Subsistence and substitutability in consumer preferences

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Abstract: We propose a representation of individual preferences with a subsistence requirement in consumption in an otherwise standard constant-elasticity-of-substitution (CES) utility specification. We study how substitutability between the subsistence good, e.g. environmental services, and another good depends on the subsistence requirement and the level of consumption. We find that the Hicksian elasticity of substitution is zero below the subsistence income level, strictly monotonically increases with income above this threshold, and approaches the standard non-subsistence CES value as income goes to infinity. Whether the two goods are market substitutes depends on, besides the CES-substitutability parameter, the level of income and the subsistence requirement. Our key result that with a subsistence requirement substitutability between different consumption goods is non-constant but increases with individual income has important implications for growth, development and in particular environmental policy.

JEL-Classification: D11, I31, O12, Q01, Q56

Keywords: elasticity of substitution, Stone-Geary function, subsistence in consumption, substitutability, sustainability

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1 Introduction

We develop a general and formal conceptual framework to examine substitutability between two goods – think of environmental services and manufactured goods – in the presence of a subsistence requirement in the consumption of one of these goods.

The economic study of subsistence requirements in consumption dates back to Klein and Rubin (1947/48), Samuelson (1947/48), Geary (1950) and Stone (1954). More recently, subsistence requirements have been shown to be relevant, inter alia, in the growth, development and environmental economics literature (e.g. Garner 2010, Heal 2009a,b, King and Rebelo 1993, Kraay and Raddatz 2007, Matsuo and Tomoda 2012, Pezzey and Anderies 2003, Ravn et al. 2008, Steger 2000, Strulik 2010). Despite extensive discussion, there is no consensus yet on the appropriate definition of subsistence (Alkire 2002, Heal 2009a,b, Max-Neef et al. 1991, Rauschmayer et al. 2011, Sharif 1986).[1]

Here, we understand subsistence to capture more than mere survival, but to encompass a homogeneous composite good to which an individual attaches absolute priority before considering trade-offs with other goods. This certainly includes the consumption of a certain amount of food, water and air, but may also include immaterial components.

Substitutability between different kinds of capital stocks as well as consumption goods plays a crucial role in environmental and resource economics (Markandya and Pedroso-Galinato 2007, Neumayer 2010, Traeger 2011). While subsistence requirements have not been a focus in the study of substitutability, Hicks and Allen (1934b: 199) already had it in mind when formulating the elasticity of substitution (EoS) in consumption. Recently, Heal (2009a,b) proposed to examine substitutability between an environmental and a manufactured good in the presence of a subsistence requirement in terms of the environmental good by extending a constant-elasticity-of-substitution (CES) utility function through including a survival threshold. Without providing a formal examination, Heal (2009a: 279) conjectures that "the elasticity of substitution is not constant but depends on and increases with welfare levels."

[1]For instance, while Sharif (1986) argues that subsistence includes both physical and mental health, Heal (2009a: 279) conceptualizes a survival threshold in terms of "water, air, and basic foodstuffs".

In this paper, we generalize and formalize Heal's (2009a,b) proposal by incorporating a subsistence requirement in an otherwise standard CES utility function. We find that the Hicksian EoS is non-constant and, above the subsistence threshold, strictly monotonically increases with income. However, whether the goods are market substitutes does not only depend on the Hicksian EoS but also on the level of income and the subsistence requirement.

2 Model and definitions

There are two composite goods, S with a subsistence requirement $\overline{S}$ (think of environmental services), and X (think of manufactured consumption goods). The consumer's preferences are represented by a utility function

$$U(S,X) = \begin{cases} U_l(S) & \text{for } S \leq \overline{S} \\ U_h(S,X) & \text{else} \end{cases} \tag{1}$$

where $U_l(S)$ strictly monotonically increases with S and $U_h(S,X)$ is twice continuously differentiable, strictly monotonic in both arguments and strictly quasi-concave. Furthermore, the individual always prefers to be in the domain where the subsistence requirement is satisfied, i.e.

$$\inf_{S>\overline{S},X\geq 0} U_h(S,X) > \sup_{0\leq S\leq \overline{S}} U_l(S) \ . \tag{2}$$

This represents the idea of a subsistence requirement in consumption: for $S \leq \overline{S}$ the individual is not willing to consider trade-offs between the subsistence good S and the other good X, but lexicographically prefers more of S. Only if subsistence consumption is satisfied, i.e. for $S > \overline{S}$, is she willing to consider trade-offs between S (insofar it exceeds $\overline{S}$) and X. Overall, she prefers to be in the domain where the subsistence requirement is satisfied.[2]

[2] Applying critical-level utilitarianism (Blackorby et al. 1995) for social decision making with endogenous population size, the utility level $\inf_{S>\overline{S},X\geq 0} U_h(S,X)$ is a natural candidate for the critical utility level – assuming that utility is cardinally measurable and unit-comparable across individuals, and with adequately defined level of consumption for the manufactured good, X.

As an interesting and handy specification of $U_h(\cdot,\cdot)$ we suggest, following Heal's (2009a,b) idea, a generalized modification of the Stone-Geary function (Geary 1950, Stone 1954) on the one hand, and the CES-function (Solow 1956, Arrow et al. 1961) on the other, which contains these two functions as special cases:[3]

$$U_h(S,X) = \left[\alpha\left(S-\overline{S}\right)^{\theta} + (1-\alpha)X^{\theta}\right]^{1/\theta} \quad \text{with} \quad -\infty < \theta \leq +1,\ 0 < \alpha < 1\ . \tag{3}$$

For $\overline{S} = 0$, i.e. without subsistence requirement, this function reduces to the usual CES utility function which contains as special case perfect substitutes ($\theta = +1$), Cobb-Douglas ($\theta = 0$) and perfect complements ($\theta = -\infty$).

