, a higher damage directly increases the EA constraint and consequently shifts the limit for a higher n.

[6]The following parameter set was used for the numerical simulation: ($W = 1000$, $\overline{\alpha} = 0.8$, $\delta = 0.2$, $r = 0.04$, $q_1 = 100$, $q_2 = 100$, $dq_1 = -10$, $a = 50$, $b = 1.2$). Natural resource recovery is achieved in one period, i.e. $q_1 = q_2$. dq_1 depicts a temporary loss.

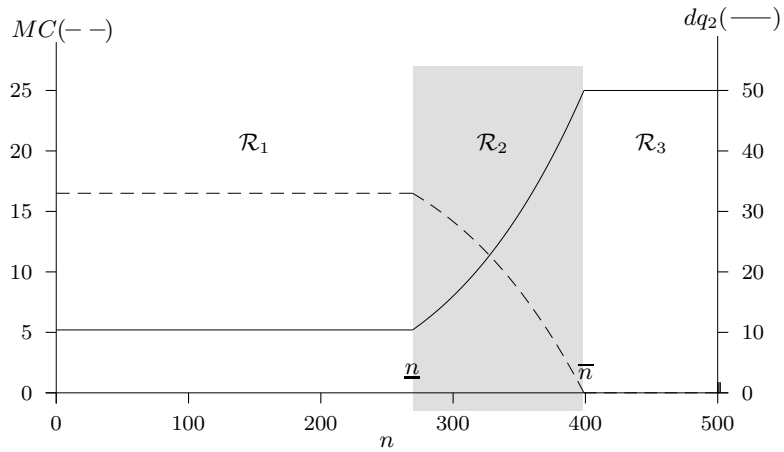


Figure 1: Optimal Compensation Scheme as a function of population size

When $n \in [\underline{n}, \overline{n}[$ [Regime 2], the EA constraint is not still bound and the level of dq_2 increases with n. This implies a corresponding decrease of the monetary compensation since MC weights more and more relatively to dq_2 in the cost function. This trend lasts till it is no more useful to use any monetary compensation that has become too heavy in the cost function. In the last regime ($n > \overline{n}$) [Regime 3], the natural compensation is the only compensating tool that is used to leave the society's welfare unchanged. For the same reasons than the value of $\underline{n}$, $\overline{n}$ increases with $\overline{\alpha}$ and $(-dq_1)$ and decreases with W and δ.

For the regime $\mathcal{R}_2$ which implies the use of both types of compensation ($n \in [\underline{n}, \overline{n}[$), the comparative static analysis gives the following relations

$$\frac{\partial\,(dq_2)}{\partial\overline{\alpha}} < 0;\ \frac{\partial\,(dq_2)}{\partial W} > 0;\ \frac{\partial\,(dq_2)}{\partial\delta} > 0;\ \frac{\partial(dq_2)}{\partial dq_1} = 0$$

and

$$\frac{\partial MC}{\partial dq_1} > 0;\ \frac{\partial MC}{\partial\delta} < 0;\quad \left.\begin{array}{l}\frac{\partial MC}{\partial\overline{\alpha}} > 0\\ \frac{\partial MC}{\partial W} < 0\end{array}\right\} \Longleftrightarrow n > \overline{n}\left(\frac{b-1}{b}\right)^{b-1}$$

The compensation scheme is obtained by equating the ratio of the marginal variations of utilities $\left(\frac{\partial G}{\partial MC}/\frac{\partial G}{\partial dq_2} = \frac{(1+\delta)\overline{\alpha}q_2}{W\delta(1-\overline{\alpha})}\right)$ to the ratio of the marginal

costs $\left(\frac{\partial C}{\partial MC}/\frac{\partial C}{\partial dq_2} = \frac{n}{ab(dq_2)^{b-1}}\right)$. Note that none of the ratios is affected by a change in MC while an increase of dq_2 decreases the ratio of the marginal costs.

The impact of the natural damage $(-dq_1)$ on the monetary compensation is obviously positive. A higher damage implies a higher compensation. Nevertheless, it is surprisingly null on the natural compensation. Indeed, the intuitive positive effect of $(-dq_1)$ on dq_2 is offset by the trade-off effect between MC and dq_2. Why does this trade-off effect not impact MC? The explanation comes from the cost function. The marginal cost of compensation with respect to MC is constant and equal to n whatever the level of MC while the marginal cost of compensation with respect to dq_2 is increasing with dq_2 $\left(ab\,(dq_2)^{b-1}\right)$. Following a rise in $(-dq_1)$ the higher level of dq_2 due to the environmental willingness to accept increases the weight of dq_2 in the cost function and implies a trade-off in favor of MC which diminishes the required level of dq_2. As a result, following an increase of $(-dq_1)$, the monetary compensation increase and the natural compensation stays unchanged[7]. Note that in regime 3, when MC is reduced to zero, since there is no trade-off between both types of compensation, the level of dq_2 is positively linked to the natural damage.

