Perfect selectivity vs. biomass harvesting in an age-structured fishery population model. What is the gain?

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Abstract

A generic type age structured fishery population model consisting of two harvestable mature age classes and one juvenile age class is formulated. Exploitation with i) similar (uniform) fishing mortality and ii) perfect fishing selectivity among the age classes is considered. Perfect selectivity allows for different fishing mortality among the age classes when the harvest is subject to maximization in one or another respect, and the gain of this type of exploitation is compared with the uniform, or biomass, exploitation. Sustainable yield – biomass functions are developed, and the maximum sustainable yield (MSY) solutions are found under both exploitation schemes. Finally, the maximum economic yield (MEY) solutions are found and characterized.

Keywords: Fishery economics, age-structured model, biomass harvesting, perfect fishing selectivity

JEL code: Q22

1.Introduction

The working horse model in fishery economics has for a long time been the biomass model (or surplus production model, or lumped parameter model, or the ‘Clark-model’, Clark 1990). The first studies using this model were published in the mid 1950’s, and were concerned with uncontrolled, or 'open access', fishing activity. Some twenty years later the analysis was extended to account for the optimal harvesting, or fishing, activity over time and where fish was considered as capital. Within this capital theoretic framework, the fish could either be kept in the ocean, or it could be harvested and stored in ‘the bank’ as capital. It was shown that three factors played an important role determining the degree of exploitation; the price-cost ratio of the yield, the natural growth rate of the fish stock and the opportunity cost of the natural capital, i.e., the rate of interest. Additionally, when studying the biomass model with controlled versus uncontrolled harvesting, it was evident that the management structure of the fish stock, or the property rights scheme, played an important role for the exploitation pressure and the economic rent.

While the predictions and the driving forces from the Clark-model are in line with real life experience (see, e.g., Brown 2000 and the references therein), it is also clear that the biomass model generally gives too little guidance in real life management situations. What often in needed in an actual management situation is a more ‘fine-tuned’, or disaggregated, population model which have the capability to say something about fishing mortality, as well as natural growth and spawning ability, among different sub-groups, or age-classes, of the considered fish population. For example, in actual management situations, the manager is often confronted with the question of deciding what fishing gears that should be used, which fishing grounds that should be exploited, the timing of the fishing over the year cycle, and so forth. In most situations these decisions implicitly, or explicitly, are related to the question whether the ‘young’ or the ‘old’ fish of the population should be targeted and subject to fishing mortality. In actual management situations in fisheries there are hence need for a more disaggregated assessment of the fish stocks[1].

[1] In an influential article Jim Wilen (2000, p.320) writes: ‘Last, over much of this period (the last few decades, A.S.), economists continued to depict bioeconomic systems in simple analytically tractable ways, assuming homogeneous population processes captured by “lumped parameter” models. During the same period, biologists were moving away from

Analysing age, or stage, structured fishery models, i.e., models where the species are grouped in different classes according to age and sex, has a long tradition within biology. Caswell (2001) gives an in-depth overview; see also Getz and Haigh (1989). However, economics play a minor role in these works. Colin Clark has a chapter on age structured fishery models in the 1976 edition of his milestone book. This chapter is more or less left unchanged in the 1990 edition (Clark 1990, Ch. 9) where he, among others, study the conditions for fishing a single age class, or cohort, independently of other cohorts, and without taking the effects on recruitment into serious account. Another early contribution is Reed (1980) who analytically solved the maximum sustainable yield problem, and where he found that optimal harvesting includes at most two age classes. Olli Tahvonen (2009) has more recently derived both analytical and numerical results on optimal harvesting in a dynamic setting under various simplifying assumptions. Other recent contributions demonstrating analytical results in age-structured fishery models include Diekert et al. (2010), Skonhoft et al. (2012) and Quaas et al. (2013). While Tahvonen analyzes the situation where one fleet (or one agent) is targeting the various age classes of the fish stock with similar, or different, catchability coefficients, Skonhoft et al. includes two fleets targeting two different age classes with perfect as well as imperfect fishing selectivity.

In this paper a generic type age structured fishery population model consisting of two mature harvestable classes is considered. The exploitation of this fish population with perfect fishing selectivity is compared to the situation where exploitation takes place with similar (uniform) harvesting rates. Perfect targeting generally allows for different fishing mortality among the age classes when the harvest is subject to maximization in one or another respect. Uniform harvesting with no distinction between the offtake, ‘a fish is a fish’, is considered as biomass exploitation. The aim of this paper is to compare the exploitation of this fish stock under these two different harvesting

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these simple views by incorporating size and age structures and more realistic depictions of spatial processes, including heterogeneity, patchy resource abundance, dispersal, and oceanographic linkages. While biologists developed new and richer depictions of more realistic population processes with simulation modeling, calibration, and statistical estimation techniques, economists mostly continued to work with simpler models that could be analytically solved’.

schemes. We derive sustainable yield functions, and find the maximum sustainable yields (MSY). We therefore demonstrate that MSY is not a well-defined concept in a model with several harvestable age classes. Not surprisingly, we find that MSY with perfect targeting exceeds MSY with uniform harvesting, and thus represents a gain. Various factors affecting this gain are analyzed. Maximum economic yield (MEY) fishing is also included in our paper and is compared to the MSY solution. The present paper is related to Tahvonen (2009), Lawson and Hilborn (1985), Getz (1980), and Walters (1986, p. 132-140) who all, in various ways, discusses possible relationships between age structured models and biomass models. See also Flaaten (2011)[2].

The paper is organized as follows. In the next section 2, the generic population model is formulated. Section 3 demonstrates what happens when the fish stock is exploited as a biomass model with uniform fishing mortality among the two harvestable age classes. We find the sustainable yield function, and the harvesting is optimized giving the peak value of this function, the MSY. In section 4 the sustainable yield function and MSY are derived when the fish stock is exploited with perfect harvesting selectivity. In section 5 prices and costs are introduced, and maximum economic yield (MEY) harvesting is explored. Finally, section 6 summarizes our findings.

2. Population model

As already indicated, our study of harvest of different age classes of a fish population is tackled by using a quite simple ('generic') model with just three cohorts of the fish population, and is similar to that of Skonhoft et al. (2012). The youngest class is immature fish which is not harvested due to, say, gear regulations (mesh size restrictions), whereas the two older classes are harvestable and contribute both to the spawning stock. Recruitment is endogenous and density dependent, and the old matures are assumed to have higher fertility than the young matures. Natural mortalities, or equivalently the survival rates, are assumed to be fixed and density independent for all three age classes. In the single period of 1 year, three events

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[2] Flaaten (2011, p.113) rightly observes that 'when working with detailed and complex year class models we must be aware that even in such models we can find the maximum sustainable yield (MSY) and the corresponding stock size, though these characteristics are not as apparent as in the aggregated biomass models'.

happen in the following order: First, recruitment and spawning, then fishing which takes place instantaneously of both mature age classes, and finally natural mortality. The restriction of natural mortality to take place only a certain period of the year is certainly one of our generic model assumptions.