As a measure of substitutability between the two goods, we use the EoS introduced by Hicks (1932[1963]), Robinson (1933) and Hicks and Allen (1934).[4]

Definition 1

The *elasticity of substitution* is given by:

$$\sigma(S,X) := \frac{MRS}{X/S}\,\frac{d\,(X/S)}{d\,MRS}\ , \tag{4}$$

where

$$MRS := -\left.\frac{d\,X}{d\,S}\right|_{U(S,X)=\text{const.}} = \frac{\dfrac{\partial\,U(S,X)}{\partial\,S}}{\dfrac{\partial\,U(S,X)}{\partial\,X}}\ . \tag{5}$$

The Hicksian EoS measures substitutability between two goods along an indifference curve, as the elasticity of the ratio of the amounts of the two goods in a given allocation with respect to the marginal rate of substitution (MRS) between the two in that allocation. This is the most basic and a rather general measure of substitutability. In contrast to some more sophisticated measures it does not rely on further assumptions

[3] Although the CES function has originally been proposed as a production function, it is widely used as a utility function since Armington (1969). Note also that specification (3) is itself a special case of the 'affinely homothetic' S-branch utility tree (Brown and Heien 1972, Blackorby et al. 1978).

[4] Hicks (1932[1963]) and Robinson (1933) independently introduced the elasticity of substitution between two inputs to production, which has then been adapted to consumption goods by Hicks and Allen (1934a,b). For a generalization to more than two goods, see Blackorby and Russel (1989).

on individual behavior or institutional context, but characterizes individual preferences only (Bertoletti 2005, Frondel 2011, Stern 2011).

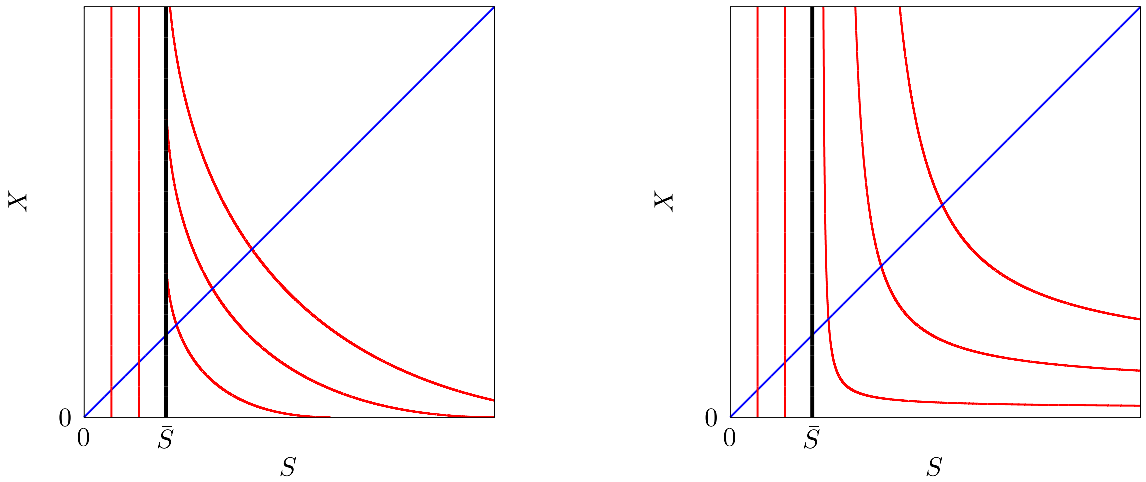

Figure 1: Sets of indifference curves for $\theta = 0.5$ (left), and $\theta = -0.5$ (right). The blue 45-degree line highlights that preferences are non-homothetic.

For utility function (1), the MRS (5) is not defined in the domain $S \leq \overline{S}$. As the individual is not willing to trade off S for X when $S \leq \overline{S}$, we plausibly extend Definition 1 by defining σ to be equal to zero for all $S \leq \overline{S}$.

Following Hicks (1932 [1936: 296]), we define the EoS for the affinely homothetic utility function (3) with respect to the true origin ($S = 0, X = 0$; see Figure 1. This is in contrast to e.g. Brown and Heien (1972) or Beckman and Smith (1993), who define the EoS with respect to the subsistence requirement bundle ($S = \overline{S}, X = 0$), thus confining themselves to a standard CES setting.

3 Results

For $\overline{S} = 0$, i.e. without subsistence requirement, utility function (1) with specification (3) reduces to the usual CES utility function with

$$\sigma(S, X) = \frac{1}{1-\theta} = \text{const.} \tag{6}$$

The parameter θ completely determines the EoS, with the special cases of perfect substitutes ($\theta = +1$), Cobb-Douglas ($\theta = 0$) and perfect complements ($\theta = -\infty$). If there is a subsistence requirement, one obtains:

Proposition 1

For $\overline{S} > 0$, the EoS of utility function (1) with specification (3) is given by

$$\sigma(S, X) = \begin{cases} 0 & \text{for } S \leq \overline{S} \\ \frac{1}{1-\theta}\left[1 - \frac{(1-\alpha)\frac{\overline{S}}{S}}{\alpha\left[\frac{S-\bar{S}}{X}\right]^{\theta} + (1-\alpha)}\right] & \text{else.} \end{cases} \tag{7}$$

Proof. See Appendix A. □

By (extended) Definition 1, the EoS is zero as long as the subsistence requirement $\overline{S}$ is not yet satisfied. In the domain where subsistence consumption is satisfied, the value of σ is determined by the parameter value of θ and the amounts consumed of both goods (S, X), and the subsistence requirement $\overline{S}$.