We first analyse the impact of wealth on compensations. On the one hand, the intertemporal wealth impacts negatively the ratio of the marginal variation of utilities while it does not affect the ratio of the marginal costs. Maintaining both ratios equal implies that a higher level of wealth induces a higher natural compensation. On the other hand, the level of wealth impacts the monetary compensation through two channels: one direct and one indirect. The direct effect can be deduced from the willingness to accept derived from Equation (18) with $dq_2 = 0$. A richer agent is inclined to require a higher amount of MC to compensate the natural damage than a poor agent[8]. The indirect effect comes from the trade-off between both compensations. A rise in dq_2 tends to diminish the level of MC (19) and this decrease is stronger with a higher W. Since the intertemporal wealth has a positive effect on dq_2, this indirect effect is negative on MC. The whole effect on MC depends on which effect dominate. The negative direct effect wins for a sufficiently high level of n. As we have shown, a rise in n modifies the compensation scheme in favor of dq_2 with respect to MC while

[7]The offset of both effects on dq_2 comes from the specification of both the utily function and the cost function.

[8]The impact of $-dq1$ on MC is given by the relation $\frac{dMC}{(-dq_1)} = \frac{(1-\overline{\alpha})W}{(1+\delta)\overline{\alpha}q_1}$

it does not modify the willingness to accept. A higher weight of dq_2 leads the direct effect on MC dominated by the indirect effect through dq_2. The whole impact of wealth on MC is then negative.

The impact of the time preference is less ambiguous. The ratios equalization links positively δ with dq_2: the more the second period is valued in the utility, the higher is the level of required natural compensation. The effect of δ on MC is also unambiguously negative. The direct effect through the willingness to accept is negative: the valuation of the damage in period one is lower with a higher δ so that the required monetary compensation is lower. In addition, the positive effect of δ on dq_2 impact negatively the monetary compensation.

The mean preference for the private good ($\overline{\alpha}$) is negatively linked to dq_2 in the equality of the marginal utilities ratio to the marginal costs ratio. The lower the weight for the environmental good in the utility (high $\overline{\alpha}$), the lower the effect of natural damage on the utility and lower the required natural compensation. Similarly to the impact of wealth, there are two effects of opposite sign of the monetary compensation. The direct effect is negative while the effect through dq_2 is positive. Again, the whole effect depends on the value of n. A high level of n implies a high weight of dq_2 and the positive effect through dq_2 dominates the direct negative effect.

As mentioned in the proof of the previous proposition, when $b < 1$, the cost function is concave which implies that the compensation that are implemented are corner solutions of the problem of cost minimization:

Proposition 2 *For $b < 1$, three solutions can emerge from the program of the social planer*

1. $dq_2 = -(1+r)dq_1$ *and* $MC = -dq_1 W \frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)} \left(\frac{1}{q_1} - \frac{\delta}{q_2}(1+r)\right)$ *iff* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *and* $n < \widehat{n}$ *(regime 1)*

   with $\widehat{n} = \dfrac{a(-dq_1)^{b-1}\left(\left(\frac{q_2}{q_1}\frac{1}{\delta}\right)^b - (1+r)^b\right)}{W\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\frac{\delta}{q_2}\left(\frac{q_2}{q_1}\frac{1}{\delta} - (1+r)\right)}$

2. $MC = 0$ *and* $dq_2 = -\frac{q_2}{q_1}\frac{1}{\delta}dq_1$ *iff* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *and* $n > \widehat{n}$ *(regime 3)*

3. $MC = 0$ *and* $dq_2 = -(1+r)dq_1$ *iff* $(1+r) > \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 4)*

Proof. Rewriting the cost function in MC according to (19) gives

$$C\left(dq_2, MC\right) = nMC + a\left(\left(\frac{-dq_1}{q_1} - MC\frac{\overline{\alpha}(1+\delta)}{(1-\overline{\alpha})W}\right)\frac{q_2}{\delta}\right)^b$$

which is clearly quasi-convex in dq_2 if and only if $b \geq 1$. For $b < 1$, minimising the cost leads to set $MC = 0$ (condition (8)). The value of dq_2 is then derived from (20) which corresponds to regime 3 if the parameters are such that $dq_2 > -(1+r)\,dq_1$ for $dq_2 = \frac{-dq_1}{q_1}\frac{q_2}{\delta}$ and to regime 4 otherwise.

Rewriting the cost function in dq_2 according to (19) gives

$$C\left(dq_2, MC\left(dq_2\right)\right) = nW\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{-dq_1}{q_1} - \frac{\delta}{q_2}dq_2\right) + a\left(dq_2\right)^b$$

which is clearly quasi-convex in dq_2 if and only if $b \geq 1$. For $b < 1$, the cost function is convex in $dq2$ so that the only solution which minises the cost is a corner solution. According to condition 7 minimising the cost requires $dq_2 = -(1+r)\,dq_1$. The value of MC is derived from (19), which corresponds to regime 1 if the parameters are such that $MC > 0$ for $MC = -dq_1 W\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{1}{q_1} - \frac{\delta}{q_2}(1+r)\right)$ and to regime 4 otherwise.

We compare regime 1 and regime 3.

Under regime 3, the cost reduces to

$$C_3\left(dq_2, MC\right) = a\left(\frac{-dq_1}{q_1}\frac{q_2}{\delta}\right)^b$$

Under regime 1, the cost reduces to

$$C_1\left(dq_2, MC\right) = n\left(-dq_1\right)W\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{1}{q_1} - \frac{\delta}{q_2}(1+r)\right) + a\left(-(1+r)dq_1\right)^b$$

$$C_3 < C_1 \Longleftrightarrow n > \frac{q_2\overline{\alpha}(1+\delta)a\left(-dq_1\right)^{b-1}\left(\left(\frac{q_2}{q_1}\frac{1}{\delta}\right)^b - (1+r)^b\right)}{W(1-\overline{\alpha})\delta\left(\frac{q_2}{q_1}\frac{1}{\delta} - (1+r)\right)} = \widehat{n}$$

■

With $b < 1$ the limit between regime 1 and 3 is given by $\widehat{n}$. As previously explained, a higher (resp. lower) level of n goes in favor of the use of natural (resp. monetary) compensation. As shown in Figure 2, contrary to the case with $b > 1$, there is no more optimal mixed compensation and regime 1 switches directly to regime 3 with the increase of n since only corner solutions enable to minimise the cost.

