Robust viable management of a harvested ecosystem model

Esther Regnier *   Michel De Lara†

August 27, 2013

Abstract

The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the ecosystem approach by 2010. In this perspective, we propose a theoretical management framework that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties. More specifically, we consider a discrete-time two-species dynamical model, where states are biomasses and where two harvesting efforts act as controls. Uncertainty takes the form of disturbances affecting each species growth factors, and are assumed to take their values in a known given set. We define the robust viability kernel as the set of initial species biomasses such that at least one harvesting strategy guarantees minimal production and preservation levels for all times, whatever the uncertainties. We apply our approach to the anchovy-hake couple in the Peruvian upwelling ecosystem. We find that accounting for uncertainty significantly reduces the robust viability kernel compared to the deterministic one (without uncertainties). We observe that the robust viability kernel is sensible to a set of worse case scenarios exclusively, which we identify. We comment on the management implications of comparing robust viability kernels (with uncertainties) and the deterministic one (without uncertainties).

Key words: optimization; viability; uncertainty; robustness; sustainability; ecosystem management; fisheries; Peruvian upwelling.

*Université Paris 1 & Paris School of Economics, 106–112 boulevard de L'Hôpital, 75647 Paris Cedex 13, France. Corresponding author: esther.regnier@univ-paris1.fr

†Université Paris–Est, CERMICS, 6–8 avenue Blaise Pascal, 77455 Marne la Vallée Cedex 2, France. delara@cermics.enpc.fr

1 Introduction

There is a growing demand for moving from single species management schemes to an ecosystemic approach of fisheries management [6]. The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of an ecosystem approach by 2010.

Furthermore, uncertainty inherent to fisheries is recognized to play an important role in the failure of management regimes. Fisheries modeling requires estimations of stock status and total withdrawal from stock; such information remains imprecise and error prone. Uncertainty can also concern the structure and dynamics of ecosystems, which are poorly known. At last, uncertain climatic hazards or technical progress are likely to affect fisheries productivity. Some claim that fishing decreases the resilience of fish populations, rendering them more vulnerable to environmental change [8], and that not accounting for uncertainty can lead to excessive harvest of a resource [7].

We propose a theoretical management framework that deals jointly with i) ecosystem dynamics, ii) conflicting issues of production and preservation and iii) robustness with respect to dynamics uncertainties.

We set forward the robust viability theory [3] as a relevant approach to address dynamical control problems under constraints with uncertainty. This approach aims at describing all the evolutions of a dynamical system under uncertainty that satisfy, at each instant, given objectives, whatever the uncertainties. Starting from a so-called *robust viable state*, there exists a control strategy guaranteeing constraints, here production and preservation objectives, for all dates of a time span, and for all uncertainties. The set of robust viable states is called the robust viability kernel. Notice that, in the robust viability theory, no trade-offs are allowed between pursued objectives or time periods: all constraints must be satisfied for all times, whatever the uncertainties.

We apply this theory to a discrete-time two-species dynamical model, where states are biomasses and where two harvesting efforts act as controls. Uncertainty takes the form of disturbances affecting each species growth factors, and are assumed to take their values in a known given set. We consider three nested sets in order to appraise the sensibility of our results to uncertainties. Constraints are imposed for each species: a minimum safe biomass level, usually identified by biologists, and a minimum required harvesting level assumed to ensure economic needs. Thus, starting from a robust viable biomass couple, it is possible to drive the system on a sustainable path along which catches and biomasses stand above production and biological minimums, despite of uncertainties.

Reducing uncertainties to zero amounts to formulate the problem within the deterministic viability framework [1]. Comparison of these deterministic viable states with the sets of robust viable states shades light on the potential interest of adopting a precautionary approach in harvested ecosystem management.

The paper is organized as follow. Section 2 introduces a generic class of harvested nonlinear ecosystem models, the sustainability constraints, and presents the concept of robust viability kernel. The deterministic viability kernel is also defined for comparison purpose. In section 3, we proceed with an application of the robust and deterministic viability analysis to the Peruvian hake-anchovy upwelling ecosystem between 1971 and 1981. We numerically compute robust viability kernels, stemming from different uncertainty sets, and compare them to the deterministic viability kernel, whose expression is obtained analytically. Section 4 concludes.

2 The Robust Viability Approach

In what follows, we present a class of generic harvested *nonlinear* ecosystem models with uncertainty. Next, we introduce the concept of robust viable state, that is, a state starting from which conservation and production constraints can be guaranteed over a given time span, despite of uncertainty. We then define the set of deterministic viable states — states guaranteeing conservation and production constraints in absence of uncertainties — for which we are able to provide an analytical expression.

2.1 A generic ecosystem model with uncertainty and the associated sustainability constraints

For simplicity, we consider a dynamical model with two species, each targeted by a specific fleet, but this approach can be easily extended to more than two species in interaction. Each species is described by its biomass: the two–dimensional state vector (y, z) represents the biomasses of both species. The two–dimensional control vector (v_y, v_z) comprises the harvesting effort for each species, respectively, each lying in $[0, 1]$. The two terms ε_y and ε_z correspond to uncertainties affecting each species biomass, respectively. The discrete-time control dynamical system we consider is given by

$$\begin{cases} y(t+1) &= y(t)\mathcal{R}_y\big(y(t), z(t), \varepsilon_y(t)\big)\big(1 - v_y(t)\big)\ , \\ z(t+1) &= z(t)\mathcal{R}_z\big(y(t), z(t), \varepsilon_z(t)\big)\big(1 - v_z(t)\big)\ , \end{cases} \tag{1}$$

where t stands for time (typically, periods are years), and ranges from the initial time t_0 to the time horizon T (where $T \geq t_0 + 2$). The two functions $\mathcal{R}_y : \mathbb{R}^2 \to \mathbb{R}$ and $\mathcal{R}_z : \mathbb{R}^2 \to \mathbb{R}$ represent biological growth factors. The property that the growth factor $\mathcal{R}_y(y, z, \varepsilon_y)$ of species y depends on the other species biomass z (and vice versa) captures ecosystemic features of species interactions. Furthermore, these interactions are complicated by uncertainties. After two periods, $\varepsilon_y(t)$ indirectly impacts $z(t + 2)$ through $y(t + 1)$, so that both disturbances affect both species. According to the nature of the interaction between y and z, uncertainties affecting one of the species will constitute lagged positive or negative externalities for the other species. Catches are given by $v_y y\mathcal{R}_y\big(y, z, \varepsilon_y\big)$ and $v_z z\mathcal{R}_z\big(y, z, \varepsilon_z\big)$ (measured in biomass). This model is generic in that no explicit or analytic assumptions are made on how the growth factors $\mathcal{R}_y$ and $\mathcal{R}_z$ indeed depend upon both biomasses (y, z).