Proposition 2

For $S > \overline{S}$ and $X > 0$, $\sigma(S, X)$ (Equation 7) has the following properties:

$$0 < \sigma(S, X) < \frac{1}{1-\theta} \quad \text{for all } (S > \overline{S}, X > 0) \tag{8}$$

$$\frac{d\,\sigma(S, X)}{d\,S} > 0 \quad \text{for } \theta \geq 0 \tag{9}$$

$$\sigma(S, X) = \begin{Bmatrix} 0 \\ \alpha \\ \frac{1}{1-\theta} \end{Bmatrix} \quad \text{for } S \to \overline{S} \quad \text{and} \quad \theta \gtreqless 0 \tag{10}$$

$$\sigma(S,X) \rightarrow \frac{1}{1-\theta} \quad \text{for } S \rightarrow \infty \tag{11}$$

$$\frac{d\,\sigma(S,X)}{d\,\overline{S}} < 0 \quad \text{for } \theta \geq 0 \tag{12}$$

$$\sigma(S,X) \rightarrow \frac{1}{1-\theta} \quad \text{for } \overline{S} \rightarrow 0 \tag{13}$$

$$\sigma(S,X) \rightarrow \infty \quad \text{for } \theta \rightarrow 1 \tag{14}$$

Proof. See Appendix B. □

The EoS is positive, but strictly smaller than the standard non-subsistence CES value (Property 8). That is, the subsistence requirement shifts the relationship between the two goods towards complementarity.

For non-negative values of θ the EoS in any given allocation is the greater, the higher the consumption of S (Property 9). As the consumption of S approaches the subsistence requirement from above, the EoS approaches zero for $\theta > 0$, is equal to the share α of S for $\theta = 0$, and approaches the CES value for $\theta < 0$ (Property 10). As consumption of the subsistence good goes to infinity, the EoS approaches the CES value (Property 11).

For non-negative values of θ the EoS is the smaller, the higher the subsistence requirement (Property 12). In the limit case where no subsistence requirement exists, the EoS approaches the CES value (Property 13). When θ approaches unity, the EoS becomes infinite, irrespectively of the level of consumption of S or X (Property 14).

Figure 2 shows $\sigma(S,X)$ (Equation 7) as a function of S for different signs of θ. For $\theta > 0$ (Figure 2 left), a subsistence good that is a substitute to the other good when available in large quantity (high S), can change to become a complement as its availability is reduced (low S). In contrast, for $\theta < 0$ (Figure 2 right) the subsistence good is a complement to the other good at all levels of S.

Having analyzed the elasticity of substitution in pure preference space, we now add institutional context to the analysis by invoking a market setting, i.e. given income m and prices p_S and p_X. The utility-maximizing allocation of the two goods under the budget constraint is then given by the respective Marshallian demand functions $S^\star(m, p_S, p_X)$ and $X^\star(m, p_S, p_X)$.

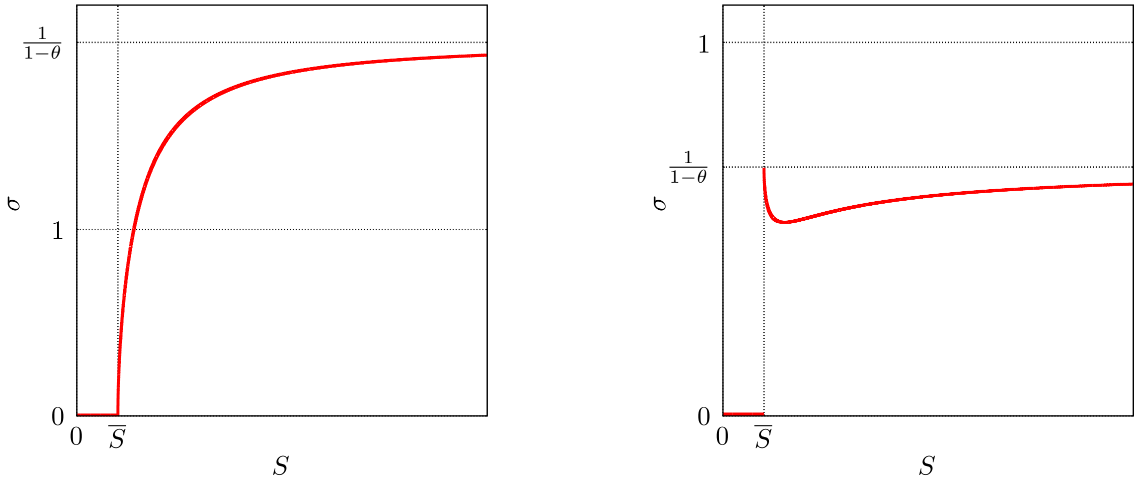

Figure 2: Elasticity of substitution $\sigma(S, X)$ (Equation 7) as a function of consumption of the subsistence good S for $\theta = 0.5$ (left) and $\theta = -0.5$ (right).