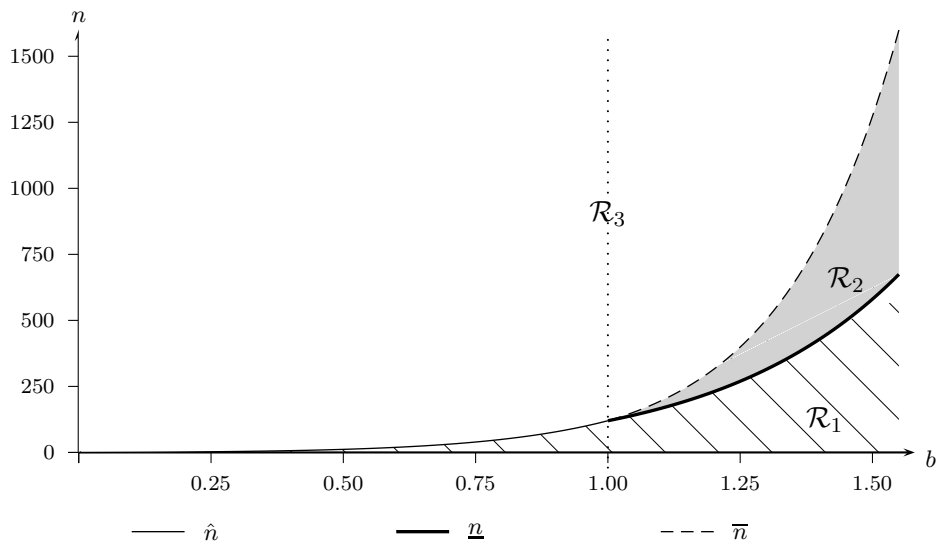


Figure 2: Regimes depending on the values of b and n

3.1.2 Cost and welfare analyses

In this section, we investigate the cost associated with alternative compensation schemes and their welfare implications. First, recall that for low marginal rate of substitution of the environmental good between period 1 and 2, the compensation scheme reduces to Regime 4 (no monetary compensation and a minimal natural compensation driven by the EA constraint whatever the level of n). The change of the aggregate welfare is positive as well as every individual welfare variation[9]. The agent that values the environmental good the most (lowest α_i) wins the most. Let denote by CS^* the optimal Compensation Scheme given by either Propositions 1 and 2. We now introduce three other compensation schemes that could be referred as benchmark cases. The first one (CS_0) combines monetary and natural compensation without EA constraint. As already seen, the compensation scheme (CS_0) is composed of two regimes characterized by:

- $dq_2 = \left(\frac{(1-\overline{\alpha})nW\delta}{\overline{\alpha}(1+\delta)q_2ab}\right)^{\frac{1}{b-1}}$ and $MC = \frac{(1-\overline{\alpha})W\left(\frac{-dq_1}{q_1}\right)}{(1+\delta)\overline{\alpha}} - \left(\frac{\delta(1-\overline{\alpha})W}{q_2(1+\delta)\overline{\alpha}}\right)^{\frac{b}{b-1}}\left(\frac{n}{ab}\right)^{\frac{1}{b-1}}$ iff $\overline{n} > n$

[9] $dV_i = (1-\alpha_i)dq_1\left[-\frac{1}{q_1} + \frac{\delta}{q_2}(1+r)\right] > 0 \quad \forall i$ under Regime 4.

- $MC = 0$ and $dq_2 = -\frac{q_2}{q_1}\frac{1}{\delta}dq_1$ iff $n > \overline{n}$

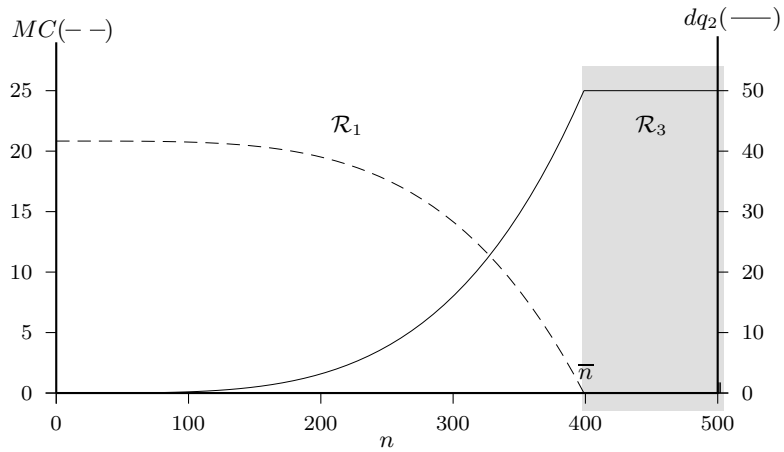


Figure 3: Compensation scheme without EA constraint (CS_0)

As shown in Figure 3 without EA constraint $dq_2 > 0$ whatever the value of n. Nevertheless the level of natural compensation is very low ($dq_2 \approx 0$) for small n.