The fish population in number of individuals at time t (year) is structured as recruits $X_{0,t}$ ($yr < 1$), young mature fish $X_{1,t}$ ($1 \le yr < 2$) and old mature fish $X_{2,t}$ ($2 \le yr$). The number of recruits is governed by the recruitment function:

(1) $$X_{0,t} = R(X_{1,t}, X_{2,t})$$

where $R(\cdot\cdot)$ is assumed to be of the Beverton-Holt type characterized by $R(0,0) = 0$ and $\partial R / \partial X_{i,t} = R'_i > 0$, together with $R''_i < 0$ ($i = 1,2$) (see also below). As we assume higher fertility of the old than the young spawning age-class, $R'_2 > R'_1$ holds as well. The number of young mature fish follows next as:

(2) $$X_{1,t+1} = s_0 X_{0,t},$$

where s_0 is the fixed natural survival rate. Finally, the number of old mature fish is described by:

(3) $$X_{2,t+1} = s_1(1 - f_{1,t})X_{1,t} + s_2(1 - f_{2,t})X_{2,t}$$

where $f_{1,t}$ and $f_{2,t}$ are the total fishing mortalities of the young and old mature stage, respectively, while s_1 and s_2 are the natural survival rates of these mature stages, assumed to be fixed and density independent. Typically, these survival rates do not differ too much. Eq. (3) describing the relationship between the two mature age classes is notified as the spawning constraint. When combining Eqs. (1) and (2) we find:

(4) $$X_{1,t+1} = s_0 R(X_{1,t}, X_{2,t}),$$

which in our model is the recruitment constraint.

The weight of the young and old fish (kg per fish) are denoted by w_1 and w_2, respectively, and with $w_2 > w_1$. The biomass harvested in year t and the standing biomass are thus defined through:

(5) $$H_t = (w_1 X_{1,t} f_{1,t} + w_2 X_{2,t} f_{2,t}),$$

and

(6) $B_t = (w_1 X_{1,t} + w_2 X_{2,t})$,

respectively, as fish abundance is measured in the beginning of the year, before fishing, and fishing takes place before natural mortality.

The population equilibrium for *fixed* fishing mortalities $f_{i,t} = f_i$, $t = 0,1,2...$, is defined by $X_{i,t+1} = X_{i,t} = X_i$ $(i = 1,2)$ such that Eqs. (3) and (4) hold as:

(3') $X_2 = s_1(1 - f_1)X_1 + s_2(1 - f_2)X_2$

and

(4') $X_1 = s_0 R(X_1, X_2)$.

An internal equilibrium is shown in Figure 1 where the recruitment is specified as the Beverton-Holt function $R(X_1, X_2) = \dfrac{a(X_1 + \alpha X_2)}{b + (X_1 + \alpha X_2)}$ with $a > 0$ as the scaling parameter, $b > 0$ as the shape parameter and $\alpha > 1$ as the parameter indicating higher fertility of the old mature fish. Higher fishing mortalities shift down the spawning constraint and hence lead to smaller equilibrium stocks. At the same, the mature fish stock composition changes such that the abundance of young mature stock increases relatively to the old mature stock. Notice that this happens also if only the fishing mortality of the young stock shifts up. Notice also that an internal equilibrium is defined for $0 \le f_1 < 1$ only; that is, not all the young mature fish can be harvested. Thus, with $f_1 = 1$ and $0 \le f_2 \le 1$ the spawning constraint becomes flat and we find $X_2 = 0$ together with the recruitment constraint as $X_1 = s_0 R(X_1, 0) > 0$. Is this case, the equilibrium number of fish harvested of the young mature category is $f_1 X_1 = X_1$.

Figure 1 about here

3. Uniform harvesting and exploiting the fish stock as a biomass model

3.1 The sustainable yield function

We start to analyze the exploitation of our fish population as a biomass model in biological equilibrium with similar, or uniform, fishing mortalities among both age classes, i.e., $f_1 = f_2 = f$. Uniform exploitation may typically be interpreted as (totally) imperfect harvesting selectivity; that is, one fishing fleet, or agent, is targeting both stocks with identical catchability coefficients (see also section 5 below). Our first task then is to solve for the biomass harvest Eq. (5), defined as $H = (w_1 X_1 + w_2 X_2) f$, and for the standing biomass Eq, (6), defined as $B = (w_1 X_1 + w_2 X_2)$, for the whole range of possibly fishing mortalities with the population defined by Eqs. (3’) and (4’), and where Eq. (3’) now reads $X_2 = (s_1 X_1 + s_2 X_2)(1 - f)$.

For the above specified Beverton – Holt recruitment function, and our baseline parameter values (Table A1 in Appendix), H and B are depicted as functions of the fishing mortality in Figure 2. The unexploited biomass, implying $f = H = 0$, represents the carrying capacity, B_K. Moreover, we find the smallest standing biomass when $f = 1$, but still with some harvest of the young mature stock and also positive biomass, $H = B_{\min} = w_1 X_1 > 0$. The reason for positive biomass when harvesting all the mature fish is that there is no harvest of immature fish. Not surprisingly, because the recruitment function is not peak-valued (i.e., not of the Ricker-type), we also find the size of the standing biomass to decline monotonically with the exploitation pressure and higher fishing mortalities, while the harvested biomass first increases, reach a peak value and then declines.

Figure 2 about here

In Figure 3 we have plotted the functional relationship between H and B, $H = F(B)$, for identical fishing mortalities. This peak valued curve $F(B)$ thus represents the sustainable (equilibrium) yield – biomass relationship, or the sustainable yield function, in our age structured fishery model when exploitation is governed by uniform fishing mortalities. We find the maximum sustainable yield (MSY) at $H^{msy} = 1127$ (tonne), reached at a biomass level $B^{msy} = 2253$ (tonne) when $f = f^{msy} \approx 0,50$ (see also Figure 2). With $f = 1$ we have $H = F(B_{\min}) = B_{\min} = 800$ (tonne). The similarities as well as the differences compared to the standard logistic growth biomass function are apparent. Our sustainable yield function is left-hand skewed, and the MSY biomass size is located well below half of the biomass carrying capacity. The most striking difference is, however, the restricted definition set as $F(B)$ is not defined below that of the biomass level $B_{\min}$ governed by $f=1$.