An uncertainty *scenario* is defined as a sequence of uncertainty couples of length $T - t_0$,

$$\big(\varepsilon_y(\cdot), \varepsilon_z(\cdot)\big) = \big((\varepsilon_y(t_0), \varepsilon_z(t_0)), \ldots, (\varepsilon_y(T-1), \varepsilon_z(T-1))\big) \in \prod_{t=t_0}^{T-1} \mathbb{S}(t)\ , \tag{2}$$

where uncertainties $(\varepsilon_y(t), \varepsilon_z(t))$ are assumed to take there values in a two-dimensional set:

$$(\varepsilon_y(t), \varepsilon_z(t)) \in \mathbb{S}(t) \subset \mathbb{R}^2. \tag{3}$$

We now propose to define sustainability as the ability to respect preservation and production minimal levels for all times, building upon the original approach of [2]. For this purpose, we consider:

- on the one hand, *minimal biomass levels* $y^\flat \geq 0$, $z^\flat \geq 0$, one for each species,
- on the other hand, *minimal catch levels* $Y^\flat \geq 0$, $Z^\flat \geq 0$, one for each species.

2.2 The robust viability kernel

A control strategy γ is defined as a sequence of mappings from biomasses towards efforts as follows:

$$\gamma = \{\gamma_t\}_{t=t_0,\ldots,T-1}, \quad \text{with} \quad \gamma_t : \mathbb{R}^2 \to [0, 1]^2\ . \tag{4}$$

A control strategy γ as in (4) and the dynamic model (1) jointly produce state paths by the closed-loop dynamics

$$\begin{cases} y(t+1) &= y(t)\mathcal{R}_y\big(y(t), z(t), \varepsilon_y(t)\big)\big(1-\gamma_t(y(t), z(t))\big) \;, \\ z(t+1) &= z(t)\mathcal{R}_z\big(y(t), z(t), \varepsilon_z(t)\big)\big(1-\gamma_t(y(t), z(t))\big) \;, \end{cases} \tag{5}$$

and control paths by

$$(v_y(t), v_z(t)) = \gamma_t(y(t), z(t)) \;, \quad t = t_0, \ldots, T-1 \;. \tag{6}$$

Notice that, as in (6), controls $(v_y(t), v_z(t))$ are determined by constantly adapting to the state $(y(t), z(t))$ of the system, itself affected by uncertainties $(\varepsilon_y(t-1), \varepsilon_z(t-1))$.

The *robust viability kernel* $\mathbb{V}^R(t_0)$ [3] is the set of initial states $(y(t_0), z(t_0))$ — called *robust viable states* — for which there exists a control strategy γ as in (4) producing state paths $\{(y(t), z(t))\}_{t=t_0,\ldots,T}$ as in (5), and control paths $\{(v_y(t), v_z(t))\}_{t=t_0,\ldots,T-1}$ as in (6), such that, for all uncertainty scenarios $(\varepsilon_y(\cdot), \varepsilon_z(\cdot)) \in \prod_{t=t_0}^{T-1} \mathbb{S}(t)$ in (2), the following goals are satisfied:

- preservation (minimal biomass levels)

$$y(t) \geq y^\flat \;, \quad z(t) \geq z^\flat \;, \quad \forall t = t_0, \ldots, T \;, \tag{7}$$

- production requirements (minimal catch levels)

$$v_y(t)y(t)\mathcal{R}_y\big(y(t), z(t), \varepsilon_y(t)\big) \geq Y^\flat \;, \quad v_z(t)z(t)\mathcal{R}_z\big(y(t), z(t), \varepsilon_z(t)\big) \geq Z^\flat \;, \quad \forall t = t_0, \ldots, T-1 \;. \tag{8}$$

Characterizing robust viable states makes it possible to test whether or not minimal biomass and catch levels can be guaranteed for all time, despite of uncertainty. By *guaranteed* we mean that biomasses and catches never fall below the minimal thresholds as in the inequalities (7) and (8).

The robust viability kernel can be computed numerically by means of a dynamic programming equation associated with dynamics (1), state constraints (7) and control constraints (8) [3] (see §B). We performed the numerical simulations with the scientific software Scicoslab.

2.3 The deterministic viability kernel

The deterministic version of the framework exposed in §2.2 corresponds to the case where the uncertainties $(\varepsilon_y(t), \varepsilon_z(t)) = (0, 0)$, for all $t = t_0, \ldots, T-1$, that is, the uncertainty sets in (3) are reduced to the singleton $\mathbb{S}(t) = \{(0,0)\}$. Then, the robust viability kernel coincides with the so-called *viability kernel* [1], defined in §A.

The following Proposition 1 gives an analytical expression of the deterministic viability kernel under some conditions on the guaranteed levels in (7) and (8). The proof, adapted from [4] is given in §A.

Proposition 1 *If the minimal biomass thresholds $y^\flat$, $z^\flat$ and catch thresholds $Y^\flat$, $Z^\flat$ are such that*

$$y^\flat \mathcal{R}_y\big(y^\flat, z^\flat, 0\big) - y^\flat \geq Y^\flat \ \text{ and } \ z^\flat \mathcal{R}_z\big(y^\flat, z^\flat, 0\big) - z^\flat \geq Z^\flat \ , \tag{9}$$

the deterministic viability kernel is given by

$$\mathbb{V}(t_0) = \Big\{ \ (y,z) \in \mathbb{R}^2_+ \mid y \geq y^\flat, z \geq z^\flat, y\mathcal{R}_y\big(y,z,0\big) - y^\flat \geq Y^\flat, z\mathcal{R}_z\big(y,z,0\big) - z^\flat \geq Z^\flat \ \Big\} \ . \tag{10}$$

Conditions (9) mean that, at the point $(y^\flat, z^\flat)$ of minimum biomass thresholds, the surplus $y^\flat \mathcal{R}_y\big(y^\flat, z^\flat, 0\big) - y^\flat \geq Y^\flat$ and $z^\flat \mathcal{R}_z\big(y^\flat, z^\flat, 0\big) - z^\flat \geq Z^\flat$ are at least equal to the minimum catch thresholds $Y^\flat$ and $Z^\flat$, respectively.