Proposition 3

The EoS in the utility-optimal allocation, $\sigma^\star := \sigma(X^\star, S^\star)$, has the following properties:

$$\sigma^\star = 0 \quad \text{for} \quad m \leq p_S \overline{S} \ . \tag{15}$$

For $m > p_S\overline{S}$, the following holds:

$$0 \quad < \quad \sigma^\star \quad < \quad \frac{1}{1-\theta} \tag{16}$$

$$\frac{d\,\sigma^\star}{d\,m} \quad > \quad 0 \tag{17}$$

$$\sigma^\star \quad \to \quad \frac{\alpha}{\alpha + (1-\alpha)\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{\theta}{\theta-1}}} \frac{1}{1-\theta} < \frac{1}{1-\theta} \quad \text{for} \quad m \to p_S\,\overline{S} \tag{18}$$

$$\sigma^\star \quad \to \quad \frac{1}{1-\theta} \quad \text{for} \quad m \to \infty \tag{19}$$

$$\frac{d\,\sigma^\star}{d\,\overline{S}} \quad < \quad 0 \tag{20}$$

For $\theta > 0$, there is a threshold value of income

$$m^P = p_S \, \overline{S} \, \frac{1-\theta}{\theta} \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\theta}} \left(\frac{p_S}{p_X}\right)^{\frac{\theta}{1-\theta}} \tag{21}$$

such that

$$\sigma^\star \lesseqqgtr 1 \quad \text{for} \quad m \lesseqqgtr m^P. \tag{22}$$

Proof. See Appendix C. □

Property (15) is by (extended) Definition 1: the EoS between the two goods in the optimal allocation is zero as long as income is not high enough to afford a consumption level satisfying the subsistence requirement.

Properties (16) – (20) correspond to the statements of Proposition 2 about $\sigma(S, X)$ but are considerably stronger, as they now hold for all values of the CES-parameter θ. In particular, for $m > p_S\overline{S}$ the EoS in the optimal allocation strictly monotonically increases with income (Property 17) and decreases with the subsistence requirement (Property 20).

Again, the EoS approaches the standard CES value as income goes to infinity (Property 19). For finite income, the EoS between the two goods is strictly lower in the presence of a subsistence requirement compared to the CES case (Property 16).

For $\theta > 0$, there is a threshold level of income m^P (Equation 21), so that the two goods are substitutes (complements) in terms of the Hicksian EoS for incomes above (below) m^P (Property 22). This threshold increases with the subsistence requirement $\overline{S}$ and the price of the subsistence good p_S, it decreases with the price of the other good p_X.

Finally, we analyze whether the two goods are market substitutes (complements) in the sense that if the price of one good increases the demand for the other goods increases (decreases):

Proposition 4

For $m > p_S\overline{S}$, the cross-price effects on the Marshallian demand of the two goods are as follows:

$$\frac{d\,S^\star}{d\,p_X} \gtreqless 0 \qquad \text{for} \quad \theta \gtreqless 0 \;, \tag{23}$$

$$\frac{d\,X^\star}{d\,p_S} < 0 \qquad \text{for} \quad \theta \leq 0 \;. \tag{24}$$

For $\theta > 0$, there exists a threshold value of income

$$m^M = \frac{p_S\,\bar{S}}{\theta} + m^P \tag{25}$$

such that

$$\frac{d\,X^\star}{d\,p_S} \lesseqgtr 0 \quad \text{for} \quad m \lesseqgtr m^M. \tag{26}$$

Proof. See Appendix D. □

Whether S is a market-substitute for X only depends on the sign of the CES parameter θ (Result 23). The effect of an increase of p_S on the demand for X is more nuanced: X is a market-complement for S if $\theta \leq 0$ (Result 24). For $\theta > 0$, whether X is a market-substitute for S depends on all model parameters,[5] and in particular on income: X is a substitute for S only for sufficiently high income, and a complement otherwise (Result 26).

The threshold value of income m^M (Equation 25) that determines market substitutability is a simple additive extension to the income threshold that determines the Hicksian EoS (Equation 21). As $m^P > 0$ and $0 < \theta < 1$, one has that $m^M > p_S\overline{S}$. m^M increases with the subsistence requirement $\overline{S}$ and the price of the subsistence good p_S, it decreases with the price of the other good p_X. The intuition behind Result (25) is that as the price of the subsistence good increases, it requires higher income to meet the subsistence requirement, thus also shifting the market-substitutability relationship towards complementarity (see Figure 3).

[5]Except for $\theta \to 1$, where the two are perfect market-substitutes irrespective of any other parameter values.