Figure 3 about here

In Table A2 (Appendix), we have also included the size of the fish stocks. A striking feature with the stock evolvement along $H = F(B)$ when moving from B_K , is the rapid reduction in number of old mature fish with increasing fishing mortality. The ratio of the old to the young mature fish is defined through Eq. (3') when $f_1 = f_2 = f$ as $X_2 / X_1 = s_1(1-f)/[1-s_2(1-f)]$. For the unexploited fish stock; that is, at the biomass carrying capacity, we find 2.33 ($X_2 / X_1 = s_1/(1-s_2) = 0.7/0.3 = 1840/789$) under the baseline parameter values scenario (Tables A1 and A2). This ratio reduces to 0.54 at B^{msy} . With $f=1$, we find, as already indicated, $X_2 / X_1 = 0$, and only juveniles not exploited entering the young mature class contributes to the standing biomass.

Lawson and Hilborn (1985) also constructed a sustainable yield function based on uniform harvesting for a fish population model including several age classes. The recruitment function was of the Beverton – Holt type (but they also used the Ricker function), and found the maximum sustainable yield function to have strong similarities with the standard logistic growth function. In an *ad hoc* manner they also fitted their function to the standard logistic function and calculated the maximum specific growth rate. See also Tahvonen (2009) who study uniform harvesting, as well as quasi uniform harvesting where the fishing mortalities are fixed through the ratio of the catchability coefficients using the standard Schaefer harvesting function.

3.2 Optimizing the harvest

We now proceed to characterize the maximum sustainable biomass level B^{msy} and the maximum sustainable harvest H^{msy} (MSY) with uniform fishing mortalities. The problem of finding the uniform fishing mortality maximizing the equilibrium biomass harvest is stated as $H^{msy} = \max[(w_1 X_1 + w_2 X_2) f]$ subject to the spawning constraint (3') with $f_1 = f_2 = f$, and the recruitment constraint (4'). The lagrangian of this problem may be written as $L = (w_1X_1 + w_2X_2)f - \lambda[X_1 - s_0R(X_1, X_2)]$ $-\mu[X_2 - (s_1X_1 + s_2X_2)(1-f)]$ where $\lambda > 0$ and $\mu > 0$ are the shadow prices of the recruitment constraint and spawning constraint, respectively. The first order necessary conditions (with $X_i > 0$, $i = 1, 2$) are:

(7) $\partial L / \partial f = (w_1X_1 + w_2X_2) - \mu(s_1X_1 + s_2X_2) = 0\,; 0 < f < 1,$

(8) $\partial L / \partial X_1 = w_1 f - \lambda + \lambda s_0 R'_1 + \mu s_1(1-f) = 0$

and

(9) $\partial L / \partial X_2 = w_2 f + \lambda s_0 R'_2 - \mu + \mu s_2(1-f) = 0$.

These equations together with the biological constraints define the fishing mortality f^{msy}, the number of fish X_i^{msy} ($i = 1, 2$) the maximum sustainable yield H^{msy} (MSY), and also B^{msy} through Eq. (6). Control Eq. (7) indicates that fishing should take up to the point where the total marginal biomass gain equalizes the total marginal biomass loss due to reduced spawning of both age classes, evaluated by the spawning constraint shadow price. When rewriting this condition as $(w_1 - \mu s_1)X_1 + (w_2 - \mu s_2)X_2 = 0$ and taking into account that we have $w_2 / s_2 > w_1 / s_1$, as $w_2 > w_1$ and the survival rates typically do not differ too much (section 2), we thus find that the marginal biomass gain of fishing the old mature age class must exceed its marginal loss given by the 'biological discounted' spawning constraint shadow price, i.e., $w_2 > \mu s_2$. The opposite holds for the young mature class, $w_1 < \mu s_1$. Rewriting the stock condition (8) as $\lambda = w_1 f + \lambda s_0 R'_1 + \mu s_1(1-f)$, it states that the young mature fish should be maintained so that its shadow price equalizes its marginal biomass gain value plus the marginal reduced recruitment $\lambda s_0 R'_1$ loss plus the marginal reduced spawning $\mu s_1(1-f)$ loss, both loss factors evaluated at their respective shadow pieces. Condition (9) can be given a similar interpretation.

While our interpretation of the conditions characterizing H^{msy} make good economic sense in terms of marginal gains and marginal losses, the above necessary conditions are not particularly helpful to indicate how the various biological forces influence the value of f^{msy} and the location of B^{msy} in this uniform harvesting situation. However, not surprisingly, numerical simulations demonstrate that higher weights of the fish stocks and higher fertility increase H^{msy} while higher natural mortalities, or lower natural survival rates, work in the opposite direction. We also find that the value of f^{msy} is only modestly affected when, say, both fishing weights are doubled. However, when the ratio of the fish weights shifts, substantial changes may take place. See Table 1below.

4. Harvesting the fish stock with perfect selectivity

We now proceed to study harvest of our fish population when both age classes are perfectly targeted with separate harvest controls, or fishing mortalities. We first find and characterize the MSY harvest and the associated standing biomass level. Next, we construct the sustainable yield function. While constructing the sustainable yield function with uniform fishing mortality is straightforward, the construction is more unclear when the fishing mortalities are different simply because it is 'more to choose about'. More details on this below.

The problem of finding the maximum sustainable yield (MSY) is now defined by $H^{msy} = \max(w_1X_1f_1 + w_2X_2f_2)$ subject to the spawning constraint (4') and recruitment constraint (3'). The Lagrangian of this problem is written as $L = (w_1X_1f_1 + w_2X_2f_2) - \lambda[X_1 - s_0R(X_1, X_2)]$ $-\mu[X_2 - (s_1X_1(1-f_1) + s_2X_2(1-f_2)]$. Following the Kuhn-Tucker theorem the first order necessary conditions (assuming $X_i > 0$, $i = 1,2$) are:

(10) $\partial L / \partial f_1 = X_1(w_1 - \mu s_1) \leq 0$; $0 \leq f_1 < 1$,

(11) $\partial L / \partial f_2 = X_2(w_2 - \mu s_2) \overset{\geq}{\underset{<}{=}} 0$; $0 \leq f_2 \leq 1$,

(12) $\partial L / \partial X_1 = w_1f_1 - \lambda + \lambda s_0R'_1 + \mu s_1(1 - f_1) = 0$

and

(13) $\partial L / \partial X_2 = w_2f_2 + \lambda s_0R'_2 - \mu + \mu s_2(1 - f_2) = 0$.

The control condition (10) states that the fishing mortality of the young mature stock should take up to the point where the marginal biomass gain is equal, or below, it's biological discounted marginal biomass loss, evaluated at the spawning constraint shadow price. Because the fishing mortality of this stock must be below one to prevent stock depletion (section 2 above), the marginal biomass gain can not exceed the discounted marginal biomass loss. Control condition (11) is analogous for the old mature stock, except that the marginal biomass gain may exceed the marginal discounted biomass loss if fishing mortality equals one. Eqs. (12) and (13) again steer the shadow price values and has more or less the same content as in the uniform harvesting situation. As the maximum sustainable yield problem with perfect fishing selectivity is a less constrained than with uniform fishing, it follows from the very logic of optimization that the value of H^{msy} in this perfect selectivity situation will exceed H^{msy} found in the uniform fishing situation. See also numerical illustration below.