3 Application to the anchovy-hake couple in the Peruvian upwelling ecosystem (1971–1981)

We now apply the above robust viability analysis to the Peruvian hake-anchovy fisheries between 1971 and 1981. For this, we extend the model in [4] to the uncertain case. We compute the robust viability kernel numerically, testing different assumptions on the uncertainty sets $\mathbb{S}(t)$ in (3), to appraise the sensitivity of robust viable states to uncertainty scenarios.

3.1 Lotka-Volterra dynamical model with uncertainties

The Peruvian anchovy-hake system is modeled as prey-predator system, where the anchovy growth rate is decreasing in the hake population. We describe this interaction by the following discrete-time

Lotka-Volterra system

$$\begin{cases} y(t+1) &= y(t)\overbrace{\left(\varepsilon_y(t) + R - \frac{R}{\kappa}y(t) - \alpha z(t)\right)}^{\mathcal{R}_y\left(y(t),z(t),\varepsilon_y(t)\right)}\left(1 - v_y(t)\right) \\ z(t+1) &= z(t)\underbrace{\left(\varepsilon_z(t) + L + \beta y(t)\right)}_{\mathcal{R}_z\left(y(t),z(t),\varepsilon_z(t)\right)}\left(1 - v_z(t)\right), \end{cases} \tag{11}$$

where $R > 1$, $0 < L < 1$, $\alpha > 0$, $\beta > 0$ and $\kappa = \frac{R}{R-1}K$, with $K > 0$ the carrying capacity for the prey. The variable y stands for anchovy biomass and z for hake biomass. The purpose of this compact model is not to provide biological "knowledge" on the Peruvian upwelling ecosystem, but rather to capture the essential features of the system in what concerns decision making.

The five parameters of the deterministic version (20) of model (11) have been estimated in [4], based on 11 yearly observations of the Peruvian anchovy-hake biomasses and catches over the time period 1971–1981. Their values are given in Table 1.

Parameters	**Estimates**
R	2.25 $year^{-1}$
L	0.945 $year^{-1}$
κ	67113 10^3 $tons$
K	37285 10^3 $tons$
α	1.220 10^6 $tons^{-1}$
β	4.845 10^{-8} $tons^{-1}$

Table 1: Parameter estimates of the deterministic Lotka-Volterra model (20)

3.2 Choice of uncertainty sets

We are now more specific about the uncertainty sets $\mathbb{S}(t)$ in (3), in which the uncertainties $\varepsilon_y(t)$ and $\varepsilon_z(t)$ in (11) take their values. For the sake of simplicity, we consider stationary uncertainty sets $\mathbb{S} = \mathbb{S}(t)$.

First, we form an uncertainty set $\mathbb{S}_E$ with empirical values. Second, we refine this set. Third, we identify and only consider the worst-case scenarios.

3.2.1 Empirical uncertainties set

Figure 1 represents the observed biomasses of Peruvian anchovy and hake over the years 1971–1981 and the simulated biomasses with the deterministic version of the Lotka-Volterra model (that is, with $\varepsilon_y(t) = 0$ and $\varepsilon_z(t) = 0$ in the dynamical system (1)).

(a) Anchovy (b) Hake

Figure 1: Observed and simulated biomasses over 1971–1981

The time period 1971–1981 is denoted by $t = t_0, \dots, T$, with $t_0 = 0$, and $T = 10$. Let $(\overline{y}(t), \overline{z}(t))_{t=t_0,\dots,T}$ and $(\overline{v}_y(t), \overline{v}_z(t))_{t=t_0,\dots,T-1}$ denote the observed biomass and effort trajectories. We set $\overline{\varepsilon}_y(t)$ and $\overline{\varepsilon}_z(t)$ implicitly defined by

$$\begin{cases} \overline{y}(t+1) &= \overline{y}(t)\big(\overline{\varepsilon}_y(t) + R - \frac{R}{\kappa}\overline{y}(t) - \alpha\overline{z}(t)\big)\big(1 - \overline{v}_y(t)\big) \\ \overline{z}(t+1) &= \overline{z}(t)\big(\overline{\varepsilon}_z(t) + L + \beta\overline{y}(t)\big)\big(1 - \overline{v}_z(t)\big), \end{cases} \tag{12}$$

so that (11) is satisfied. Figure 2 displays the points $\{(\overline{\varepsilon}_y(t), \overline{\varepsilon}_z(t)) | t = t_0, \dots, T-1\}$, (there are 10 points as 1971 observations are used as starting points for simulating biomasses). We denote $\overline{\varepsilon}_y^{min} = \min_{t=t_0,\dots,T-1} \overline{\varepsilon}_y(t) = -0.25$, $\overline{\varepsilon}_y^{max} = \max_{t=t_0,\dots,T-1} \overline{\varepsilon}_y(t) = 1.54$, $\overline{\varepsilon}_z^{min} = \min_{t=t_0,\dots,T-1} \overline{\varepsilon}_z(t) = -0.38$ and $\overline{\varepsilon}_z^{max} = \max_{t=t_0,\dots,T-1} \overline{\varepsilon}_z(t) = 0.088$.

