# Continuous cover forestry vs. clearcuts with optimal carbon storage

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Abstract

This study applies a novel forest economic model to analyze the effect of Pigouvian carbon subsidies on the economically optimal choice between clearcuts and continuous cover forestry. Unlike previous studies, we determine the optimal management system endogenously, by optimization. We show analytically that subsidized carbon sequestration postpones thinning and increases optimal stand volume along the rotation. A very high carbon price makes it optimal to postpone thinning until the stand volume has surpassed the growth-maximizing level. In this case, the scarce resource is not wood but the remaining capacity for carbon sequestration, and the goal of thinning is to maintain optimal stand growth and carbon subsidies. Carbon subsidization favors continuous cover management by increasing the present value of revenues from thinning and by decreasing clearcut net revenues, and disfavors it by increasing bare land value. Numerical results show that carbon prices within a realistic range may switch the optimal management system from clearcuts to continuous cover management. The optimal choice between the two regimes is found to be sensitive to the level of timber product decay and subsequent cuts to the subsidy payments. We also show that a higher interest rate can lead to a higher stand volume and a longer optimal rotation, which contrasts the results of the classical Faustmann model. Additionally, we show that carbon pricing is likely to increase wood supply.

Keywords: Uneven-aged forestry, carbon sequestration, optimal rotation, Faustmann formula, dynamic optimization

# 1. Introduction

As societies face the necessity of addressing climate change, the capacity of forests to act as carbon sinks gains in importance (IPCC 2000, 2014). During the past few decades, as much as 30 % of annual global anthropogenic $CO_2$ emissions have been absorbed by forests (Pan et al. 2011). Further, the world's forest ecosystems together hold more than double the amount of carbon in the atmosphere (FAO 2006). Thus carbon sequestration benefits provided by forests constitute an immense economic externality (Canadell and Raupach 2008) that calls for Pigouvian policies. As of yet, forest management is not targeted by any climate policy instruments in the European countries. However, New Zealand has since 2008 applied a system where forest owners can earn carbon credits for the growth of their forests within the framework of the internationally linked New Zealand Emissions Trading Scheme[1] (see Adams and Turner 2012, Tee et al. 2014). Our study applies a novel forest economic model to analyze the effect of optimal carbon storage on one of the fundamental questions in forestry today: the choice between plantation type of forestry based on clearcuts and management that maintains forest cover continuously.

In most Nordic countries, forestry has since the 1930s been based on an officially promoted rotation regime, where forest stands are artificially regenerated, thinned, and finally clearcut, resulting in even-aged stands (Siiskonen 2007, Kotilainen and Rytteri 2011, Lundmark et al. 2013). Also Canadian forestry has traditionally been strongly oriented toward even-aged management (Gauthier et al. 2009). However, criticism of clearcuts and discussion on management alternatives, such as continuous cover forestry, has recently been on the rise in all of these countries (Lämås and Fries 1995, Puettmann et al. 2009, Valkeapää and Karppinen 2013). On the British Isles, forest authorities explicitly encourage foresters to replace even-aged systems with more complex and diversified forests (Mason 2015). Continuous cover forestry (or uneven-aged forest management) targets harvesting to the largest tree-classes and utilizes natural regeneration, resulting in a heterogeneous size distribution and lacking the expensive initial investment in artificial regeneration. Continuous cover management attracts interest as an approach that allows a forest owner – or the society at large – to combine timber production objectives with other goals: recreational use, collecting non-timber forest products and maintenance of biodiversity (Cedergren 2008, Thompson et al. 2009). In addition to being more favorable to many forest-dwelling species (Dahlberg 2011, Calladine et al. 2015), uneven-aged forests are likely to be more resilient against

[1] Earning of carbon credits is restricted to forests that were not forest land in 1989 or were deforested between 1990 and 2007. Older forests are subject to emission offset regulation if they are deforested. (New Zealand Ministry for Primary Industries 2013 a, b.)

climate change and other threats than even-aged forests (Thompson et al. 2009, Ray et al. 2010). In Finland, the Forest Act reform in January 2014 (MAF 2014) expanded the set of permissible management options to allow continuous cover management as an alternative to clearcuts. Meanwhile in Sweden, the forest authorities are working to produce and disseminate information on the applicability of continuous cover management in private as well as state owned forests (Cedergren 2008).