From the above control conditions, it is now clear that differences in the biological present-value biomass content w_i / s_i ($i = 1, 2$) steers the fishing priority. These factors were also prevalent in the uniform harvesting situation, but now the implications are much clearer. With $w_2 / s_2 > w_1 / s_1$ the above Kuhn-Tucker conditions indicate that the fishing mortality should be higher for the old than the young mature stock; that is, $f_2 > f_1$. There are then generally three possibilities, all corner solutions, to meet the above control conditions (10) and (11); Case i) $0 < f_1 < 1$ and $f_2 = 1$, Case ii) $f_1 = 0$ and $f_2 = 1$, and Case iii) $f_1 = 0$ and $0 < f_2 < 1$. The size of both mature stocks will be higher in Case iii) than in Case ii), which again will be higher than in Case i). Therefore, B^{msy} will be higher in Case iii) than in Case ii), which again will be higher than in Case i).

For our baseline parameter values (again, see Table A1 Appendix), we find that Case ii) with old mature fishing only is optimal and hence $f_2^{msy} = 1$ and $f_1^{msy} = 0$. Therefore, the spawning constraint Eq. (3') as $X_2 = s_1 X_1$ together the recruitment constraint (4') define X_1^{msy} and X_2^{msy} in this optimum. For these stock sizes, the maximum sustainable yield reads $H^{msy} = w_2 X_2^{msy}$, or $H^{msy} = w_2 s_1 X_1^{msy}$, while the associated biomass level becomes $B^{msy} = w_1 X_1^{msy} + w_2 X_2^{msy} = (w_1 + w_2 s_1) X_1^{msy}$. We thus find that the maximum sustainable yield – biomass ratio simply may be written as $H^{msy} / B^{msy} = w_2 s_1 / (w_1 + w_2 s_1)$.

Having successfully been able to characterize the maximum sustainable yield and the related sustainable biomass level in this perfect selectivity fishing situation (see also Skonhoft et al. 2012), the next question is how to construct the related sustainable yield function, $H = F(B)$. There are various possibilities, but the following two steps procedure is adapted. In the first step, starting at the carrying capacity B_K with $f_1 = 0$ and $f_2 = 0$, $f_1 = 0$ is kept fixed while f_2 is increased gradually up to $f_2 = 1$ where H^{msy} and the associated biomass level B^{msy} are reached. In step 2 starting at this point with $f_1 = 0$ and $f_2 = 1$, $f_2 = 1$ is now kept fixed while f_1 increases gradually up to $f_1 = 1$. We then have the smallest standing biomass, but still with some harvest of the young mature stock and $H = B_{\min} = w_1 X_1 > 0$ (see above). Figure 4 depicts the outcome where also the sustainable yield function with uniform fishing mortalities from Figure 3 is redrawn. While the two end-points of these two maximum sustainable yield functions are identical, $H = F(B)$ will elsewhere in the perfect selectivity case exceed that of the uniform case. Therefore, for all biomass levels $B_{\min} < B < B_K$, perfect fishing selectivity represents a more efficient way to exploit the fish stock. See Appendix for a proof.

Figure 4 about here

Table 1 gives a numerical illustration. The maximum sustainable yield with perfect targeting is 23 percent higher (1,378/1,127) than the uniform harvesting situation under the baseline parameter values scenario. In other words, the maximum biomass offtake gain is thus about 23 percent when the fishing mortalities are composed in an optimal way. Somewhat surprisingly, we find that the size of the old mature stock is higher in the perfect selectivity situation. In this table we have also included results from a scenario where the weight of the old mature stock is reduced from its baseline value of 3 to 2.3 (kg/fish). As a result, the optimal harvesting scheme with perfect selectivity shifts from Case ii) to Case i) with $0 < f_1^{msy} = 0.16 < 1$ and $f_2^{msy} = 1$. Hence, the exploitation becomes more aggressive. Fishing becomes also more aggressive in the uniform fishing situation, and where we now find $f^{msy} = 0.55$.

The most striking change is, however, that the H^{msy} difference reduces quite dramatically, and the gain of perfect fishing selectivity under this scenario is just 5 % (1,061/1,016). Therefore, as the fish stocks become more homogenous, here in terms of weights, the gain of exploiting the fish population with perfect fishing selectivity reduces. Indeed, it can be shown that if we hypothetically have $w_2 / s_2 = w_1 / s_1$ the maximum sustainable yield will be similar under these two harvesting schemes, and that H^{msy} with perfect targeting can be met by an infinite number of combinations of fishing mortalities. The intuition is that both control conditions (10) and (11) in this situation must hold as equations, and thus give the same information. Accordingly, it will be one degree of freedom in our system of equations determining the fishing mortalities. On the other hand, it can also be shown that with $w_2 / s_2 = w_1 / s_1$ we still find an unique solution in the uniform fishing situation.

Table 2 about here

5. The maximum economic yield (MEY) solution

We now proceed to study the maximum economic yield (MEY) problem, and compare this solution to that of the MSY problem. We first consider the perfect selectivity fishing situation, where two fleets, or two agents, are targeting the fish population. With $E_{i,t}$ as the effort use of fleet i at time t and assuming a standard Schaefer harvesting function, the harvest, or catch, of the two fleets is thus defined as:

(14) $$h_{i,t} = q_i E_{i,t} X_{i,t}\,;\ i = 1,2\,,$$

where q_i are the catchability coefficients. Therefore, $f_{i,t} = q_i E_{i,t}$ yields the fishing mortalities under our assumption that fishing takes place before natural mortality. The spawning constraint Eq. (3) now becomes:

(15) $$X_{2,t+1} = s_1(1 - q_1 E_{1,t})\,X_{1,t} + s_2(1 - q_2 E_{2,t})\,X_{2,t}\,.$$

With p_1 and p_2 as the fish prices (Euro/kg), assumed to be fixed over time and not influenced by the size of the catch, and where the old mature fish typically is more valuable than the young fish ('larger filets') $p_2 \geq p_1$, and c_i as the unit effort cost (Euro/effort) $(i = 1, 2)$, also assumed to be fixed,

$\pi_t = \left(p_1 w_1 q_1 X_{1,t} - c_1 \right) E_{1,t} + \left(p_2 w_2 q_2 X_{2,t} - c_2 \right) E_{2,t}$ describes the current joint profit of the two fleets with perfect selectivity. The MEY problem is thus stated as