The *empirical uncertainties set*

$$\mathbb{S}_E = \{(\overline{\varepsilon}_y(t), \overline{\varepsilon}_z(t)) | t = t_0, \dots, T-1\} \cup \{(0,0)\} \tag{13}$$

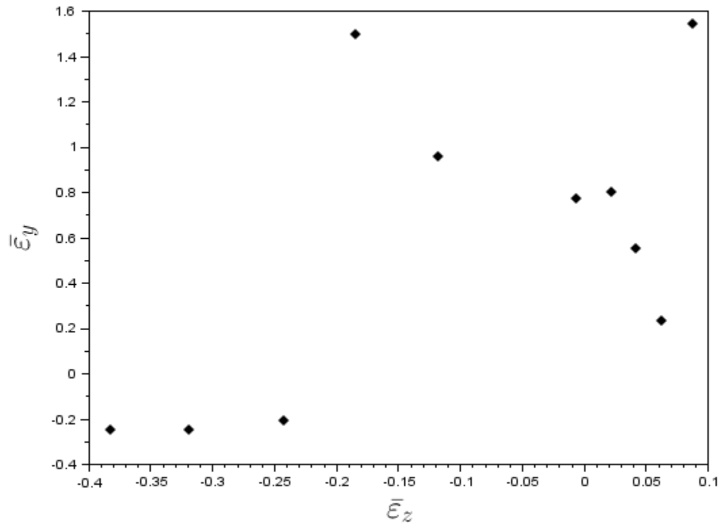

Figure 2: Empirical distribution of $(\overline{\varepsilon}_y(t), \overline{\varepsilon}_z(t))_{t=t_0,\dots,T-1}$ characterized by (12)

is made of the ten empirical uncertainty couples (see diamonds in Figure 2) and the uncertainty couple $(\varepsilon_y, \varepsilon_z) = (0, 0)$ (corresponding to the deterministic case).

3.2.2 Refinement of the empirical uncertainties set

The set $\mathbb{S}_{ER}$ is made of 400 uncertainty couples delineated by a 30×30 grid over the surface $[\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_y^{max}] \times [\overline{\varepsilon}_z^{min}, \overline{\varepsilon}_z^{max}]$, including all the uncertainty couples of $\mathbb{S}_E$ (see the grid in Figure 4).

Since $\{(0,0)\} \subset \mathbb{S}_E \subset \mathbb{S}_{ER}$, the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}_{ER}^R(t_0) \subset \mathbb{V}_E^R(t_0) \subset \mathbb{V}(t_0) \; . \tag{14}$$

Indeed, the larger the uncertainty sets $\mathbb{S}(t)$ in (3), the smaller the robust viability kernel $\mathbb{V}^R(t_0)$, because the constraints are much harder to satisfy, since they have to be respected for all uncertainties. However, we observed that the robust viability kernels $\mathbb{V}_{ER}^R(t_0)$ and $\mathbb{V}_E^R(t_0)$ are practically equal. Our assumption was that, by exposing the ecosystem dynamics to a denser set of scenarios, sensibly fewer initial states should be likely to allow for an effort strategy guaranteeing all sustainability constraints at all times. In fact, whatever the density of the grid that we tested, adding uncertainty couples with values in the rectangle $[\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_y^{max}] \times [\overline{\varepsilon}_z^{min}, \overline{\varepsilon}_z^{max}]$ had no incidence on the robust viability kernel. In fact, the robust viability kernel is sensitive to some extreme points of

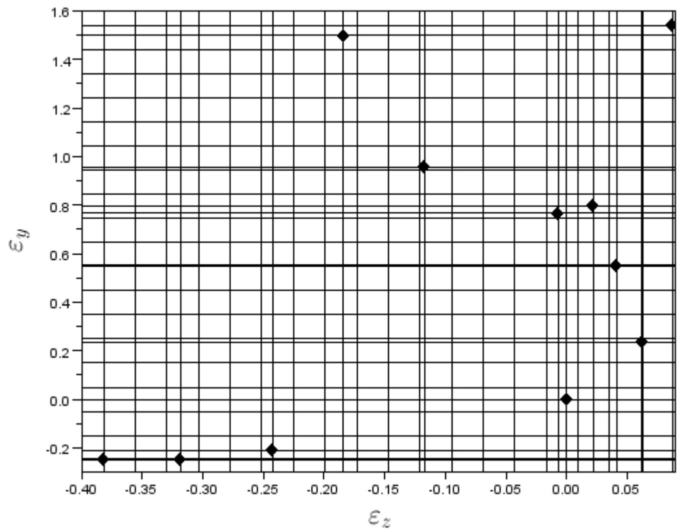

Figure 3: Uncertainty sets $\mathbb{S}_E$ (diamonds) and $\mathbb{S}_{ER}$ (grid)

the uncertainty set. This is why, we now turn to three new uncertainty sets built upon "worst-case uncertainties".

3.2.3 Worst-case uncertainties

Our numerical simulations led us to consider the three following uncertainty sets.

- The set $\mathbb{S}_L$ is composed of uncertainty couples defined by the limit values (i.e. min and max) of the empirical uncertainty, divided by two (see the crosses in Figure 4):

$$\mathbb{S}_L = \{(\frac{\overline{\varepsilon}_y^{min}}{2}, \frac{\overline{\varepsilon}_z^{min}}{2}), (\frac{\overline{\varepsilon}_y^{min}}{2}, \frac{\overline{\varepsilon}_z^{max}}{2})\} \ . \tag{15}$$

- The set $\mathbb{S}_M$ is composed of limit empirical uncertainty couples (see the diamonds in Figure 4):

$$\mathbb{S}_M = \{(\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_z^{min}), (\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_z^{max})\} \ . \tag{16}$$

  The first uncertainty couple $(\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_z^{min})$ corresponds to low growth factor for both species. The second uncertainty couple $(\overline{\varepsilon}_y^{min}, \overline{\varepsilon}_z^{max})$ affects negatively the prey growth and positively the predator growth. The fact that the latter uncertainty couple produces worse adverse ecological and economic consequences is quite intuitive whereas it is less obvious for the first one, given the non-linear relationship linking both species. We observed that considering more extreme empirical uncertainty couples does not affect the robust viability kernel. Keeping only one of the two increases the robust viability kernel. This is why those two couples deserve the label of worst-case uncertainties.