Recent research suggests that uneven-aged management may be economically competitive compared to even-aged forestry (Haight and Monserud 1990, Tahvonen 2009, Tahvonen et al. 2010, Kuuluvainen et al. 2012). However, the common conception among silviculturalists and many forest economists is that while continuous cover management may be preferable in ecological and social terms, it remains economically inferior compared to clearcut management (Cedergren 2008). This discrepancy largely follows from divergent research methods. First, silvicultural studies, such as those referred in Cedergren (2008), typically do not optimize uneven-aged management at all but merely simulate stand development under *ad hoc* management strategies. Second, economic optimization of uneven-aged forestry has suffered from prolonged theoretical confusions. The economics of uneven-aged forestry was perhaps first studied by numerical nonlinear optimization in Adams and Ek (1974). Their paper and following ones by various authors apply predetermined endpoints that the stand must reach through a transition path (see discussion in Tahvonen 2009). This approach is criticized by Haight (1985) and Haight and Getz (1987), who present the uneven-aged problem as an infinite horizon problem without endpoint restrictions. Since then, research on uneven-aged forestry has branched off into two distinct approaches: the general dynamic approach in the vein of Getz and Haight (1989), and the static investment efficient approach introduced in Adams (1976).

The general dynamic approach consists of optimizing the management of a stand over the infinite time horizon given any initial state. In contrast, the "investment efficient" (IE) approach aims to define the optimal steady state stand stocking by understanding the stumpage value of trees left unharvested in continuous cover cuttings as an investment cost similarly as regeneration cost in even-aged forestry (Adams 1976). However, the stumpage value of the residual trees represents an opportunity cost that occurs given the choice is a continuous cover harvest instead of a clearcut. While the opportunity cost is relevant when comparing the profitability of uneven- and even-aged management, it should not be included in the optimization model when specifying economically optimal continuous cover management (Rämö and Tahvonen 2014). Various other theoretical problems of the IE approach have been discussed in Haight (1985), Getz and Haight (1989, 287–

295), Tahvonen and Viitala (2006) and Tahvonen (2011). Yet, the IE approach is widespread in silvicultural literature on continuous cover forestry; see e.g. Schütz (2012, 24–26).

Our study follows the general dynamic optimization train of thought. The general dynamic approach has been combined with an individual-tree model (Haight and Monserud 1990) and applied to examine the optimal choice between even- and uneven-aged forestry (Tahvonen 2009). It has also been extended to an individual-tree model of Norway spruce with five thermal zones (Tahvonen 2011) as well as single species Scots pine and birch forests in Fennoscandia (Rämö and Tahvonen 2014).

The economic research on continuous cover forestry has been purely numerical but very recently it has turn out that the problem can also be studied analytically, either with a discrete time size-structured model (Tahvonen 2015) or with a continuous time biomass model (Tahvonen 2014). The latter approach extends the model by Clark (1976, p. 263) and enables one to study analytical features of uneven-aged forestry and the choice between these forest management alternatives. Our study extends this model by including subsidized carbon sequestration.

Carbon sequestration has been studied mainly by the classical optimal rotation model. Van Kooten et al. (1995) examine the effect of carbon taxes and subsidies on optimal rotation and supply of carbon services. The authors write that the internalization of carbon benefits generally increases rotation ages only moderately but might in some cases yield the result that it is optimal never to harvest (clearcut) the stand. The study by van Kooten et al. (1995) and most of their followers apply the simple rotation model where forests can be harvested by clearcutting only. In e.g. Nordic context this is a strong simplification since more than 40% of bare land value may originate from partial harvesting (i.e. thinning) before the clearcut (Niinimäki et al. 2013). Recent research on economically optimal carbon storage include Niinimäki et al. (2013) for even-aged Norway spruce and Pihlainen et al. (2014) for even-aged Scots pine, both computed using a detailed process-based model. These studies show that changing thinning strategies is at least as important as lengthening the rotation period for economically efficient carbon storage.

Research on uneven-aged forest management with carbon sequestration is also emerging. Goetz et al. (2010) present an integrated biophysical and economic model for determining the optimal selective management (harvesting and planting) regime for Scots pine in Spain when considering timber and carbon sequestration in the biomass, wood products and forest soil. They conclude that an increase in carbon price leads to a notable increase in the number of trees, and that sequestration costs are significantly lower for changes in forest management than for a change in land use.

Pukkala et al. (2011) compare uneven- and even-aged management systems “in spruce and pine stands in terms of timber, carbon, and bilberry benefits”. Buongiorno et al. (2012) present a compromise policy that maximizes carbon storage while maintaining a rate of return on the capital of standing trees equal to the interest rate. However, both Pukkala et al. (2011) and Buongiorno et al. (2012) apply the economically flawed IE approach. Parajuli and Chang (2012), in turn, apply what they call “the generalized Faustmann formula” for uneven-aged management with carbon sequestration. The formulation of the optimization problem is a variation of the IE approach. According to their results, carbon subsidies will not alter the optimal solutions for uneven-aged loblolly pine stands significantly.