$\max_{E_{1,t}, E_{2,t}} \sum_{t=0}^{\infty} \rho^t \pi_t$ subject to the biological constraints (15) and (4) together with the effort use constraint, $0 \leq E_{i,t} \leq 1/q_i \, (i = 1, 2)$. In addition, the initial stock sizes $X_{i,0}$ have to be known. $\rho^t = 1/(1+\delta)^t \leq 0$ is the discount factor with $\delta \geq 0$ as the (constant) discount rent. The lagrangian of this problem may be written as

$$L = \sum_{t=0}^{\infty} \rho^t \{ \left[\left(p_1 w_1 q_1 X_{1,t} - c_1 \right) E_{1,t} + \left(p_2 w_2 q_2 X_{2,t} - c_2 \right) E_{2,t} \right] - \rho \lambda_{t+1} \left[X_{1,t+1} - s_0 R(X_{1,t}, X_{2,t}) \right]$$
$$- \rho \mu_{t+1} \left[X_{2,t+1} - s_1 \left(1 - q_1 E_{1,t} \right) X_{1,t} - s_2 \left(1 - q_2 E_{2,t} \right) X_{2,t} \right] \}$$

. Following the Kuhn-Tucker theorem the first order necessary conditions are now (when assuming $X_{i,t} > 0$, $i = 1, 2$):

(16) $\partial L / \partial E_{1,t} = \rho^t (p_1 w_1 q_1 X_{1.t} - c_1 - \rho \mu_{t+1} s_1 q_1 X_{1.t}) \underset{<}{\overset{\geq}{=}} 0$; $0 \leq E_{1,t} \leq 1/q_1$, $t = 0, 1, 2, \ldots$,

(17) $\partial L / \partial E_{2,t} = \rho^t (p_2 w_2 q_2 X_{2.t} - c_2 - \rho \mu_{t+1} s_2 q_2 X_{2.t}) \underset{<}{\overset{\geq}{=}} 0$; $0 \leq E_{2,t} \leq 1/q_2$, $t = 0, 1, 2, \ldots$,

(18) $\partial L / \partial X_{1,t} = \rho^t [p_1 w_1 q_1 E_{1,t} + \rho \lambda_{t+1} s_0 R'_1 - \lambda_t + \rho \mu_{t+1} s_1 \left(1 - q_1 E_{1,t} \right)] = 0$, $t = 1, 2, 3, \ldots$

and

(19) $\partial L / \partial X_{2,t} = \rho^t [p_2 w_2 q_2 E_{2,t} + \rho \lambda_{t+1} s_0 R'_2 + \rho \mu_{t+1} s_2 \left(1 - q_2 E_{2,t} \right) - \mu_t] = 0$, $t = 1, 2, 3, \ldots$.

Notice that while the fishing mortality of the young mature class must be below one to sustain the old mature population in biological equilibrium, or steady state, this is not necessary so in a dynamic setting as $E_{1,t} = 1/q_1$ may hold for some periods without depletion the old mature stock. In light of the previous section, the interpretation of the control conditions (16) and (17) should be clear. There are only two differences; economic discounting, ρ, and not only biological discounting plays a role, and the cost parameters are included. The dynamic stock Eqs. (18) and (19) steer the shadow price values, and have straightforward interpretations (see also the above section 3.2).

As the profit functions are linear in the controls, economic theory suggests that fishing should be adjusted to lead the fish stocks to steady state as fast as possible; that is, the Most Rapid Approach Path (MRAP) dynamics. However, the MRAP is not a regular one in our age-structured fish population, among others because the steady state will be a corner solution (see below). The age structure also implies that the population could be above that of the optimal steady state level for one age class and at the same time lower than the optimal steady state level for the other age class. Since fishing is confined to two age classes, the MRAP may imply a large harvest in one period and small, or zero, harvest in the next (see Skonhoft and Gong 2014 for a parallel discussion).

In what follows we analyze steady states where all variables are constant over time. Omitting the time subscript and combining the control conditions (16) and (17) gives a possible solution where both controls are interior. This equation, which is notified as a separating condition (see below), is written as:

(20) $$(1/s_1)(p_1w_1 - c_1/q_1X_1) = (1/s_2)(p_2w_2 - c_2/q_2X_2),$$

and is an increasing, concave curve in the (X_1, X_2)-plane starting from the origin. See Figure 1. Because $p_2w_2/s_2 > p_1w_1/s_1$ we may suspect that with $c_2/q_2 > c_1/q_1$ this interior solution can be an optimal option. As demonstrated in Skonhoft et al. (2012) in a static version of this model, however, this steady state outcome is a saddle point of the objective function. Therefore, possible optimal steady state solutions satisfying the above necessary conditions (16) - (19) indicate different marginal profit (Euro/fish) among the two fish stocks, and in an optimum at least one of the controls must be set to its lower, or upper, bound. Recognizing that E_1 cannot be set to its maximum in steady state, there are now four possible cases to be considered. We have the same three cases as above (section 4), now stated as Case i) with

$0 < E_1 < 1/q_1$ and $E_2 = 1/q_2$, Case ii) with $E_1 = 0$ and $E_2 = 1/q_2$, Case iii) with $E_1 = 0$ and $0 < E_2 < 1/q_2$. Additionally, Case iv) with $0 < E_1 < 1/q_2$ and $E_2 = 0$ is also a possible optimal option. From the control conditions (16) and (17) we find that the separating condition (20) holds as $(1/s_2)\left(p_2 w_2 - c_2 / q_2 X_2\right) > (1/s_1)\left(p_1 w_1 - c_1 / q_1 X_1\right)$ in Cases i) – iii) and the opposite in the last Case iv). Therefore, the solution in Cases i) – iii) is located with an interaction between the recruitment constraint Eq. (4') and the spawning constraint (3') to the north-west of this separating condition Eq. (20). On the contrary, the solution in Case iv) is located to the south-east of the separating condition. See Figure 1, which illustrates either Case i), Case ii) or Case iii) as the optimal option.

In our baseline parameter value scenario (Table A1) with similar fishing costs of the two fleets, $c_1 / q_1 = c_2 / q_2$ (Euro), we find that fishing the old mature stock only and Case ii) is optimal. That is, the solution of the MEY problem coincides with the solution of the MSY problem. This result is not in accordance with the standard biomass model (the Clark-model). While discounting pulls in the direction of an optimal biomass level below that of B^{msy} in the Clark-model, stock dependent harvesting costs pull in the other direction. In our age-structured model the discount rent plays a more indirect role as it generally influences which of the cases i), ii), iii) or iv) that will be most profitable, while the structure of the harvest costs may work in the opposite direction of that of the Clark-model. For example, as already indicated, we find that higher fleet 2 costs may change the optimal harvest option from Case ii) to Case iv) accompanied by smaller fish stocks and a smaller standing biomass level. Table 2 illustrates. The effects of changing discounting are also demonstrated, and where doubling the discount rent does not change the optimal harvest option in this perfect fishing selectivity MEY problem.