The two other compensation schemes are either natural compensation (CS_{nat}) or monetary compensation (CS_{mon}). They are defined as follows:

- (CS_{nat}): $dq_2 = -\frac{q_2}{q_1}\frac{1}{\delta}dq_1$ and $MC = 0$
- (CS_{mon}): $MC = W\frac{(1-\overline{\alpha})}{\overline{\alpha}(1+\delta)}\left(\frac{-dq_1}{q_1}\right)$ and $dq_2 = 0$

Note that CS_{nat} and CS_{mon} are fixed and do not vary with n.

Figure 4 shows the costs associated with these compensation schemes (CS_0, CS_{nat}, CS_{mon}) and with the previously presented one (CS^* which is composed of regimes 1, 2 and 3). From a cost minimization perspective, we observe that for $n < \underline{n}$ the compensation scheme described by regime 1 (thick line) is not the least costly possible option. The EA constraint imposes an additional cost. Without this constraint, there would exist two better options: monetary compensation (dashed line) for $n < (\underline{n}/b)$ and monetary compensation associated with natural compensation at a level lower than $(-dq_1)(1 + r)$ (dotted line) for $n < \underline{n}$. For $n \geq \underline{n}$, CS^* is the less costly

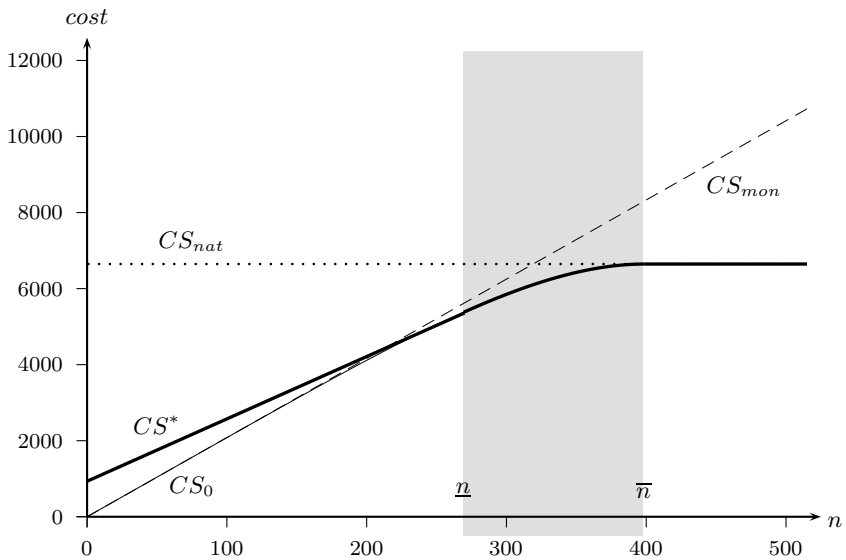
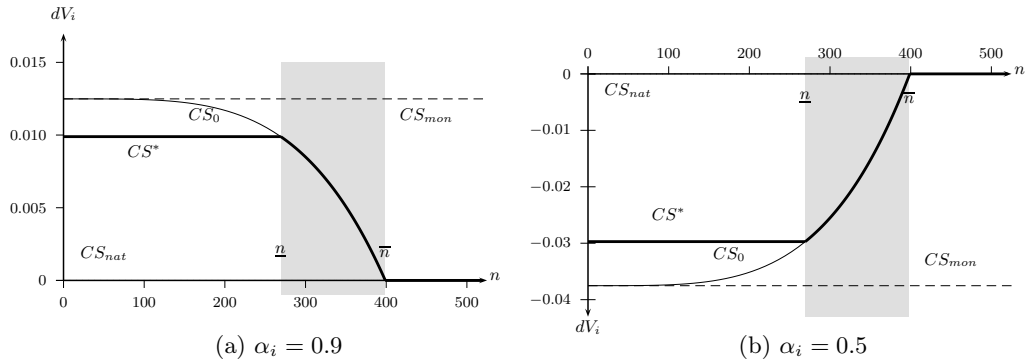


Figure 4: Costs associated with the four compensation schemes

option (jointly with CS_0 for $\underline{n} \leq n \leq \overline{n}$ and jointly with CS_0 and CS_{nat} for $n \geq n$).

If either regimes 1, 2 and 3 leave the social welfare unchanged, it is not true for individual ones. As shown in Figures 5.a and 5.b, compensation may result in some losers and winners. This inequity reduces as the share of the natural compensation grows (Regime 2).

When expressed - totally or partially - in money terms, necessary compensation has to be large for people who value money the less (low α_i) (Brekke, 1997). Under regimes 1 and 2, individuals with $\alpha_i = \overline{\alpha}$ do not support any individual welfare variations whereas inviduals with $\alpha_i < \overline{\alpha}$ incur a loss of welfare decreasing with α_i and n and individuals with a $\alpha_i > \overline{\alpha}$ benefit from a gain of welfare. This gain increases with α_i and decreases with n (Figure 5.a). Both cost and welfare analyses highlight that Regime 1 is worth in terms of cost compared to a compensation scheme without EA constraint (CS_0) but better in terms of inequity. As suggested by both figures 5.a and 5.b, when the EA constraint applies, it limits the gains for the winners but also the losses for the losers. In the trade-off between efficiency and equity, the EA constraint diminishes the cost efficiency of the compensation but also lowers inequity between agents. In that context, while the



(a) $\alpha_i = 0.9$ (b) $\alpha_i = 0.5$

Figure 5: Individual welfare gain/loss (heterogeneity in α_i)

primary justification of the EA constraint is based on environmental criteria, it may also be supported for equity purposes. Figures 5.a and 5.b also show that the monetary compensation (CS_{mon}) is the worst in terms of equity compared to the other compensation schemes.