In short, there is a solid body of research on economically optimal even-aged forestry with carbon sequestration with and without optimized thinning. For uneven-aged forestry, the number of such studies is quite limited, and most of the existing studies apply optimization methods that leave considerable room for improvement. Furthermore, no analytical solutions have been presented for the problem of uneven-aged forestry with carbon sequestration. Also completely missing is a theoretically sound comparison of the optimality of the two management systems when carbon sequestration is subsidized. The present paper aims to contribute to these questions.

The economic model presented in this study is an extension of the model that was introduced in Kilkki and Väisänen (1969), further developed in Clark (1976) and revisited in Tahvonen (2014). The model combines elements of the Schaefer biomass harvesting model with Faustmann rotation (Schaefer 1957, Faustmann 1849). In Tahvonen (2014), the assumptions on the aging function are revised to account for natural regeneration: the decreasing growth of the aging original trees is supplemented by the density dependent growth of new, naturally regenerated trees. Thus, by approximating the dynamics of uneven-aged forests, the stylized model enables the analysis of both uneven-aged forestry and the choice between forest management alternatives. Our study includes subsidized carbon sequestration by modifying and extending the van Kooten et al. (1995) formulation, where carbon benefits are a function of the change in biomass.

This is the first study to endogenously determine the optimal forest management system under carbon subsidization. We consider different carbon subsidy policies, where timber harvesting causes no release of carbon and hence no subsidy subtractions, or where the partial or complete release of the carbon content of each harvested unit causes a corresponding subsidy cutback. We show analytically and numerically that subsidized carbon sequestration postpones the start of thinning and increases optimal stand volume before the possible clearcut. If the carbon price is very high relative to stumpage price, it is optimal to postpone thinning until the stand volume has surpassed the growth-maximizing level. In this case, the shadow price of stand volume is negative because the scarce resource is not wood but the remaining capacity for carbon sequestration. Hence the stand is thinned to maintain optimal stand growth and carbon sequestration revenues, despite negative direct net revenues from thinning. Carbon subsidization favors continuous cover management by increasing the present value of revenues from the thinning period and by decreasing clearcut net revenues, and disfavors it by increasing bare land value. According to numerical results, carbon prices within a realistic range may switch the optimal management system from clearcuts to continuous cover management, but only if timber harvesting causes subsidy subtractions. We also show that a higher interest rate can lead to a higher stand volume and a longer optimal rotation. The latter result is in stark contrast to the one yielded by the established Faustmann rotation approach. Additionally, we show that carbon pricing is likely to increase wood supply.

## 2. Biomass model for economically optimal thinning and rotation

Studying the effects of carbon subsidization on optimal uneven-aged forest management, including the choice between continuous cover forestry and clearcuts, necessitates a model that can describe both management types and allows the superiority of either one to be determined endogenously, by optimization. Such a model, a reformulation of the one introduced in Kilkki and Väisänen (1969) and further developed by Clark (1976), is presented in Tahvonen (2014).

In this optimal rotation and thinning model, the goal of the forest owner is to maximize the net present value of the next and all future rotations. Let *x(t)* denote the stand volume ($m^3$ $ha^{-1}$) and *h(t)* the rate of harvested volume ($m^3$ $a^{-1}$ $ha^{-1}$) in thinning. Parameter $x_0$ denotes the initial stand volume while $t_0 = 0$ is the initial moment. Additionally, *w* is the regeneration cost, $\delta$ the interest rate, and $p_1$ and $p_2$ the stumpage prices for thinning and clearcut, respectively. The stand can be thinned continuously until the moment *T*, when the stand is clearcut. The optimal rotation *T*, however, might be infinitely long. The optimization problem takes the form

$$\max_{\{h(t),T\}} V = -w + \int_0^T p_1 h(t) e^{-\delta t} dt + e^{-\delta T}\left[p_2 x(T) + V\right],$$

subject to

$$\dot{x} = g(t)\, f\left(x(t)\right) - h(t),\ \ x(t_0) = x_o,$$

where

$$\mathrm{V} = \frac{-w + \int_0^T p_1 h(t) e^{-\delta t} dt + e^{-\delta T} p_2 x(T)}{1 - e^{-\delta T}}$$

denotes the value of bare land. The differential equation for stand volume describes stand growth as a product of aging $g(t)$ and density dependent growth $f(x)$. Clark (1976) assumes that $g'(t) < 0$ and $g(t) \to 0$ as $t \to \infty$ and that $f$ is a single-peaked function. These assumptions on aging are suitable for pure plantation forestry and yield optimal finite rotation periods. Here we specify the functions to include natural regeneration, i.e. $g$ remains strictly positive as $t \to \infty$. Thus, the growth and aging functions are assumed to satisfy