Table 2 about here

We then consider the MEY problem with one fleet. The harvest functions are then generally defined as:

(21) $h_{i,t} = q_i E_t X_{i,t}\,;\; i = 1, 2\,.$

With similar catchability coefficients, $q_1 = q_2 = q$ and (perfect) uniform fishing mortalities, the spawning constraint reads:

(22) $X_{2,t+1} = s_1(1 - qE_t)\,X_{1,t} + s_2(1 - qE_t)\,X_{2,t}$

while the current profit is described by $\pi_t = \left(p_1 q X_{1,t} + p_2 q X_{2,t} - c\right)E_t$. The MEY problem is thus now stated as $\max_{E_t} \sum_{t=0}^{\infty} \rho^t \left(p_1 q X_{1,t} + p_2 q X_{2,t} - c\right)E_t$ subject to constraints (4) and (22) together with the restrictions on the fishing mortality and effort use.

Again we assume that it is economically beneficial to fish and also not to deplete the fish stocks. We also now only consider steady state with all variables constant over time. The numerical results with the baseline parameter values are shown in Table 2 (row four). The solution here differs from the MSY solution (Table 1), and the fishing mortalities become lower. Therefore, under this scenario we find that the 'cost effect' dominates the 'discounting effect' such that this solution is located to the right hand side of the MSY solution with less aggressive harvesting and a larger standing biomass. In this baseline parameter value scenario with $c_1 = c_2 = c$, we also find that the steady state profit in this uniform harvesting problem is lower than in the perfect selectivity situation. This comes not as a surprise as the parameter values are similar, and we again have that the MEY problem with perfect selectivity is a less constrained maximization problem than with uniform fishing. We also find that the stock sizes and yield becomes lower with uniform harvesting. Increasing the harvesting cost works now just as in the Clark model with reduced effort use, smaller fishing mortalities and a higher standing biomass. On the other hand, a more myopic fishing policy; that is, increasing the discount rent, gives a more aggressive harvesting compared to the baseline scenario. Therefore, with uniform harvesting in our age structured model, discounting works also in the same direction as in the Clark-model.

6. Concluding remarks

This paper has compared the exploitation of an age structured fish population under perfect fishing selectivity with similar (uniform) fishing mortalities for the harvestable stocks. We find that the maximum sustainable yield (MSY) will be highest in the perfect selectivity case. This follows from the very logic of optimization, as this situation represents a less constrained problem. We also find that the sustainable yield function in this situation will exceed the sustainable yield function with uniform harvesting in the whole range of possible fishing mortalities. With our illustrative baseline data, we find the MSY to be substantial higher (23 percent) in the perfect selectivity situation. This difference reduces when the harvestable fish stocks become more homogeneous through, say, smaller weight discrepancies.

In contrast to the standard biomass model, or the Clark model, our analysis demonstrates that MSY is not a well-defined concept in a real world situation where the fish stock is composed of different age classes. The MSY, as well as the sustainable yield function, will depend on the degree of fishing selectivity and the prioritizing of the harvest among the harvestable age classes. The European Community (EU) with its new Common Fishery Policy (CFP) states that '*Fishing sustainably means fishing at levels that do not endanger the reproduction of stocks and at the same time maximises catches for fishermen. This level is known as the 'maximum sustainable yield' (MSY). Under the new CFP, stocks must be fished at these levels…The new CFP shall set the fishing levels at MSY levels by 2015 where possible, and at the latest by 2020 for all fish stocks'* **(**http://europa.eu/rapid/press-release_MEMO-13-1125_en.htm). However, without further specifications this goal is of restricted value.

Maximum economic yield fishing MEY is also studied in our age structured fishery model. With perfect selectivity we find that the location of the MEY solution compared to the MSY generally will differ from what we find in the Clark model, and that the driving forces may be different. For example, in our numerical illustration, we find that higher fishing costs may lead to more aggressive fishing and a higher exploitation pressure, while increasing the discount rate may not change the optimal MEY policy. On the other hand, with uniform fishing mortalities our numerical illustrations indicate similar driving forces as in the Clark model. For example, a higher discount rate yields a more aggressive steady state harvesting policy.

Literature

Brown, G., 2000. Renewable natural resource management and use with and without markets. Journal of Economic Literature 37, 875 - 914

Caswell, H., 2001. Matrix Population Models: Construction, analysis and interpretation (2th. Ed.). Sinauer, Boston.

Clark, C., 1990. Mathematical Bioeconomics (2th. Ed). Wiley, New York.

Diekert, F., D. Hjermann, E. Nævdal and N. C. Stenseth, 2010. Non-cooperative exploitation of multi-cohort fisheries-the role of gear selectivity in the north-east arctic cod fishery. Resource and Energy Economics 32, 78 - 92.

Flaaten, O., 2011, Fishery Economics and Management. http://munin.uit.no/bitstream/handle/10037/2509/book.pdf?sequence=3

Getz, A. and R. G. Haigh, 1989. Population Harvesting. Princeton University Press, Princeton.

Lawson, T. and R. Hilborn, 1985. Equilibrium yields and yield isopleths from a general age-structured model of harvested populations. Canadian Journal of Aquatic Science 42, 1766 - 1771

Quaas, M., T. Requate, K. Ruckes, A. Skonhoft, N. Vestergaard and R. Voss, 2013. Incentives for optimal management of age-structured fish populations. Resource and Energy Economics 35, 113 - 134

Skonhoft, A., N. Vestergaard and M. Quaas, 2012. Optimal harvest in an age structured model with different fishing selectivity. Environmental and Resource Economics 51, 525 - 544

Skonhoft, A. and P. Gong, 2014. Wild salmon fishing. Harvesting the old or the young? Resource and Energy Economics 36, 417 - 435

Tahvonen, O., 2009. Economics of harvesting age-structured fish populations. Journal of Environmental Economics and Management 58, 281 - 299.

Walters, C., 1986. Adaptive Management of Renewable Resources. Macmillian Publishing, London.

Wilen, J., 2000. Renewable Resource Economists and Policy: What Differences Have We Made? Journal of Environmental Economics and Management 39, 306 - 327

Appendix

Data and additional results

The numerical simulations are merely an illustration and the parameter values are not related to any particular fisheries. Table A1 shows the baseline parameter values. The same natural survival rates for the young and old mature age class is assumed while the weight of the old age class is assumed 50 % higher than the young mature age class. Similar catchability coefficients and unit effort costs of the two fleets are assumed. The fishing prices are assumed to be similar as well.

Table A1 about here

Table A2 gives detailed results from the uniform sustainable yield harvesting situation with the baseline parameter values.