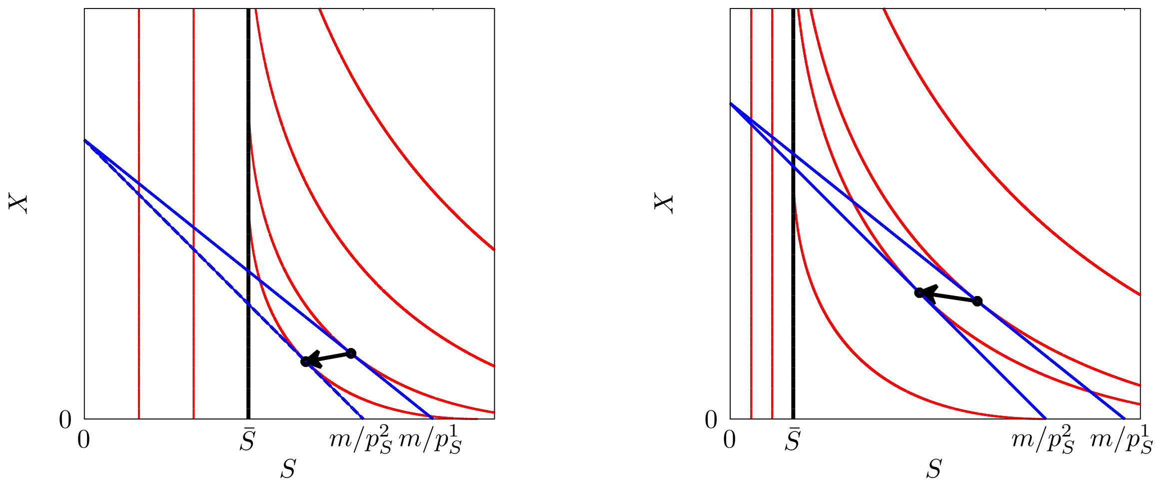

Figure 3: Optimal consumption of S and X for low income (left, $m = 1.7$) and high income (right, $m = 5$) in the case $\theta = 0.5$ and $\overline{S} = 1$. Indifference curves are depicted in red, budget constraints in blue for $p_X = 1$ and two values of p_S, $p_S^1 = 0.8$ and $p_S^2 = 1$. At low income (left), an increase of p_S (from p_S^1 to p_S^2) decreases demand for X; at high income (right), the same increase of p_S increases demand for X.

4 Conclusions

We have proposed a formal description of individual preferences with a subsistence requirement in consumption in an otherwise standard CES utility specification. We have studied how substitutability between the subsistence good and another good depends on the subsistence requirement and the level of consumption of the two goods.

We find that (i) a subsistence requirement shifts the substitutability relationship between goods towards complementarity; (ii) the Hicksian EoS is equivalent to the non-subsistence CES value only if the subsistence good or income is available in an infinite amount; (iii) above the subsistence threshold, the EoS strictly monotonically increases with income; (iv) whether the two goods are market substitutes or complements depends on, besides the CES-substitutability parameter, the level of income and the subsistence requirement. While our analysis is set in the framework of a specific and relevant functional form – a simple extension of an otherwise standard CES utility function – our main results remain valid under more general preference specifications.

Our key result that with a subsistence requirement, substitutability between different consumption goods is non-constant but increases with individual income, has important implications for growth, development and environmental policy. These need to be explored by further research, and we can think of several fruitful areas:

First, the role of income distribution for the Pareto-efficiency of market equilibrium allocations needs to be reassessed for preferences characterized by a subsistence requirement. One could examine a two-household model in which one household may have an insufficient budget to meet the subsistence requirement. This may lead, inter alia, to poverty traps in subsistence economies.

Second, since substitutability between manufactured goods and environmental services is a key issue in the appraisal of climate policies (Heal 2009a,b, Sterner and Persson 2008), the application of our model will have directly relevant implications for the optimal management of climate change.

Third, the standard result of growth and resource economics by Solow (1974) that a constant consumption path forever is feasible even if production essentially depends on a non-renewable resource needs to be qualified: if the Solowesque constant consumption level is below the subsistence threshold, one would not like to think of that solution as "sustainable".

Fourth, the explicit discussion of environmental services that are necessary to meet human subsistence needs is related to and may contribute to the discussion of critical natural capital and safe minimum standards (Ciriacy-Wantrup 1952, Bishop 1978, Ekins 2003). Moreover, our results are analogously relevant for substitutability in production, when some part of natural capital is 'critical'.

Fifth, in the discussion of sustainable development, the distinction between weak and strong sustainability is important and vindicated on the grounds of different degrees of substitutability between human-made and natural capital and the respective services (Neumayer 2010, Trager 2011). With our preference-model in a general-equilibrium setting, the distinction becomes endogenous, as the elasticity of substitution depends on income.

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Appendix

A Proof of Proposition 1

With utility function (3), the marginal rate of substitution for $S > \overline{S}$ (Equation5) is

$$MRS = \frac{\frac{\partial U_h(S,X)}{\partial S}}{\frac{\partial U_h(S,X)}{\partial X}} = \frac{\alpha}{1-\alpha}\left[\frac{S-\overline{S}}{X}\right]^{\theta-1} = \frac{\alpha}{1-\alpha}\left[\frac{S}{X}\left(1-\frac{\overline{S}}{S}\right)\right]^{\theta-1}, \tag{A.1}$$

so that

$$\frac{MRS}{(X/S)} = \frac{\alpha}{1-\alpha}\left(\frac{S}{X}\right)^{\theta}\left(1-\frac{\overline{S}}{S}\right)^{\theta-1} \tag{A.2}$$

and

$$\frac{d\,MRS}{d\,(X/S)} = \frac{\alpha(\theta-1)}{1-\alpha}\left[\frac{S}{X}\left(1-\frac{\overline{S}}{S}\right)\right]^{\theta-2}\left\{\frac{d(S/X)}{d(X/S)}\left(1-\frac{\overline{S}}{S}\right)+\frac{S}{X}\frac{d\left(1-\frac{\overline{S}}{S}\right)}{d(X/S)}\right\} \tag{A.3}$$

$$= \frac{\alpha(\theta-1)}{1-\alpha}\left[\frac{S}{X}\left(1-\frac{\overline{S}}{S}\right)\right]^{\theta-2}\left\{-\left(\frac{X}{S}\right)^{-2}\left(1-\frac{\overline{S}}{S}\right)+\frac{S}{X}\frac{d\left(1-\frac{\overline{S}}{S}\right)}{d(X/S)}\right\} \tag{A.4}$$