Finally, under regime 3 where the only compensation is the natural one, every individual welfare losses from the damage are offset by the natural compensation. For this regime, the compensation granted to all individuals corresponds to a pure intertemporal compensation with a good similar to the damaged one. WTA_i^q does not vary with n and is similar for all individuals, i.e. $WTA_i^q = \frac{-dq_1}{q_1}\frac{q_2}{\delta} = dq_2 \ \forall i$. From a welfare perspective, natural compensation is the most appropriate solution since there are no welfare losses at aggregate and individual levels. Nevertheless, Figure 4 shows that for a low n the cost of natural compensation is, in our example, between 2 or 3 times higher than the cost associated with other compensation schemes.

3.2 Heterogeneity in wealth

In this section, we assume that agents are differentiated according to their wealth, W_i. The social welfare function writes:

$$\mathcal{W} = \mathcal{W}\big[v(W_1, q_1, q_2), \ldots, v(W_n, q_1, q_2)\big]$$

It can be rewritten:

$$\begin{aligned}\mathcal{W} &= \sum_{i=1}^{n} v(W_i, q_1, q_2) \\ &= \alpha \sum_{i=1}^{n} \ln\left(\frac{W_i}{(1+\delta)(1+r)}\right) + n(1-\alpha)\ln q_1 + \delta\alpha \sum_{i=1}^{n} \ln\left(\delta \frac{W_i}{(1+\delta)}\right) + n\delta(1-\alpha)\ln q_2\end{aligned}$$

and condition (6) becomes:

$$d\mathcal{W} = d\sum_{i=1}^{n} V_i = \frac{\alpha(1+\delta)}{\overline{W}} MC \sum_{i=1}^{n} \frac{\overline{W}}{W_i} + \frac{n(1-\alpha)}{q_1} dq_1 + n\delta \frac{(1-\alpha)}{q_2} dq_2 = 0 \tag{21}$$

where $\overline{W} = \frac{\sum_{i=1}^{n} W_i}{n}$, so that

$$dq_2 = \left(\frac{-dq_1}{q_1} - MC \frac{\alpha(1+\delta)}{(1-\alpha)\overline{W}} \frac{1}{n} \sum_{i=1}^{n} \frac{\overline{W}}{W_i}\right) \frac{q_2}{\delta}$$

and

$$MC = \frac{1}{\frac{1}{n}\sum_{i=1}^{n} \frac{\overline{W}}{W_i}} \frac{(1-\alpha)}{\alpha(1+\delta)} \left(\frac{-dq_1}{q_1} - \frac{\delta}{q_2} dq_2\right) \overline{W}$$

where $\frac{1}{n}\sum_{i=1}^{n} \frac{\overline{W}}{W_i} = I_W \geq 1$ is a measure of the average wealth inequality in the society. An increase in I_W implies a greater wealth inequality in the society ($I_W = 1$ means no inequality).[10]

Similarly to the heterogeneous preferences case, we distinguish two different cases according to the value of b with respect to 1.

Proposition 3 *For $b \geq 1$, four solutions can emerge from the program of the decision-maker*

1. $dq_2 = -(1+r)dq_1$ *and* $MC = (-dq_1)\frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\overline{W}}{I_W}\left(\frac{1}{q_1} - \frac{\delta}{q_2}(1+r)\right)$ *iff* $n < \underline{\underline{n}}$ *and* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 1)*

[10] When considering the special case where $dq_2 = 0$, in analogy with Medin et al. (2001), MC corresponds to the per person 'benefit' when equal marginal utility of the environmental good is assumed. It is defined by $MC = \frac{n}{\sum_{n=1}^{n}\left(\frac{\partial v}{\partial W_i} / \frac{\partial v}{\partial q_1}\right)}(-dq_1)$. If equal marginal utility of income is assumed (i.e $I_W = 1$ in our case), then we have $MC = \frac{1}{n}\sum_{n=1}^{n}\left(\frac{\partial v}{\partial q_1} / \frac{\partial v}{\partial W_i}\right)(-dq_1) = \frac{1}{n}\sum_{n=1}^{n} WTA_i^W$.