Table A2 about here

The sustainable yield functions

The biomass Eq. (6) (when omitting the time subscript) may be rewritten as $X_2 = B / w_2 - (w_1 / w_2) X_1$. For a fixed value of B this equation represents an iso-biomass line, and its intersection with the recruitment constraint Eq. (4') thus gives the unique stock size combination conditioned upon the fixed biomass level B. Therefore, every biomass level B along the horizontal axis in Figure 4 in the domain $B_{\min} < B < B_K$ define a unique combination of X_1 and X_2. In other words, for the same standing biomass level the number of fish will be identical at the sustainable yield functions in the uniform and the perfect selectivity fishing situation.

When the harvest is located to the right hand side of the peak value with perfect selectivity and Case ii) is the optimal harvest option ('step 1', section 4), the sustainable yield difference between perfect selectivity and uniform harvesting writes:

(A1) $$\Delta = w_2 X_2 f_2 - (w_1 X_1 + w_2 X_2) f .$$

With uniform harvesting, the spawning constraint Eq. (3') may be written as $X_2 = \frac{s_1(1-f)X_1}{(1-s_2(1-f))}$ while it is defined as $X_2 = \frac{s_1 X_1}{(1-s_2(1-f_2))}$ with perfect selectivity when $f_1 = 0$. Therefore, the relationship between the fishing mortalities is described by $f_2 = \frac{f}{s_2(1-f)}$. Inserted into (A1) yields:

(A2) $$\Delta = w_2(\frac{f}{s_2(1-f)} - f)X_2 - w_1 X_1 f$$

after some small rearrangements. When next inserting for $X_1 = \frac{(1-s_2(1-f))X_2}{s_1(1-f)}$, we find:

(A3) $$\Delta = [w_2(\frac{1-s_2(1-f)}{s_2(1-f)} - 1) - w_1 \frac{(1-s_2(1-f))}{s_1(1-f)}]fX_2 > 0$$

with $s_2 \le s_1$ and $w_2 > w_1$. Therefore, the sustainable yield is higher with perfect selective fishing than with uniform fishing when being to the RHS of the MSY with perfect selectivity. Similar outcome can be demonstrated when being to the left hand side of MSY of the sustainable yield function ('step 2' section 4).

Figure 1. Biological equilibrium with fixed fishing mortalities ($0 \le f_1 < 1$ and $0 \le f_2 \le 1$**). Beverton-Holt recruitment function.**

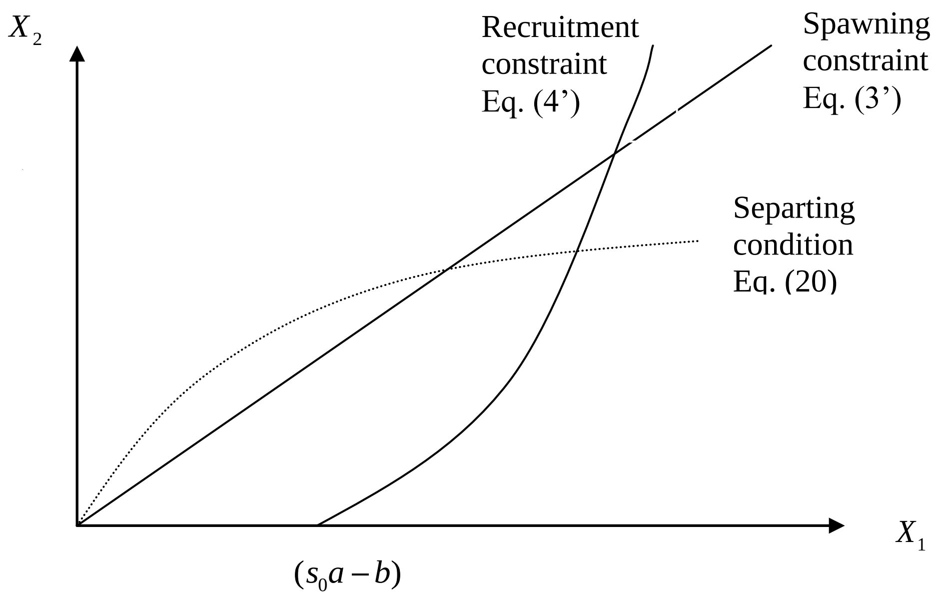

Figure 2. Sustainable biomass (serie1) and sustainable yield (serie 2). In tonne. Uniform fishing mortalities. Baseline parameter values

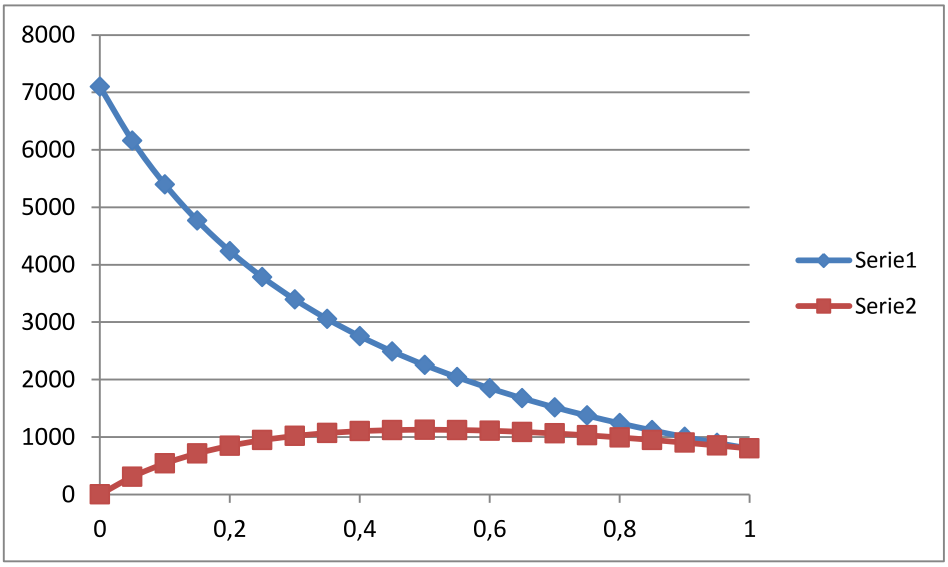

Figure 3. Sustainable yield function, $H = F(B)$. Uniform fishing mortalities. Biomass B (tonne) horizontal axis and harvest H (tonne) vertical axis. Baseline parameter values