$$f(0) \geq 0,\ f(\underline{x}) = 0,\ f''(x) < 0,\ f'(\hat{x}) = 0,\ 0 < \hat{x} < \underline{x}, \tag{A1}$$

$$g(0) > 0, g'(t) < 0, g''(t) > 0, \lim_{t \to \infty} g(t) = \tilde{g} > 0, \tag{A2}$$

$$\tilde{g}f'(0) > \delta, \tag{A3}$$

as in Tahvonen (2014). Natural regeneration implies that even if the stand is managed by clearcuts, it will generally not be even-aged but consist of trees of different ages. An example of such a growth function is shown in Figures 1 and 2, based roughly on data from Bollandsås et al. (2008). If left undisturbed, the stand will approach a volume of 370 $m^3$ $ha^{-1}$. The growth of the stand is maximized at the volume of approximately 181 $m^3$ $ha^{-1}$ and the long run maximum sustained yield is about 6.4 $m^3$ $a^{-1}$ $ha^{-1}$.

Next, we include carbon sequestration in the model. Following van Kooten et al. (1995), we study a policy where the society pays the forest owner a Pigouvian subsidy for the carbon that is sequestered by the stand as it grows. Here the external benefit is a function of the change in stand volume, not of the stand volume itself as in the Hartman formulation (van Kooten et al. 1995, Hartman 1976). In order to achieve cost-effectiveness, i.e. equal marginal costs for $CO_2$ abatement across the economy, the forestry carbon subsidy can be linked to a carbon price – assumed to be formed in an emissions trading market – by multiplying the carbon price by the amount of carbon stored in a wood volume unit. Present attempts to implement emission trading include various problems and lately the EU ETS carbon price level has remained below €10 $tCO_2^{-1}$ (Edenhofer 2014). Influential estimates of the social cost of carbon, i.e. marginal economic damage caused by an additional ton of carbon dioxide emissions (Nordhaus 2014), range from \$15 to \$33–106 $tCO_2^{-1}$, all expected to increase towards the year 2050 (Tol 2005, Stern and Dietz 2014, respectively)[2]. However, costs arising from the small probability of catastrophic damages are difficult to capture in such estimates, implying that the true social cost might be even higher (Pindyck 2013, Weitzmann 2014). In this study, to cover the middle ground between the estimated social cost and the current

[2] In 2014 US dollars, converted from 2004 (Tol; assumed because not reported) and 2012 (Stern & Dietz) US dollars.

EU ETS carbon price, most computations are carried out assuming a carbon price of €0–60 $tCO_2^{-1}$. For simplicity, the carbon price is assumed to be constant over time.

Harvesting wood from the stand may result in at least a part of the stored carbon being released, and the subsidy should be cut (subtracted) accordingly. Thus we include a negative term, where the harvest rate is multiplied by the carbon price, the carbon content of a wood volume unit, and a constant $\alpha$ which denotes the rate of carbon release at harvest. (In the formulation of van Kooten et al. (1995), the forest owner has to pay a tax for the released carbon, that is, the part of carbon that is not "pickled".) The value of $\alpha$ is defined as $0 \leq \alpha \leq 1$. If the society acknowledges exclusively the carbon stored in the forest stand, it is reasonable to subsidize the increase in stand volume net of harvesting. This implies that the subsidies will be subtracted for the full amount of harvested volume ($\alpha = 1$), which we call a *net subsidy system*. The underlying assumption, also employed in The New Zealand subsidy system (Manley and Maclaren 2010), is that harvesting wood biomass causes the instant release of all carbon stored within it. This could be the case if the wood is burned for bioenergy and no substitution effects are taken into account.