- The uncertainty set $\mathbb{S}_H$ is composed of limit empirical uncertainty couples increased by 10% (see the triangles in Figure 4):

$$\mathbb{S}_H = 1.1 * \mathbb{S}_M \ . \tag{17}$$

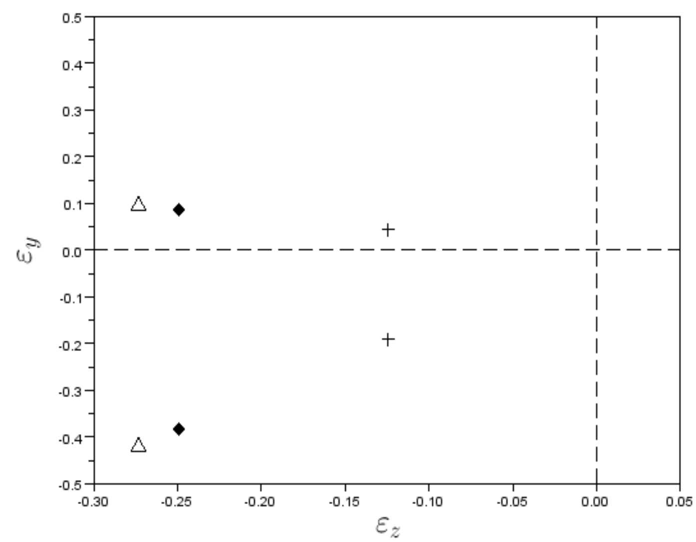

Figure 4: Uncertainty sets $\mathbb{S}_L$ (crosses), $\mathbb{S}_M$ (diamonds) and $\mathbb{S}_H$ (triangles)

Since $\{(0,0)\} \subset \mathbb{S}_L \subset \mathbb{S}_M \subset \mathbb{S}_H$, the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}_H^R(t_0) \subset \mathbb{V}_M^R(t_0) \subset \mathbb{V}_L^R(t_0) \subset \mathbb{V}(t_0) \ . \tag{18}$$

Practically, the robust viability kernels $\mathbb{V}_M^R(t_0)$ and $\mathbb{V}_E^R(t_0)$ are equal.

3.3 Robust and deterministic viability kernels

We consider the minimal biomasses $y^\flat = 7,000,000$ tons and $z^\flat = 200,000$ tons, and minimal catches $Y^\flat = 2,000,000$ tons and $Z^\flat = 5,000$ tons (IMARPE, 2000, 2004). The condition (9) in Proposition 1 is satisfied for the above minimal threshold values and for the Lotka-Volterra model coefficient estimates in Table 1.

Figure 5 displays the three robust viability kernels associated with dynamics (11), constraints (7) and (8), and with the uncertainty sets $\mathbb{S}_L$, $\mathbb{S}_M$ and $\mathbb{S}_H$, respectively. As expected, the set of robust viable states is decreasing with the size of the uncertainty set in (18). The robust kernel is quite sensitive to changes in the worst-case uncertainties. The uncertainty set $\mathbb{S}_H$ has been retained because numerical computation shows that, for larger extreme points, the robust kernel is empty.

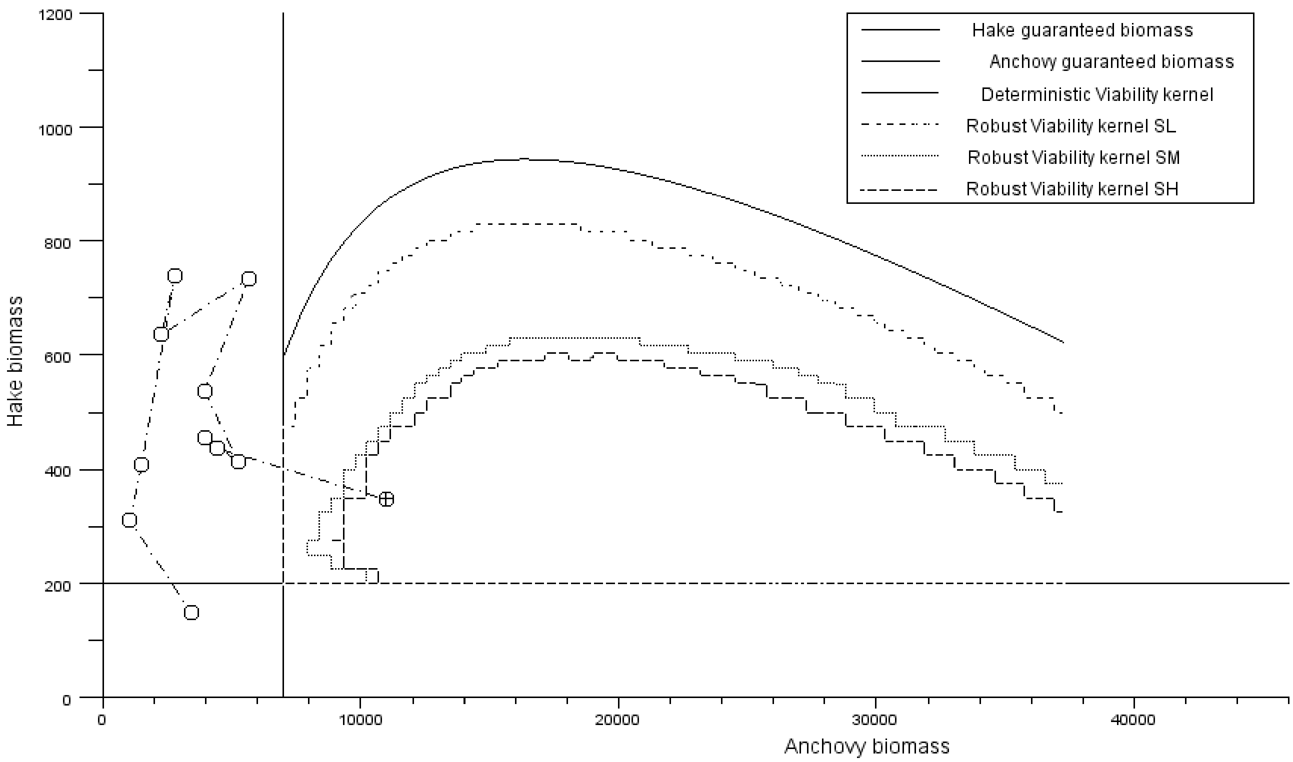

Figure 5: Comparing robust and deterministic viable states

Replacing the growth rates $\mathcal{R}_y$ and $\mathcal{R}_z$ in (10) by their expressions (11) yields the expression of the deterministic viability kernel:

$$\mathbb{V}(t_0) = \left\{ \ (y, z) \mid y \geq y^\flat, z^\flat \leq z \leq \frac{1}{\alpha}[R - \frac{R}{\kappa}y - \frac{y^\flat + Y^\flat}{y}] \ \right\}. \tag{19}$$