2. $dq_2 = \left[\frac{n(1-\alpha)\delta}{ab\alpha(1+\delta)q_2}\frac{\overline{W}}{I_W}\right]^{\frac{1}{b-1}}$ *and* $MC = \frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\overline{W}}{I_W}\frac{-dq_1}{q_1} - \left(\frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\overline{W}}{I_W}\frac{\delta}{q_2}\right)^{\frac{b}{b-1}}\left[\frac{n}{ab}\right]^{\frac{1}{b-1}}$ *iff* $\overline{\overline{n}} > n > \underline{\underline{n}}$ *(regime 2)*

3. $MC = 0$ *and* $dq_2 = -\frac{q_2}{q_1}\frac{1}{\delta}dq_1$ *iff* $n > \overline{\overline{n}}$ *and* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 3)*

4. $MC = 0$ *and* $dq_2 = -(1+r)dq_1$ *iff* $(1+r) > \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 4)*

Proof. Conditions (7) and (8) imply

$$dq_2 > -(1+r)\,dq_1 \Longleftrightarrow n > \frac{\overline{\alpha}}{(1-\overline{\alpha})}\frac{(1+\delta)}{\delta}q_2 ab\frac{I_W}{\overline{W}}\left((1+r)(-dq_1)\right)^{b-1} = \underline{\underline{n}}$$

$$MC > 0 \Longleftrightarrow n < ab\left(\frac{(1+\delta)\,\alpha}{(1-\alpha)}\frac{I_W}{\overline{W}}\right)\left(\frac{q_2}{\delta}\right)^b\left(\frac{-dq_1}{q_1}\right)^{b-1} = \overline{\overline{n}}$$

both conditions can be fulfilled iff

$$\overline{\overline{n}} > \underline{\underline{n}} \Longleftrightarrow \frac{q_1}{q_2} < \frac{1}{\delta(1+r)}$$

■

The comments about each regime are quite similar to those for heterogeneous preferences. Here we concentrate on the distinctions between both cases. The values of MC and dq_2 show that the heterogeneity in wealth introduces the expression $\overline{W}/I_W$ instead of W with no heterogeneity. This expression highlights two different elements in the wealth heterogeneity: the value of the average wealth (how rich the society is), and the distribution effect (how unequal the society is).

The comparative static analysis for regime 2 gives the following relations

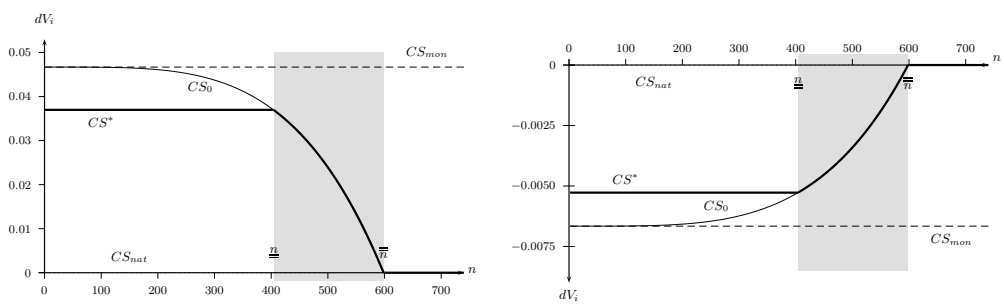
$$\frac{\partial(dq_2)}{\partial dq_1} = 0;\ \frac{\partial(dq_2)}{\partial\alpha} < 0;\ \frac{\partial(dq_2)}{\partial\overline{W}} > 0;\frac{\partial(dq_2)}{\partial\delta} > 0;\ \frac{\partial(dq_2)}{\partial(I_W)} < 0$$

and

$$\frac{\partial MC}{\partial dq_1} < 0;\ \frac{\partial MC}{\partial\delta} < 0;\quad \left.\begin{array}{l}\frac{\partial MC}{\partial\alpha} > 0\\ \frac{\partial MC}{\partial\overline{W}} < 0\\ \frac{\partial MC}{\partial I_W} > 0\end{array}\right\} \Longleftrightarrow n > \overline{\overline{n}}\left(\frac{b-1}{b}\right)^{b-1}$$

The impacts of dq_1, δ are similar to the case with heterogeneity in preferences, even for α, instead of $\overline{\alpha}$. Here the impact of $\overline{W}$ can be compared to the impact of W in the previous case. A richer society is inclined to require a higher amount of MC to compensate the natural damage than a poor

society, but the trade-off between both compensations implies that for a sufficiently high level of n the indirect effect from dq_2 on MC dominates and the whole impact of the average wealth on $\underline{\underline{n}}$ is positive. The impact of I_W is of opposite sign. An increase in I_W implies a higher income inequality in the population[11]. On one hand, since poorer agents value more a monetary compensation (V_i is concave in W_i), a lower monetary compensation is required for any $I_W > 1$ than when there is no inequality in wealth ($I_W = 1$). On the other hand, the natural compensation is identically valued whatever the level of wealth of the agents (income inequality plays no role). As a result, the frontier which separates Regime 1 and 2 and Regime 2 and 3 are moved for higher levels of n and the scales for which Regime 1 and 2 apply become larger. Conversely, since $\overline{\overline{n}}$ increase with I_W, the scale of Regime 3 reduces since natural compensation is not increasingly valued by poor people.



(a) $W_i = 200$ (b) $W_i = 1000$

Figure 6: Individual welfare gain/loss (heterogeneity in W_i)

As already stressed in previous subsection, monetary compensation will be in favor of individuals that value money the most. As shown in Figure 6.a the poorest individuals ($W_i < \overline{W}/I_W$) are the winners[12].

Proposition 4 *For $b < 1$, three solutions can emerge from the program of the decision-maker*

1. $dq_2 = -(1+r)dq_1$ *and* $MC = (-dq_1)\frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\overline{W}}{I_W}\left(\frac{1}{q_1} - \frac{\delta}{q_2}(1+r)\right)$ *iff* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *and* $n < \widetilde{n}$ *(regime 1)*

[11] A rise in I_W is here defined as a mean preserving spread in the sense that the ratio I_W rises but the mean wealth stays unchanged, so that wealth inequality clearly increases in the population.