With (A.2) and (A.4), the EoS (4) becomes

$$\sigma(S,X) = \frac{MRS}{X/S}\,\frac{d\,(X/S)}{d\,MRS} = \frac{MRS}{X/S}\left(\frac{d\,MRS}{d\,(X/S)}\right)^{-1} \tag{A.5}$$

$$= \left(\frac{S}{X}\right)^{\theta}\left(1-\frac{\overline{S}}{S}\right)^{\theta-1}\frac{1}{\theta-1}\left[\frac{S}{X}\left(1-\frac{\overline{S}}{S}\right)\right]^{2-\theta}\times$$

$$\times\left\{-\left(\frac{X}{S}\right)^{-2}\left(1-\frac{\overline{S}}{S}\right)+\frac{d\left(1-\frac{\overline{S}}{S}\right)}{d(X/S)}\frac{S}{X}\right\}^{-1} \tag{A.6}$$

$$= \frac{1}{1-\theta}\left\{1-\frac{\frac{X}{S}}{\left(1-\frac{\overline{S}}{S}\right)}\frac{d\left(1-\frac{\overline{S}}{S}\right)}{d(X/S)}\right\}^{-1}. \tag{A.7}$$

To calculate the remaining derivative, we transform the problem from the standard variables (S, X) into the following variables (w, v):

$$w := \frac{X}{S} \tag{A.8}$$

$$v := \alpha\left(S-\overline{S}\right)^{\theta} + (1-\alpha)X^{\theta}, \tag{A.9}$$

where w is the ratio of the two consumption goods and v is a monotonic transformation of the utility function, so that $v = constant$ is equivalent to $U(S,X) = constant$. Derivatives under the constraint $U(S,X) = const.$, i.e. along an indifference curve, are now taken along $v = const.$, or $dv = 0$.

From (A.9), using (A.8), we have

$$\left(1 - \frac{\overline{S}}{S}\right) = \left[\frac{v}{\alpha S^\theta} - \frac{1-\alpha}{\alpha} w^\theta\right]^{\frac{1}{\theta}} \quad , \tag{A.10}$$

such that

$$\frac{d\left(1 - \frac{\overline{S}}{S}\right)}{d(X/S)} = \frac{d\left[\frac{v}{\alpha S^\theta} - \frac{1-\alpha}{\alpha} w^\theta\right]^{\frac{1}{\theta}}}{dw} \tag{A.11}$$

$$= -\left[\frac{v}{\alpha S^\theta} - \frac{1-\alpha}{\alpha} w^\theta\right]^{\frac{1}{\theta}-1} \left(\frac{v}{\alpha S^{\theta+1}} \frac{dS}{dw} + \frac{1-\alpha}{\alpha} w^{\theta-1}\right) \tag{A.12}$$

Totally differentiating (A.9), and using $dv = 0$, yields

$$0 = \theta \left[\alpha \left(S - \overline{S}\right)^{\theta-1} + (1-\alpha) w^\theta \, S^{\theta-1}\right] dS + \theta(1-\alpha)\, S^\theta \, w^{\theta-1}\, dw \tag{A.13}$$

$$\Leftrightarrow \quad \frac{dS}{dw} = -\frac{(1-\alpha) S^\theta \, w^{\theta-1}}{\alpha \left(S - \overline{S}\right)^{\theta-1} + (1-\alpha) w^\theta \, S^{\theta-1}} \tag{A.14}$$

Using (A.12) and (A.14), we have:

$$-\frac{\frac{X}{S}}{\left(1-\frac{\overline{S}}{S}\right)}\frac{d\left(1-\frac{\overline{S}}{S}\right)}{d(X/S)} = \frac{\frac{X}{S}}{\left(1-\frac{\overline{S}}{S}\right)}\left[\frac{v}{\alpha S^{\theta}} - \frac{1-\alpha}{\alpha}w^{\theta}\right]^{\frac{1}{\theta}-1}\left(\frac{v}{\alpha S^{\theta+1}}\frac{dS}{dw} + \frac{1-\alpha}{\alpha}w^{\theta-1}\right) \tag{A.15}$$

$$= -\frac{X}{S}\left(1-\frac{\overline{S}}{S}\right)^{-\theta}\left(\frac{v}{\alpha S^{\theta+1}}\frac{(1-\alpha)S^{\theta}\,w^{\theta-1}}{\alpha\left(S-\overline{S}\right)^{\theta-1}+(1-\alpha)S^{\theta-1}\,w^{\theta}} - \frac{1-\alpha}{\alpha}w^{\theta-1}\right) \tag{A.16}$$

$$= -\left(\frac{X}{S}\right)^{\theta}\left(1-\frac{\overline{S}}{S}\right)^{-\theta}\left[\frac{v(1-\alpha)}{\alpha^2 S\left(S-\overline{S}\right)^{\theta-1}+\alpha(1-\alpha)X^{\theta}} - \frac{1-\alpha}{\alpha}\right] \tag{A.17}$$