This set $\mathbb{V}(t_0)$ is delineated by the outer dashed curve in Figure 5. We observe an important gap between the deterministic kernel and the robust ones. A share of the states identified as viable by the deterministic approach is in fact excluded when introducing uncertainty. Indeed, there exists no effort strategy capable of guaranteeing preservation and production minima for biomass couples standing outside the robust kernels, given the chosen scenarios sets and time horizon. Furthermore, we cannot tell whether the effort strategies advocated by the deterministic approach for an initial biomass couple belonging to the robust kernels may guarantee sustainability objectives over time in presence of uncertainty.

The horizontal and vertical lines represent the minimal biomass safety levels and the circles indicate the biomass observations of the anchovy-hake couple over 1971–1981. Only one circle, marked by a cross, stands within the robust sets of states, corresponding to the initial biomass couple observed in 1971. Starting from that date, there theoretically existed a control strategy providing, for the next 10 years, at least the sustainable yields $Y^\flat$ and $Z^\flat$, and guaranteeing biomasses over the preservation thresholds $y^\flat$, $z^\flat$, whatever the scenario stemming from $\mathbb{S}_H$. In reality, the catches of year 1971 were very high, and the biomass trajectories were well below the biological minimal levels for 14 years.

4 Conclusion

This work is a theoretical and practical contribution to ecosystem sustainable management under uncertainties. The robust viable kernel is an interesting mean to display the impact of uncertainty on the possibility of a sustainable management. Wherever a fishery stands, the set of robust states enables to predict whether target objectives can be guaranteed over a time span despite of uncertainty.

By exposing the ecosystem dynamics to uncertainties, fewer initial states are likely to allow for effort strategies guaranteeing all sustainability constraints at all times, compared to the deterministic framework.

In addition, we have been able to shed light on the uncertainties that really matter for a precautionary approach. Indeed, by computing robust viable kernels, we realized that they were sensitive to few important uncertainties, corresponding to some of the worst-case ones. Identifying them intuitively a priori was not obvious. This may help the decision-maker to focus on those essential scenarios for sustainable management.

In rather common situations where very little is known about uncertainties, the robust framework contents itself of poor assumptions on sets rather than possibly unjustified probabilistic ones. However, we have seen that the robust viability kernel can be empty. To account for less radical analysis, the viability stochastic theory is an alternative approach to address dynamical control problems under uncertainty and constraints. This approach allows for constraints violations with a low probability. Thus, it can be used to affect to each biomass couples taken as potential initial states, the maximum probability with which constraints may be satisfied at all times. This issue is under current investigation.

A The deterministic viability kernel

The *deterministic viability kernel* $\mathbb{V}(t_0)$ is the set of viable states defined as follows. A couple (y_0, z_0) of initial biomasses is said to be a *viable state* if there exist a trajectory of harvesting efforts (controls) $\big(v_y(t), v_z(t)\big) \in [0,1]$, $t = t_0, \dots, T-1$, such that the state path $\{\big(y(t), z(t)\big)\}_{t=t_0,\dots,T}$, and control path $\{\big(v_y(t), v_z(t)\big)\}_{t=t_0,\dots,T-1}$, solution of[1]

$$
\begin{cases}
y(t+1) &= y(t)\overbrace{\Big(R - \frac{R}{\kappa} y(t) - \alpha z(t)\Big)}^{\mathcal{R}_y\big(y(t),z(t)\big)}\big(1 - v_y(t)\big) \\
z(t+1) &= z(t)\underbrace{\big(L + \beta y(t)\big)}_{\mathcal{R}_z\big(y(t),z(t)\big)}\big(1 - v_z(t)\big)
\end{cases}
\tag{20}
$$

starting from $\big(y(t_0), z(t_0)\big) = (y_0, z_0)$ satisfy the following goals:

- preservation (minimal biomass levels)

$$
y(t) \geq y^\flat \ , \quad z(t) \geq z^\flat \ , \quad \forall t = t_0, \dots, T \ , \tag{21}
$$

- and production requirements (minimal catch levels)

$$
v_y(t) y(t) \mathcal{R}_y\big(y(t), z(t)\big) \geq Y^\flat \ , \quad v_z(t) z(t) \mathcal{R}_z\big(y(t), z(t)\big) \geq Z^\flat \ , \quad \forall t = t_0, \dots, T-1 \ . \tag{22}
$$

We now turn to the proof of Proposition 1.

Proof. Consider $y^\flat \geq 0$, $z^\flat \geq 0$, $Y^\flat \geq 0$, $Z^\flat \geq 0$. We set

$$
\mathbb{V}_0 = \Big\{ (y,z) \in \mathbb{R}_+^2 \, \Big| \, y \geq y^\flat, z \geq z^\flat \Big\}
$$

and we define a sequence $(\mathbb{V}_k)_{k \in \mathbb{N}}$ inductively by

$$
\begin{aligned}
\mathbb{V}_{k+1} = \{ \ (y,z) \in \mathbb{V}_k \mid \exists (v_y, v_z) \in [0,1] \ \text{such that} \ & y v_y \mathcal{R}_y(y,z) \geq Y^\flat, z v_z \mathcal{R}_z(y,z) \geq Z^\flat, \\
\text{and} \ & y' = y \mathcal{R}_y(y,z)(1 - v_y), \ z' = z \mathcal{R}_z(y,z)(1 - v_z), \\
& \text{are such that} \ (y', z') \in \mathbb{V}_k \} \ .
\end{aligned}
$$

[1]Equation (20) is (11) with the uncertainty couple $(\varepsilon_y, \varepsilon_z) = (0,0)$ (corresponding to the deterministic case).