[12] The following parameter set was used for the numerical simulation: ($\overline{W} = 1000$, $I_W = 1.5$, $\overline{\alpha} = 0.8$, $\delta = 0.2$, $r = 0.04$, $q_1 = 100$, $q_2 = 100$, $dq_1 = -10$, $a = 50$, $b = 1.2$).

2. $MC = 0$ *and* $dq_2 = -\frac{q_2}{q_1}\frac{1}{\delta}dq_1$ *iff* $(1+r) < \frac{q_2}{q_1}\frac{1}{\delta}$ *and* $n > \widetilde{n}$ *(regime 3)*

3. $MC = 0$ *and* $dq_2 = -(1+r)dq_1$ *iff* $(1+r) > \frac{q_2}{q_1}\frac{1}{\delta}$ *(regime 4)*

with $\widetilde{n} = \frac{a(-dq_1)^{b-1}\left(\left(\frac{q_2}{q_1}\frac{1}{\delta}\right)^b - (1+r)\right)}{I_W \overline{W} \frac{(1-\alpha)}{\alpha(1+\delta)} \frac{\delta}{q_2}\left(\frac{q_2}{q_1}\frac{1}{\delta} - (1+r)\right)}$

Proof. Similar to proposition 1 with the comparison of cases 2 and 3 that yields:

$$\widetilde{C}_3 < \widetilde{C}_1 \Longleftrightarrow n > \frac{\overline{W}}{I_W} \frac{a\left(-dq_1\right)^{b-1}\left(\left(\frac{q_2}{q_1}\frac{1}{\delta}\right)^b - (1+r)\right)}{\frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\delta}{q_2}\left(\frac{q_2}{q_1}\frac{1}{\delta} - (1+r)\right)} = \widetilde{n}$$

■

For $b < 1$, the level of n separating both regimes 1 and 3, i.e. $\tilde{n}$, decreases with I_W. Then heterogeneity in wealth goes in favor of a natural compensation since the borders of this regime are extended.

4 Concluding remarks

While the European Directive 2004/35/EC precludes the use of monetary compensation in response to an environmental damage, this article reintroduces the monetary compensation as a potential compensating tool complementing a natural compensation. We explore which satisfactory compensation can be provided at a minimal cost under an ecological constraint (here EA constraint). The results feature that the best way to provide compensation for ecological damage at a minimal cost may be sensitive to several parameters: nature of heterogeneity, number of agents, relative costs of monetary and natural compensations.

More precisely, we show that when the population affected by the natural damage is small, the equivalency constraint implies the use of a minimal natural resource quantity that would not be provided without this constraint for cost reason. But this constraint enables to diminish the inequity generated by the environmental damage on the heterogeneous population. Although the main purpose of enforcing an ecological constraint is an environmental one (i.e. "*no net loss*" principle) it also has welfare and cost implications. In that sense, a key result of our paper is to find the optimal balance between equity and cost efficiency considerations.

However, to go further, some results of our paper may be linked to prevention issues. For instance, we show that a poor population (low mean income) values more the monetary compensation than a rich and as a consequence, accepts a lower level of money to compensate the damage it supports. This mechanism extends the use of monetary compensation. Moreover, if this poor affected population is relatively small, the polluter can consider that the cost of compensation it should support in case of damage is sufficiently low to not undertake any prevention measure that could avoid any natural damage. Facing this kind of possible behaviors, the use of a minimal ecological constraint is strongly justified to avoid them.

Work still remains to be done to get a better understanding of all the implications of providing compensation for an environmental damage. In particular, a better consideration of natural resource dynamics as well as a deeper study of redistributive effects of the trade-off between money and nature should be considered in a next step. Heterogeneous discount rates would be another interesting question.

Appendix

A. Comparative statics for heterogeneity in preferences

$$\begin{aligned}
\frac{\partial MC}{\partial \overline{\alpha}} &= \left(-\frac{1}{\alpha^2}\right)\left(\frac{(-dq_1)\,W}{(1+\delta)\,q_1} - \frac{b}{b-1}\left(\frac{\delta W}{q_2\,(1+\delta)}\right)^{\frac{b}{b-1}}\left(\frac{(1-\overline{\alpha})}{\overline{\alpha}}\right)^{\frac{1}{b-1}}\left(\frac{n}{ab}\right)^{\frac{1}{b-1}}\right) \\
\frac{\partial MC}{\partial \overline{\alpha}} &> 0 \Longleftrightarrow \frac{\overline{\alpha}\,(1+\delta)}{W(1-\overline{\alpha})}\left(\frac{(-dq_1)}{q_1}\right)^{b-1}\left(\frac{q_2}{\delta}\right)^{b}\left(\frac{(b-1)}{b}\right)^{b-1} ab < n \\
&\Longleftrightarrow \overline{n}\left(\frac{b-1}{b}\right)^{b-1} < n
\end{aligned}$$