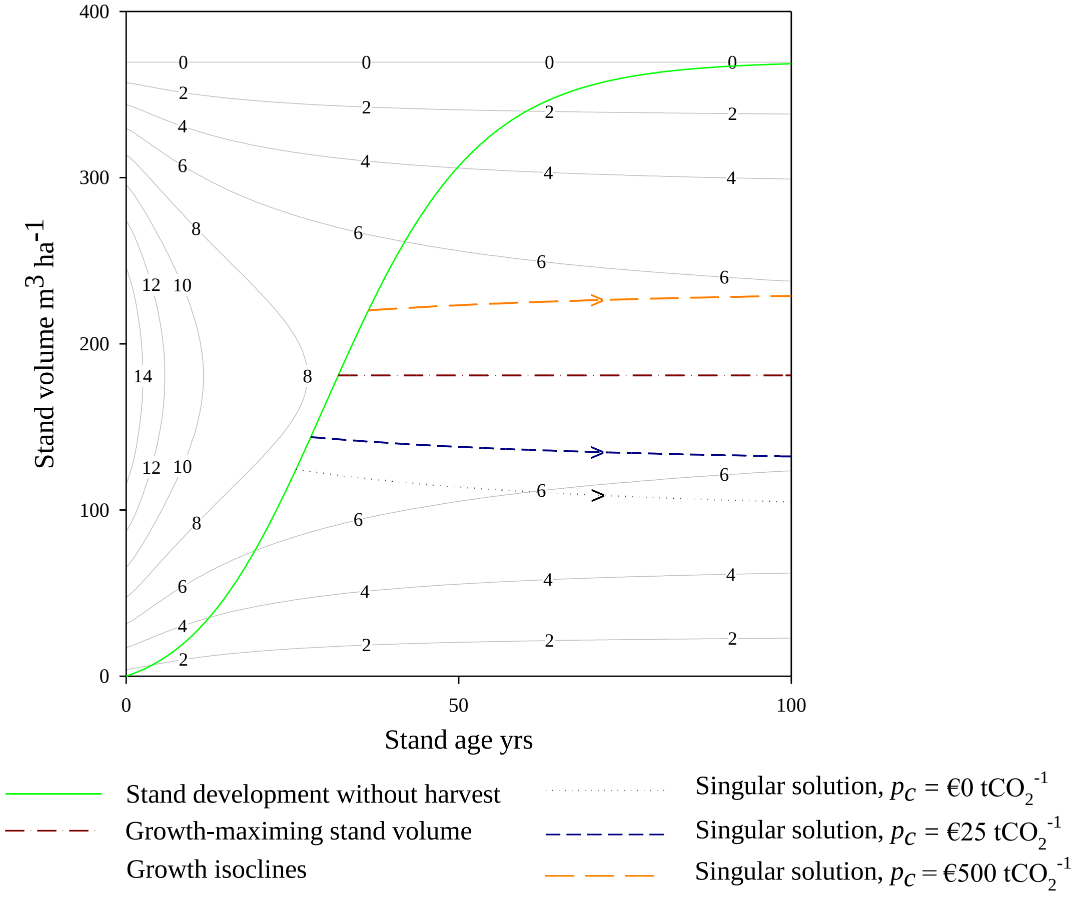


Figure 1. The growth function and singular solutions.
Note: $p$ = €40 $m^{-3}$, $\delta = 0.03$, $\alpha = 0.5$, $\mu = 0.7$.

However, if the carbon that remains stored in wood products is considered as well, harvesting is punished only partially or not at all ($0 \le \alpha < 1$). If $\alpha$ is zero, the society applies a *gross subsidy system* (Pihlainen et al. 2014). In this case, either it is known that no carbon is released into the atmosphere as a consequence of harvesting because it is permanently stored in wood products like buildings, or (more probably) the responsibility for releasing carbon is thought to lie with the user of wood material instead of the forest owner (Tahvonen 1995). Alternatively, the policy might rest on the assumption that the wood is used as a substitute for a carbon-intensive input, e.g. as an energy source instead of fossil fuels or as construction material in replacement of concrete.

Yet in reality, carbon neither stays in wood products forever nor is instantly fully released at harvest. Instead, it is gradually released as each wood product is decomposed according to its specific qualities (Pihlainen et al. 2014, cf. Goetz et al. 2010). Decisive, then, is how the flows of released carbon occurring at different points in time are valued – which depends on the interest rate (Pihlainen et al. 2014). Thus $\alpha$ becomes a function of interest rate. Further, as it is the society that defines $\alpha(\delta)$, the interest rate in question does not necessarily have to be the same as the one applied by the forest owner or any other market operator. However, to facilitate the analysis, $\alpha$ is assumed a constant and independent of any other parameters. Hence, a *product adjusted net subsidy system* is approximated by setting $0 < \alpha < 1$.