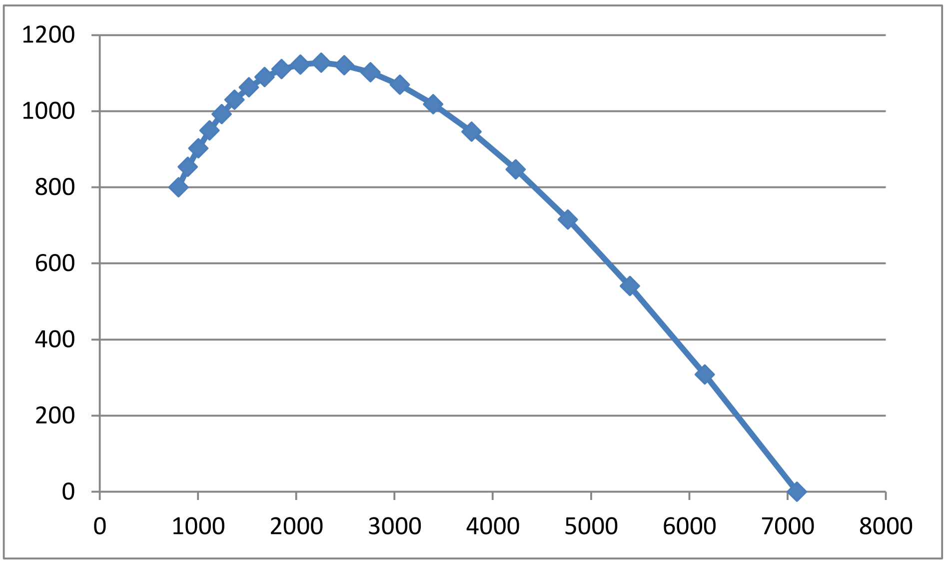

Figure 4: Sustainable yield functions. Uniform harvesting (serie 2, lower curve) and perfect selectivity (serie 1, upper curve). In tonne. Baseline parameter values

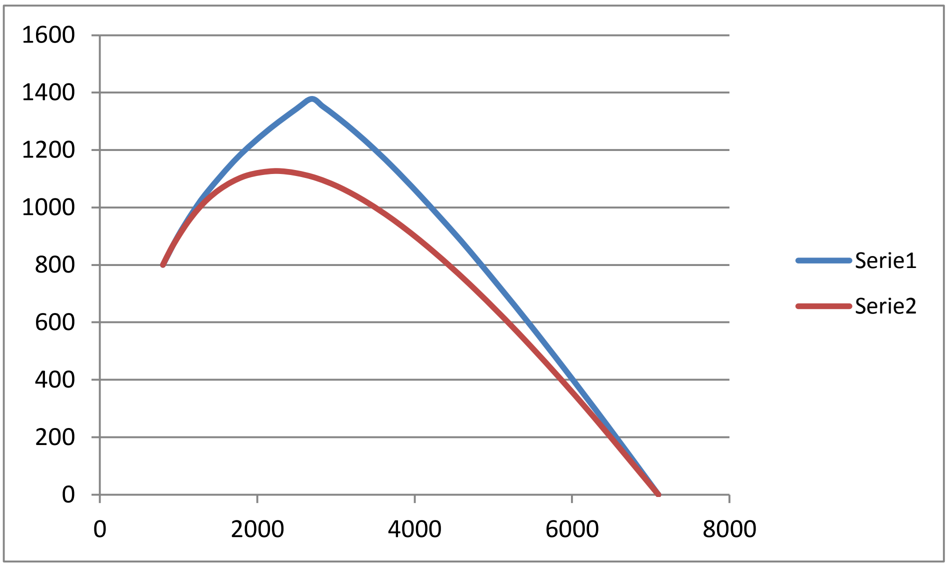

Table 1. Maximum sustainable yield (MSY) uniform harvesting and perfect selectivity. H^{msy} **(in tonne) and associated biomass level** B^{msy} **(in tonne) and fish stocks (in 1,000 # of fish)**

	f_1^{msy}	f_2^{msy}	B^{msy}	H^{msy}	X_1^{msy}	X_2^{msy}
Uniform harvesting. Baseline parameter values	0.50	0.50	2,253	1,127	623	336
Uniform harvesting, $w_2 = 2.3$ (kg/fish)	0.55	0.55	1,847	1,016	604	278
Perfect selectivity. Baseline parameter values	0	1.00	2,690	1,378	656	459
Perfect selectivity, $w_2 = 2.3$ (kg/fish)	0.16	1.00	2,129	1,061	635	374

Table 2. Steady state maximum economic yield (MEY) solution perfect fishing selectivity and uniform harvesting. H^{msy} (in tonne) and associated biomass level B^{msy} (in tonne) and fish stocks X_1^{mey} and X_2^{mey} (in 1,000 # of fish)

	f_1^{mey}	f_2^{mey}	X_1^{mey}	X_2^{mey}	B^{mey}	H^{mey}
Perfect selectivity. Baseline parameter values	0	1.00	656	459	2,690	1,378
Perfect selectivity. Fleet 2 harvesting cost up, $c_2 = 30$	0.84	0	577	211	1,786	973
Perfect selectivity. Discount rent doubled, $\delta = 0.10$	0	1.00	656	459	2690	1378
Uniform harvesting. Baseline parameter values	0.48	0.48	633	368	2369	1125
Uniform harvesting. Harvesting cost up, $c = 30$	0.40	0,40	660	478	2755	1102
Uniform harvesting. Discount rent doubled, $\delta = 0.10$	0,50	0,50	623	336	2253	1127

Table A1. Biological and economic baseline parameter values

Parameter	Description	Baseline value
s_0	Natural survival rate recruits	0.6
s_1	Natural survival rate young mature	0.7
s_2	Natural survival rate of mature	0.7
a	Scaling parameter recruitment function	1,500 (1,000 # of fish)
b	Shape recruitment function	500 (1,000 # of fish)
α	Fertility parameter	1.5
w_1	Weight young mature	2 (kg/fish)
w_2	Weight old mature	3 (kg/fish)
q_1	Catchability coefficient fleet one	0.01 (1/effort)
q_2	Catchability coefficient fleet two	0.01 (1/effort)
p_1	Fish price young mature	1 (Euro/kg)
p_2	Fish price old mature	1 (Euro/kg)
c_1	Effort cost fleet one	10 (Euro/effort)
c_2	Effort cost fleet two	10 (Euro/effort)
δ	Discount rent	0.05

Table A2: Uniform fishing. Sustainable yield (in tonne), biomass (in tonne) and fish stocks (1,000 # of fish). Baseline parameter values

f	X_1	X_2	B	H
0	789	1840	7099 (B_K)	0
0.1	759	1293	5397	540
0,2	728	926	4236	847
0.3	695	668	3394	1018
0.4	660	478	2555	1102
0.5	623	336	2253 (B^{msy})	1127 (H^{msy})
0.6	584	227	1850	1110
0.7	542	144	1517	1062
0.8	489	81	1240	992
0.9	451	34	1003	902
1.0	400	0	800 ($B_{\min}$)	800

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