$$\begin{aligned}
\frac{\partial MC}{\partial W} &= \frac{\frac{(1-\overline{\alpha})}{q_1}\,(-dq_1)}{(1+\delta)\,\overline{\alpha}} - \frac{b}{b-1}\left(W\frac{\delta\frac{(1-\overline{\alpha})}{q_2}}{(1+\delta)\,\overline{\alpha}}\right)^{\frac{b}{b-1}}\frac{1}{W}\left(\frac{n}{ab}\right)^{\frac{1}{b-1}} \\
\frac{\partial MC}{\partial W} &> 0 \Longleftrightarrow \frac{(1+\delta)\,\overline{\alpha}}{W(1-\overline{\alpha})}\left(\frac{(-dq_1)}{q_1}\right)^{b-1}\left(\frac{q_2}{\delta}\right)^{b}\left(\frac{b-1}{b}\right)^{b-1} ab > n \\
&\Longleftrightarrow \overline{n}\left(\frac{b-1}{b}\right)^{b-1} > n
\end{aligned}$$

$$\frac{\partial MC}{\partial \delta} = -\frac{\frac{(1-\overline{\alpha})}{q_1}\,(-dq_1)}{(1+\delta)^2\,\frac{\overline{\alpha}}{W}} - \frac{b}{b-1}\left(\frac{n}{ab}\frac{\delta\frac{(1-\overline{\alpha})}{q_2}}{(1+\delta)\,\frac{\overline{\alpha}}{W}}\right)^{\frac{1}{b-1}}\frac{\frac{(1-\overline{\alpha})}{q_2}}{\frac{\overline{\alpha}}{W}}\frac{\delta}{(1+\delta)^2} < 0$$

B. Comparative statics for heterogeneity in wealth

$$\frac{\partial dq_2}{\partial \alpha} = -\frac{1}{\alpha^2}\frac{1}{b-1}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{2-b}{b-1}}\left[\frac{n\delta I_W}{ab(1+\delta)q_2}\right]^{\frac{1}{b-1}} < 0$$

$$\frac{\partial dq_2}{\partial \delta} = \frac{1}{(1+\delta)^2}\frac{1}{b-1}\left(\frac{\delta}{1+\delta}\right)^{\frac{2-b}{b-1}}\left[\frac{n(1-\alpha)I_W}{ab\alpha q_2}\right]^{\frac{1}{b-1}} > 0$$

$$\frac{\partial dq_2}{\partial n} = 2\frac{1}{b-1}\frac{1}{n}\left[\frac{n(1-\alpha)I_W}{ab\alpha(1+\delta)q_2}\right]^{\frac{1}{b-1}} > 0$$

$$\frac{\partial MC}{\partial n} = \frac{(1-\alpha)}{\alpha(1+\delta)}I_W\overline{W}\left(-\frac{\delta}{q_2}\frac{1}{b-1}\frac{1}{n}\left[\frac{n(1-\alpha)\delta}{ab\alpha(1+\delta)q_2}\frac{\overline{W}}{I_W}\right]^{\frac{1}{b-1}}\right) < 0$$

$$\begin{aligned}
\frac{\partial MC}{\partial \alpha} &= (-\frac{1}{\alpha^2})\left[\frac{(-dq_1)}{q_1} - \frac{b}{b-1}(\frac{1-\alpha}{\alpha})^{\frac{1}{b-1}}\frac{\delta}{q_2}\left(\frac{n\delta}{ab(1+\delta)q_2}\frac{\overline{W}}{I_W}\right)^{\frac{1}{b-1}}\right] \\
\frac{\partial MC}{\partial \alpha} &> 0 \Leftrightarrow \frac{(-dq_1)}{q_1} < \frac{b}{b-1}(\frac{1-\alpha}{\alpha})^{\frac{1}{b-1}}\frac{\delta}{q_2}\left(\frac{n\delta}{ab(1+\delta)q_2}\frac{\overline{W}}{I_W}\right)^{\frac{1}{b-1}} \\
&\iff n > \left(\frac{b-1}{b}\right)^{\frac{b-1}{2}}\overline{\overline{n}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial MC}{\partial I_W} &= -\frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\overline{W}}{I_W^2}\left(\frac{1}{q_1}(-dq_1) - \frac{\delta}{q_2}\left[\frac{n(1-\alpha)\delta}{ab\alpha(1+\delta)q_2}\frac{\overline{W}}{I_W}\right]^{\frac{1}{b-1}}\left(\frac{b}{b-1}\right)\right) > 0 \\
&\iff n > \left(\frac{b-1}{b}\right)^{\frac{b-1}{2}}\overline{\overline{n}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial MC}{\partial \overline{W}} &= \frac{(1-\alpha)}{\alpha(1+\delta)}I_W\left(\frac{1}{q_1}(-dq_1) - \frac{\delta}{q_2}\left[\frac{n(1-\alpha)\delta}{ab\alpha(1+\delta)q_2}I_W\overline{W}\right]^{\frac{1}{b-1}}\left(\frac{b}{b-1}\right)\right) > 0 \\
&\iff n < \left(\frac{b-1}{b}\right)^{\frac{b-1}{2}}\overline{\overline{n}}
\end{aligned}$$

$$\frac{\partial MC}{\partial \delta} = -\frac{(1-\alpha)}{\alpha(1+\delta)^2}\frac{\overline{W}}{I_W}\frac{-dq_1}{q_1} - \frac{1}{\delta\,(1+\delta)}\left(\frac{(1-\alpha)}{\alpha(1+\delta)}\frac{\overline{W}}{I_W}\frac{\delta}{q_2}\right)^{\frac{1}{b-1}}\left[\frac{n}{ab}\right]^{\frac{1}{b-1}} < 0$$

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