For $k=0$, we obtain

$$\begin{aligned}
\mathbb{V}_1 &= \left\{ (y,z) \left| \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y,v_z)\in[0,1], \\ v_y y\mathcal{R}_y(y,z) \geq Y^\flat, v_z z\mathcal{R}_z(y,z) \geq Z^\flat, \\ y\mathcal{R}_y(y,z)(1-v_y) \geq y^\flat, z\mathcal{R}_z(y,z)(1-v_z) \geq z^\flat \end{array} \right. \right\} \\
&= \left\{ (y,z) \left| \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ for which there exist } (v_y,v_z) \text{ such that} \\ \frac{Y^\flat}{y\mathcal{R}_y(y,z)} \leq v_y \leq \frac{y\mathcal{R}_y(y,z)-y^\flat}{y\mathcal{R}_y(y,z)} \quad \text{and} \quad 0 \leq v_y \leq 1, \\ \frac{Z^\flat}{z\mathcal{R}_z(y,z)} \leq v_z \leq \frac{z\mathcal{R}_z(y,z)-z^\flat}{z\mathcal{R}_z(y,z)} \quad \text{and} \quad 0 \leq v_z \leq 1 \end{array} \right. \right\} \\
&= \left\{ (y,z) \left| \begin{array}{l} y \geq y^\flat, z \geq z^\flat, \\ \sup\{0, \frac{Y^\flat}{y\mathcal{R}_y(y,z)}\} \leq \inf\{1, 1-\frac{y^\flat}{y\mathcal{R}_y(y,z)}\} \\ \sup\{0, \frac{Z^\flat}{z\mathcal{R}_z(y,z)}\} \leq \inf\{1, 1-\frac{z^\flat}{z\mathcal{R}_z(y,z)}\} \end{array} \right. \right\} \\
&= \left\{ (y,z) \left| y \geq y^\flat, z \geq z^\flat, \frac{Y^\flat}{y\mathcal{R}_y(y,z)} \leq \frac{y\mathcal{R}_y(y,z)-y^\flat}{y\mathcal{R}_y(y,z)}, \frac{Z^\flat}{z\mathcal{R}_z(y,z)} \leq \frac{z\mathcal{R}_z(y,z)-z^\flat}{z\mathcal{R}_z(y,z)} \right. \right\} \\
&= \left\{ (y,z) \left| y \geq y^\flat, z \geq z^\flat, Y^\flat \leq y\mathcal{R}_y(y,z)-y^\flat, Z^\flat \leq z\mathcal{R}_z(y,z)-z^\flat \right. \right\} .
\end{aligned}$$

Then, for $k=1$, we obtain

$$\begin{aligned}
\mathbb{V}_2 &= \left\{ (y,z) \left| \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y,v_z)\in[0,1], \\ v_y y\mathcal{R}_y(y,z) \geq Y^\flat, \; v_z z\mathcal{R}_z(y,z) \geq Z^\flat \\ \text{and such that } (y',z') \in \mathbb{V}_k \\ \text{where } y' = y\mathcal{R}_y(y,z)(1-v_y), \; z' = z\mathcal{R}_z(y,z)(1-v_z) \end{array} \right. \right\} \\
&= \left\{ (y,z) \left| \begin{array}{l} y \geq y^\flat, z \geq z^\flat \text{ and, for some } (v_y,v_z)\in[0,1], \\ v_y y\mathcal{R}_y(y,z) \geq Y^\flat, \; v_z z\mathcal{R}_z(y,z) \geq Z^\flat, \; y' \geq y^\flat, \; z' \geq z^\flat, \\ Y^\flat \leq y'\mathcal{R}_y(y',z')-y^\flat, \; Z^\flat \leq z'\mathcal{R}_z(y',z')-z^\flat \\ \text{where } y' = y\mathcal{R}_y(y,z)(1-v_y), \; z' = z\mathcal{R}_z(y,z)(1-v_z) \end{array} \right. \right\} .
\end{aligned}$$

We now make use of the property (see [4]) that, when the decreasing sequence $(\mathbb{V}_k)_{k\in\mathbb{N}}$ is stationary, its limit is the viability kernel $\mathbb{V}(t_0)$. Hence, it suffices to show that $\mathbb{V}_1 \subset \mathbb{V}_2$ to obtain that $\mathbb{V}(t_0) = \mathbb{V}_1$.

Let $(y,z) \in \mathbb{V}_1$. We have that

$$y \geq y^\flat, \quad z \geq z^\flat \text{ and } y\mathcal{R}_y(y,z) - y^\flat \geq Y^\flat, \quad z\mathcal{R}_z(y,z) - z^\flat \geq Z^\flat .$$

Let us set $\hat{v}_y = \frac{y\mathcal{R}_y(y,z)-y^\flat}{y\mathcal{R}_y(y,z)}$, which has the property that $y' = y\mathcal{R}_y(y,z)(1-\hat{v}_y) = y^\flat$. We prove that $\hat{v}_y \in [0,1]$. Indeed, on the one hand, we have that $\hat{v}_y \leq 1$ since $1-\hat{v}_y = y^\flat / y\mathcal{R}_y(y,z)$, where $y^\flat \geq 0$. On the other hand, since $y\mathcal{R}_y(y,z) - y^\flat \geq Y^\flat \geq 0$, we deduce that $\hat{v}_y \geq 0$. The same holds true for $\hat{v}_z$ and $z' = z\mathcal{R}_z(y,z)(1-\hat{v}_z) = z^\flat$. By (9), we deduce that

$$y'\mathcal{R}_y(y',z') - y^\flat = y^\flat\mathcal{R}_y(y^\flat,z^\flat) - y^\flat \geq Y^\flat \text{ and } z'\mathcal{R}_z(y',z') - z^\flat = z^\flat\mathcal{R}_z(y^\flat,z^\flat) - z^\flat \geq Z^\flat .$$

The inclusion $\mathbb{V}_1 \subset \mathbb{V}_2$ follows, hence $\mathbb{V}(t_0) = \mathbb{V}_1$, and (10) holds true. □

The *viable controls* attached to a given viable state $(y, z) \in \mathbb{V}(t_0)$ are the admissible controls such that the image by the dynamics is in $\mathbb{V}(t_0)$.