The problem of the forest owner is to maximize the net present value of the next and all future rotations. The control variables are the harvest (or thinning) rate *h(t)* and the rotation age *T*. The state variable is the stand volume, *x(t)*. Denote the carbon price with $p_c$ and the amount of carbon sequestered in a wood volume unit with $\mu$. Thus the problem takes the form

$$\max_{\{h(t),T\}} J = -w + \int_0^T e^{-\delta t}\left[ph(t) + p_c\mu g(t) f\left(x(t)\right) - p_c\mu\alpha h(t)\right]dt + e^{-\delta T}\left[px(T) - p_c\mu\alpha x(T) + V\right], \tag{1}$$

$$\text{s.t. } \dot{x} = g(t) f\left(x(t)\right) - h(t), \; x(t_0) = x_0 \;, \tag{2}$$

$$h \in \left[0, h_{max}\right], \tag{3}$$

where

$$\mathrm{V} = \frac{-w + \int_0^T e^{-\delta t}\left[ph(t) + p_c\mu g(t) f(x) - p_c\mu\alpha h(t)\right]dt + e^{-\delta T}\left[px(T) - p_c\mu\alpha x(T)\right]}{1 - e^{-\delta T}} \tag{4}$$

denotes the value of bare land.

In addition to assumptions (A1) – (A3) it is assumed that the stumpage price *p* in thinning equals the stumpage price in clearcut. In even-aged forestry with thinnings from below, stumpage price is lower for thinnings than for clearcuts because the former are typically more costly per m$^3$

and yield mostly pulpwood. However, in the present model thinning implicitly targets the largest trees of the stand, whereas in clearcuts small trees are harvested as well as the large ones implying higher harvesting cost per $m^3$ (Tahvonen 2011). This implies that the difference of stumpage prices is typically minor.

Moreover it is assumed that

$$\mu > 0, \tag{A4}$$

$$0 \le \alpha \le 1, \tag{A5}$$

$$\delta > 0. \tag{A6}$$

*Optimality conditions*

The Hamiltonian function and its partial derivatives read as follows.

$$H = (p - p_c\mu\alpha)h(t) + p_c\mu g(t) f(x) + \varphi(t)\left[g(t) f(x) - h(t)\right], \tag{5}$$

$$H_h = p - p_c\mu\alpha - \varphi,$$

$$H_x = g(t) f'(x)(p_c\mu + \varphi).$$

The Hamiltonian is linear in *h(t)* and the necessary optimality conditions take the form (Seierstad and Sydsæter 1987, 397: theorem 16)

$$\text{if } p - p_c\mu\alpha - \varphi < 0, \quad h = 0, \tag{6a}$$

$$\text{if } p - p_c\mu\alpha - \varphi = 0, \quad h \in [0, h_{max}], \tag{6b}$$

$$\text{if } p - p_c\mu\alpha - \varphi > 0, \quad h = h_{max}, \tag{6c}$$

$$\dot{\varphi} = \varphi\delta - H_x = \varphi\delta - g(t) f'(x)(p_c\mu + \varphi), \tag{7}$$

$$\varphi(T) - (p - p_c\mu\alpha) \ge 0, \quad x(T) \ge 0, \quad \left[\varphi(T) - (p - p_c\mu\alpha)\right]x(T) = 0, \tag{8}$$

$$\begin{aligned}&(p - p_c\mu\alpha)h(T) + p_c\mu g(T) f(x(T)) + \varphi(T)\left[g(t) f(x(T)) - h(T)\right] - \\ &\delta\left[(p - p_c\mu\alpha)x(T) + V\right] = 0.\end{aligned} \tag{9}$$

To analyze the sufficiency of the necessary conditions, derive

$$H_{hh} = 0, \; H_{xx} = g(t) f''(x)(p_c\mu + \varphi), \; H_{xh} = 0, \; H_{xx}H_{hh} - H_{xh}{}^2 = 0 .$$

It is not possible to rule out $p_c\mu + \varphi < 0$ a priori. Thus, Hamiltonian may not be concave in *h* and *x*. However, by the sufficiency theorem of Arrow (Sydsæter et al. 2008, 332) the necessary conditions are sufficient if maximized Hamiltonian is concave in $x$. By (6a, b) it follows that if optimal $h$ remains either in the singular solution or $h = 0$ regime the condition $0 < p \le p_c\mu\alpha + \varphi \le p_c\mu + \varphi$ holds true implying that necessary optimality conditions are sufficient for any fixed value of $T$. However, as emphasized by Sydsaeter et al. (2008, 336) the concavity of the maximized Hamiltonian does not imply sufficiency of the necessary conditions when the final time is free. Thus, the sufficiency of the necessary conditions in choosing the optimal $T$ will be analyzed separately.

*The properties of optimal thinning*

The switching function is $\sigma \equiv p - p_c\mu\alpha - \varphi$. To maintain the singular solution it must hold that $\sigma = \dot{\sigma} = \ddot{\sigma} = ... = 0$. Differentiating $\sigma$ with respect to time yields $-\dot{\varphi} = 0$. Utilize (7) and then (6b) to obtain

$$\varphi\delta - g(t)\, f'(x)\left(p_c\mu + \varphi\right) = 0 .$$

Rearranging yields

$$\left(p - p_c\mu\alpha\right)\delta - g(t)\, f'(x)\left[p + (1-\alpha)p_c\mu\right] = 0 . \tag{10}$$

Note that if $p_c$ is set to zero, equation (10) reads

$$p\delta - pg(t)\, f'(x) = 0 \Leftrightarrow \delta - g(t)\, f'(x) = 0$$

as in Clark (1976, 265).