$$= -X^{\theta}\left(S-\overline{S}\right)^{-\theta}\left[\frac{v(1-\alpha)-\alpha(1-\alpha)S\left(S-\overline{S}\right)^{\theta-1}-(1-\alpha)^2X^{\theta}}{\alpha^2 S\left(S-\overline{S}\right)^{\theta-1}+\alpha(1-\alpha)X^{\theta}}\right] \tag{A.18}$$

$$= -X^{\theta}\left(S-\overline{S}\right)^{-\theta}\frac{(1-\alpha)\left(S-\overline{S}\right)^{\theta}-(1-\alpha)S\left(S-\overline{S}\right)^{\theta-1}}{\alpha S\left(S-\overline{S}\right)^{\theta-1}+(1-\alpha)X^{\theta}} \tag{A.19}$$

$$= (1-\alpha)X^{\theta}\frac{\frac{S}{S-\overline{S}}-1}{\alpha S\left(S-\overline{S}\right)^{\theta-1}+(1-\alpha)X^{\theta}}. \tag{A.20}$$

Plugging this into (A.7) yields

$$\sigma(S,X) = \frac{1}{1-\theta}\left\{1 - \frac{\frac{X}{S}}{\left(1-\frac{\overline{S}}{S}\right)}\frac{d\left(1-\frac{\overline{S}}{S}\right)}{d(X/S)}\right\}^{-1} \tag{A.21}$$

$$= \frac{1}{1-\theta}\left\{1 + (1-\alpha)X^{\theta}\,\frac{\frac{S}{S-\overline{S}}-1}{\alpha S\left(S-\overline{S}\right)^{\theta-1}+(1-\alpha)X^{\theta}}\right\}^{-1} \tag{A.22}$$

$$= \frac{1}{1-\theta}\left[\frac{\alpha S\left(S-\overline{S}\right)^{\theta-1}+(1-\alpha)X^{\theta}}{\alpha S\left(S-\overline{S}\right)^{\theta-1}+(1-\alpha)X^{\theta}\,\frac{S}{S-\overline{S}}}\right] \tag{A.23}$$

$$= \frac{1}{1-\theta}\left[\frac{\alpha\left(\frac{S-\overline{S}}{X}\right)^{\theta-1}+(1-\alpha)\frac{X}{S}}{\alpha\left(\frac{S-\overline{S}}{X}\right)^{\theta-1}+(1-\alpha)\frac{X}{S-\overline{S}}}\right] \tag{A.24}$$

$$= \frac{1}{1-\theta}\left[1-\frac{(1-\alpha)\dfrac{\overline{S}}{S}}{\alpha\left[\dfrac{S-\bar{S}}{X}\right]^{\theta}+(1-\alpha)}\right]. \tag{A.25}$$

B Proof of Proposition 2

Results (8), (10), (11), (13) and (14) can easily be verified.

Proof of Result (9):

$$\frac{d\,\sigma}{d\,S} = \frac{1}{1-\theta}\,\frac{1-\alpha\overline{S}}{S^2}\,\frac{\left[\alpha\left(\dfrac{\theta S}{X}\left[\dfrac{S-\overline{S}}{X}\right]^{\theta-1}+\left[\dfrac{S-\overline{S}}{X}\right]^{\theta}-1\right)+1\right]}{\left(1-\alpha+\alpha\left[\dfrac{S-\overline{S}}{X}\right]^{\theta}\right)^{2}} > 0 \text{ for } \theta \geq 0. \tag{B.1}$$

Proof of Result (12):

$$\frac{d\,\sigma}{d\,\overline{S}} = \frac{1}{\theta-1}\,\frac{1-\alpha}{S}\,\frac{\left[\dfrac{\theta\alpha\overline{S}}{X}\left[\dfrac{S-\overline{S}}{X}\right]^{\theta-1}+(1-\alpha)+\alpha\left[\dfrac{S-\overline{S}}{X}\right]^{\theta}\right]}{\left(1-\alpha+\alpha\left[\dfrac{S-\overline{S}}{X}\right]^{\theta}\right)^{2}} < 0 \text{ for } \theta \geq 0. \tag{B.2}$$

C Proof of Proposition 3

The consumer requires $m = p_S\overline{S}$ to meet her subsistence needs $\overline{S}$. This means that up to this level of income she is not willing to substitute S for X, i.e. $\sigma^\star = 0$. For $m > p_S\overline{S}$, she faces the utility maximization problem

$$\max_{S,X} U_h(S,X) \qquad \text{s.t.} \qquad p_S S + p_X X \leq m \;. \tag{C.1}$$

The Lagrangian and first-order conditions are:

$$\begin{aligned}
\mathcal{L}(S,X,\mu) &= \left[\alpha\left(S-\overline{S}\right)^\theta + (1-\alpha)X^\theta\right]^{1/\theta} + \mu(m - p_S S - p_X X) && \text{(C.2)}\\
\frac{\partial\mathcal{L}}{\partial S} &= 0 \;\Leftrightarrow\; \alpha(S-\overline{S})^{(\theta-1)}\left[\alpha(S-\overline{S})^\theta + (1-\alpha)X^\theta\right]^{(1/\theta-1)} = \mu p_S && \text{(C.3)}\\
\frac{\partial\mathcal{L}}{\partial X} &= 0 \;\Leftrightarrow\; (1-\alpha)X^{(\theta-1)}\left[\alpha(S-\overline{S})^\theta + (1-\alpha)X^\theta\right]^{(1/\theta-1)} = \mu p_X && \text{(C.4)}\\
\frac{\partial\mathcal{L}}{\partial \mu} &= 0 \;\Leftrightarrow\; p_S S + p_X X = m && \text{(C.5)}
\end{aligned}$$