Corollary 2 *Suppose that the assumptions of Proposition 1 are satisfied. The set of viable controls is given by*

$$\mathbb{U}_{\mathbb{V}(t_0)}(y,z) = \left\{ (v_y, v_z) \in [0,1]^2 \;\middle|\; \begin{array}{l} \frac{y\mathcal{R}_y(y,z)-y^\flat}{y\mathcal{R}_y(y,z)} \geq v_y \geq \frac{Y^\flat}{y\mathcal{R}_y(y,z)}, \quad \frac{z\mathcal{R}_z(y,z)-z^\flat}{z\mathcal{R}_z(y,z)} \geq v_z \geq \frac{Z^\flat}{z\mathcal{R}_z(y,z)}, \\ y'\mathcal{R}_y(y',z') - y^\flat \geq Y^\flat, \;\; z'\mathcal{R}_z(y',z') - z^\flat \geq Z^\flat \end{array} \right\},$$

where $y' = y\mathcal{R}_y(y,z)(1-v_y),\;\; z' = z\mathcal{R}_z(y,z)(1-v_z)$.

B Numerical computation of robust viability kernels

The dynamic programming equation associated with dynamics (1) preservation (7) and production (8) minimal thresholds is given by [2]

$$\left\{ \begin{array}{rl} V_T(y,z) = & \mathbf{1}_{\mathbb{A}}(y,z), \\ V_t(y,z) = & \mathbf{1}_{\mathbb{A}}(y,z) \sup\limits_{(v_y,v_z)\in[0,1]^2} \inf\limits_{(\varepsilon_y,\varepsilon_z)\in\mathbb{S}(t)} \left[\mathbf{1}_{\mathbb{B}(y,z,\varepsilon_y,\varepsilon_z)}(v_y,v_z) V_{t+1}(G(y,z,v_y,v_z,\varepsilon_y,\varepsilon_z)) \right], \end{array} \right. \tag{23}$$

where the function G denotes the dynamics (1)

$$G(y,z,v_y,v_z,\varepsilon_y,\varepsilon_z) = \left\{ \begin{array}{l} y\mathcal{R}_y\big(y,z,\varepsilon_y\big)\big(1-v_y\big)\;, \\ z\mathcal{R}_z\big(y,z,\varepsilon_z\big)\big(1-v_z\big)\;, \end{array} \right.$$

where $\mathbb{A}$ stands for the subset of biomass satisfying conservation objectives

$$\mathbb{A} = \{(y,z) \mid y \geq y^\flat \;, z \geq z^\flat\} = [y^\flat, +\infty[\times [z^\flat, +\infty[\;,$$

and where $\mathbb{B}$ stands for the subset of catches satisfying minimal production requirements

$$\mathbb{B}(y,z,\varepsilon_y,\varepsilon_z) = \{(v_y,v_z) \mid v_y y\mathcal{R}_y(y,z,\varepsilon_y) \geq Y^\flat,\; v_z z\mathcal{R}_z(y,z,\varepsilon_z) \geq Z^\flat\}\;.$$

The notation $\mathbf{1}_{\mathbb{A}}(y,z)$ is the indicator function of the set $\mathbb{A}$: it takes the value 1 when $(y,z) \in \mathbb{A}$ and 0 else. The same holds for $\mathbf{1}_{\mathbb{B}(y,z,\varepsilon_y,\varepsilon_z)}(v_y,v_z)$. It turns out that the robust viability value function V_{t_0} at time t is the indicator function of the robust viability kernel $\mathbb{V}^R(t_0)$.

[2]Simple extension of the results in [3] and [5]

Now, we expose how we proceed to solve for the robust viability kernel numerically. We discretize biomass, harvesting effort and uncertainty values. A top loop for time steps embraces two nested loops for state variables y and z, respectively. Next, loops over uncertainties nested in loops over harvesting efforts allow us to obtain the set of images associated to a biomass couple (some of these steps are actually done through matrix computing). Images for target constraints that are not satisfied are set equal to zero. We then project these images on the grid displayed by the value function of the previous period through linear interpolation. At given effort couple, we retain the minimum value obtained over all uncertainty couples. Then, we retain the highest value produced by a effort couple among all tested. It is this value that is multiplied to the value function of the current time period, at the location of the biomass couple at stake. The robust viability kernel is defined by the set of grid points equal to 1. This implies that biomass couples for which, at a date t, all images do not fall between four 1 in the interpolation are excluded from the robust viability kernel.

Acknowledgments. The authors thank Luc Doyen (CNRS, CERSP), Vincent Martinez (INRA) and the organizers of the European Association of Environment and Resources Economics (EAERE) 20th Annual Conference (26th-29th June 2013), David Alary, Carolyne Lamy, Joël Echevarria and François Salanié. Olivier Thébaud and the organizers of the International Institution for Fisheries Economics and Trade (IIFET) 2012 Conference (16th-20th July 2012), for their comments.

References

[1] J.-P. Aubin. *Viability Theory.* Birkhäuser, Boston, 1991. 542 pp.

[2] C. Béné, L. Doyen, and D. Gabay. A viability analysis for a bio-economic model. *Ecological Economics*, 36:385–396, 2001.

[3] M. De Lara and L. Doyen. *Sustainable Management of Natural Resources. Mathematical Models and Methods.* Springer-Verlag, Berlin, 2008.

[4] M. De Lara, E. Ocaña Anaya, and J. T. Ricardo Oliveros-Ramos. Ecosystem viable yields. *Environmental Modeling & Assessment*, 17(6):565–575, 2012.

[5] L. Doyen and M. De Lara. Stochastic viability and dynamic programming. *Systems and Control Letters*, 59(10):629–634, October 2010.

[6] S. Garcia, A. Zerbi, C. Aliaume, T. D. Chi, and G. Lasserre. The ecosystem approach to fisheries. Issues, terminology, principles, institutional foundations, implementation and outlook. *FAO Fisheries Technical Paper*, 443(71), 2003.

[7] R. Hilborn and C. F. Walters. *Quantitative fisheries stock assessment. Choice, dynamics and uncertainty.* Chapman and Hall, New York, London, 1992. 570 pp.

[8] T. Lauck, C. W. Clark, M. Mangel, and G. R. Munro. Implementing the precautionary principle in fisheries management through marine reserves. *Ecological Applications*, 81, Special Issue:72–78, 1998.