To characterize the stand volume along the singular solution, rearrange (10) into

$$f'(x) = \frac{p - p_c\mu\alpha}{p + (1-\alpha)p_c\mu}\ \frac{\delta}{g(t)}, \tag{11}$$

where

$$\frac{\delta}{\left[p + (1-\alpha)p_c\mu\right]g(t)} > 0 .$$

By assumptions on $f$ we have $f'(\hat{x}) = 0$. Thus, the singular solution satisfies the properties

$$\text{if} \begin{array}{l} p > p_c\mu\alpha,\ f'(x) > 0 \Rightarrow x < \hat{x}, \\ p = p_c\mu\alpha,\ f'(x) = 0 \Rightarrow x = \hat{x}, \\ p < p_c\mu\alpha,\ f'(x) < 0 \Rightarrow x > \hat{x}. \end{array}$$

Hence the optimal volume on the singular path will be above the growth-maximizing level if the stumpage price is sufficiently low relative to the carbon subsidy cutback, i.e. the carbon price multiplied by the carbon content of wood volume unit and the rate of carbon release at harvest.

To solve the optimal harvest on the singular path, differentiate (10) with respect to time:

$$\left[-g'(t)\, f'(x) - g(t)\, f''(x)\dot{x}\right]\left[p + (1-\alpha)p_c\mu\right] = 0 .$$

Utilizing (2) yields

$$\left\{-g'(t)\,f'(x)-g(t)\,f''(x)\left[g(t)\,f(x)-h(t)\right]\right\}\left[p+(1-\alpha)p_c\mu\right]=0,$$

$$g(t)\,f''(x)h(t)-g(t)\,f''(x)g(t)\,f(x)-g'(t)\,f'(x)=0,$$

$$h(t)=g(t)\,f(x)+\frac{g'(t)\,f'(x)}{g(t)\,f''(x)}.$$

Combining with (10) yields

$$h(t)=g(t)\,f(x)+\frac{p-p_c\mu\alpha}{p+(1-\alpha)p_c\mu}\;\frac{g'(t)\delta}{g^2(t)\,f''(x)}, \tag{12}$$

where $g'(t)\delta/\left[g^2(t)\,f''(x)\right]>0$ and $p+(1-\alpha)p_c\mu>0$. The first (positive) term of (12) corresponds to stand growth at *t*. If stumpage price net of carbon subsidy cutback is positive, the second term will be positive. In this case, thinning will exceed the stand growth and thus the stand volume must decrease on the singular path (see Figure 1). If $p<p_c\mu\alpha$, thinning level will fall below the stand growth and the stand volume will increase on the singular path. The special case $p=p_c\mu\alpha$ yields a singular path where $f'(x)=0$, i.e. thinning keeps the stand volume at the level where stand growth is maximized.

If $p<p_c\mu\alpha$, then $\varphi<0$ and $f'(x)<0$ along the singular path. Thus if the carbon subsidy cutback exceeds the stumpage price, the shadow price of the stand volume is negative. This is because any increase in stand volume decreases stand growth ($f'(x)<0, x>\hat{x}$) and thus the very valuable sequestration of carbon, and provides no additional benefit as the direct net revenues from harvesting are negative. In this case, the scarce resource is not wood but the remaining capacity for carbon sequestration. The negative shadow price explains the seemingly unintuitive finding that optimal harvest rate can be positive even when the direct net revenues from harvesting are negative: the stand volume is controlled in order to maintain a sufficient rate of stand growth and carbon sequestration. As the carbon subsidy cutback $p_c\mu\alpha$ can never be larger than the carbon subsidy $p_c\mu$ that is paid for the stand growth, and the stand growth exceeds the harvest rate, the combined net revenues from harvesting and carbon sequestration will actually be positive.

Given $x_0=0$ it follows that $\dot{\varphi}(0)=\varphi\delta-g(t)\,f'(x)\left(p_c\mu+\varphi\right)<0$ by (7) and if $\varphi(0)>p-p_c\mu\alpha$ then $h(0)=0$ by (6a). The correct choice of $\varphi(0)$ implies that $\varphi(t_1)=p-p_c\mu\alpha$ at the same moment when the solution for $\dot{x}=g(t)\,f(x)$, $x_0=0$ intersects the singular solution in $x,t$ plane and $h$ jumps to the singular solution level defined by (12). This means that initially, the stand is left to grow undisturbed because the net price for harvesting is below the value of the co-state variable (the shadow price).