From conditions (C.3) and (C.4), we obtain

$$\frac{\alpha}{1-\alpha}\left[\frac{(S-\overline{S})}{X}\right]^{\theta-1} = \frac{p_S}{p_X}. \tag{C.6}$$

Rearranging gives

$$X = (S-\overline{S})\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}} \;. \tag{C.7}$$

Inserting (C.7) into (C.5) and solving for S yields the Marshallian demand function

$$S^\star(m,p_S,p_X) = \frac{m + p_X\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}}{p_S + p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}} \;. \tag{C.8}$$

Inserting (C.8) into (C.7) yields the Marshallian demand function

$$X^\star(m,p_S,p_X) = \frac{m - p_S\overline{S}}{p_X + p_S\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{-\frac{1}{\theta-1}}} \;. \tag{C.9}$$

Inserting (C.8) and (C.9) into Equation (7) yields the Hicksian EoS in the utility-optimal allocation ($X^\star$, $S^\star$):

$$\sigma^\star = \frac{1}{1-\theta}\left[1-\frac{(1-\alpha)\,\overline{S}\;\dfrac{p_S+p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}}{m+p_X\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}}}{(1-\alpha)+\alpha\left[\dfrac{\dfrac{m+p_X\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}}{p_S+p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}}-\overline{S}}{\dfrac{m-p_S\overline{S}}{p_X+p_S\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-1}{\theta-1}}}}\right]^{\theta}}\right] \tag{C.10}$$

$$= \frac{1}{1-\theta}\left[1-\frac{(1-\alpha)\,\overline{S}\left(p_S+p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)}{\left(m+p_X\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)\left(\alpha\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-\theta}{\theta-1}}+(1-\alpha)\right)}\right] \tag{C.11}$$

$$= \frac{1}{1-\theta}\left[\frac{m\left((1-\alpha)+\alpha\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-\theta}{\theta-1}}\right)}{(1-\alpha)\overline{S}\left(p_S+p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)+m\left((1-\alpha)+\alpha\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-\theta}{\theta-1}}\right)}\right] \tag{C.12}$$

Solving the equation $\sigma^\star = 1$ for $m = m^P$ we obtain (21).

From Equation (C.12) it can easily be seen that as income m goes to infinity, the EoS $\sigma^\star$ approaches standard CES result in absence of a subsistence requirement:

$$\sigma^\star \to \frac{1}{1-\theta} \quad \text{for } m \to \infty\ . \tag{C.13}$$

Furthermore, the EoS monotonically increases with income,

$$\frac{d\,\sigma^\star}{d\,m} = \frac{1}{1-\theta}\left[\frac{(1-\alpha)\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{2\theta+1}{\theta-1}}\left(p_X+p_S\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)\left((1-\alpha)+\alpha\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{\theta}{\theta-1}}\right)}{\left(\alpha m+(1-\alpha)\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{\theta}{\theta-1}}\left(m+p_S\overline{S}+p_X\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)\right)^2}\right] > 0. \tag{C.14}$$

and decreases with an increasing subsistence requirement

$$\frac{d\,\sigma^\star}{d\,\overline{S}} = \frac{1}{\theta-1}\left[\frac{(1-\alpha)m\left(p_S + p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)\left((1-\alpha)+\alpha\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{\theta}{\theta-1}}\right)}{\left(\alpha m\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{\theta}{\theta-1}} + (1-\alpha)\left(m + p_S\overline{S} + p_X\overline{S}\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)\right)^2}\right] < 0. \tag{C.15}$$

D Proof of Proposition 4

The cross-price derivatives of the Marshallian demand functions for S and X are obtained from Equations (C.8) and (C.9) and have the following properties:

$$\frac{d\,S^\star}{d\,p_X} = \frac{\theta}{\theta-1}\left[\frac{(p_S\overline{S}-m)\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}}{\left(p_S + p_X\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{1}{\theta-1}}\right)^2}\right] \gtreqless 0 \quad \text{for} \quad \theta \gtreqless 0 \tag{D.1}$$

$$\frac{d\,X^\star}{d\,p_S} = \frac{1}{\theta-1}\left[\frac{\overline{S}\left((1-\theta)p_X + p_S\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-1}{\theta-1}}\right) - \theta m\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-1}{\theta-1}}}{\left(p_X + p_S\left[\frac{\alpha}{1-\alpha}\frac{p_X}{p_S}\right]^{\frac{-1}{\theta-1}}\right)^2}\right], \tag{D.2}$$

with

$$\frac{d\,X^\star}{d\,p_S}\begin{Bmatrix} \to\infty \\ = -(1-\alpha)\frac{\overline{S}}{p_X} < 0 \\ < 0 \end{Bmatrix} \quad \text{for} \quad \theta\begin{Bmatrix} \to 1 \\ = 0 \\ < 0 \end{Bmatrix}. \tag{D.3}$$

Solving the equation $dX^\star/dp_S = 0$ for m, we obtain (25). It is further easy to verify that $\frac{d^2X^\star}{dp_S dm} > 0$ for $\theta > 0$.