Next we show how carbon subsidization changes the stand volume on the singular path. Equation (10) can be rearranged to

$$f'(x)g(t) - \frac{p - p_c \mu\alpha}{p + (1-\alpha)p_c \mu}\delta = 0. \tag{13}$$

Given $p_c = 0$ this equation reduces to $f'(x)g(t) - \delta = 0$. Since

$$\frac{p - p_c \mu\alpha}{p + (1-\alpha)p_c \mu} = \frac{p - p_c \mu\alpha}{p + p_c \mu - p_c \mu\alpha} < 1,$$

it follows by the concavity of $f$ that subsidized carbon sequestration increases the stand volume given any age of the stand. This implies that thinning must start later under carbon subsidies. Numerical examples of this result are shown in Figures 1 and 2.

*Comparative statics of the singular solution*

Define

$$\Omega \equiv \frac{(p - p_c \mu\alpha)\delta}{p + (1-\alpha)p_c \mu}. \tag{14}$$

By (13) $\partial x / \partial \Omega = 1/\left[g(t) f''\right] < 0$, i.e. the smaller is $\Omega$, the higher is the stand volume along the singular solution. Differentiation yields

$$\frac{\partial \Omega}{\partial p} = \frac{\delta p_c \mu}{\left[p + (1-\alpha)p_c \mu\right]^2} > 0.$$

Thus, $\partial x / \partial p < 0$ implying that the higher is $p$ the lower is stand volume along the singular solution. When the stumpage price rises, the profitability of timber production relative to carbon sequestration services increases. It thus becomes optimal to start the thinning earlier and reach a lower stand volume. Next we obtain

$$\frac{\partial \Omega}{\partial p_c} = \frac{-\delta p \mu}{\left[p + (1-\alpha)p_c \mu\right]^2} < 0.$$

Thus, $\partial x / \partial p_c > 0$ implying that the higher is $p_c$ the higher is stand volume along the singular solution. The effect is similar for $\mu$, $CO_2$ content of a timber volume unit. When the carbon price increases, the profitability of timber production relative to carbon sequestration is reduced, and it becomes optimal to start the thinning later and reach a higher stand volume.

Additionally, note that if $\alpha = 0$ (no carbon is released back to the atmosphere from wood products) we obtain from (13)

$$f'(x)g(t) = \frac{\delta p}{p + p_c \mu}.$$

Thus, when $p_c \to \infty$, it follows that $f' \to 0$, i.e. the stand volume approaches the level that produced the maximum growth. Thus an extremely high carbon price works in favor of a growth-maximizing thinning solution under a gross subsidy system.

However, if $0 < \alpha < 1$,

$$f'(x)g(t) = \frac{(p - p_c \mu\alpha)\delta}{[p + (1-\alpha)p_c \mu]}.$$

Now when $p_c \to \infty$, $(p - p_c \mu\alpha)\delta / [p + (1-\alpha)p_c \mu] \to -\alpha\delta/(1-\alpha) < 0$ from above. Hence, stand volume on the singular path rises above the growth-maximizing level and further towards a level where $f'(x)g(t) = -\alpha\delta/(1-\alpha)$ as the carbon price approaches infinity. In other words, under a product adjusted net subsidy system it becomes optimal to keep the stand level above the level that maximizes forest growth when carbon price is very high.

If $\alpha = 1$ (all carbon is released immediately to atmosphere),

$$f'(x)g(t) = \frac{(p - p_c \mu)\delta}{p}$$

implying that when $p_c \to \infty$, $(p - p_c \mu)\delta / p \to -\infty$. Under a net subsidy system, the stand volume on the singular path rises above the growth-maximizing level and further towards the carrying capacity as the carbon price approaches infinity, suggesting that thinning becomes suboptimal.

Next we obtain

$$\frac{\partial \Omega}{\partial \alpha} = \frac{\delta p_c \mu(-p_c \mu)}{[p + (1-\alpha)p_c \mu]^2} < 0.$$

Thus, $\partial x / \partial \alpha > 0$ implying that the higher is the rate of carbon release at harvest, the higher is stand volume along the singular solution. When harvesting releases a larger fraction of a wood unit's carbon content and thus causes a larger subsidy subtraction, the profitability of wood production relative to carbon sequestration is reduced. It becomes optimal to start the thinning later and reach a higher stand volume.

Finally,

$\dfrac{\partial \Omega}{\partial \delta} = \dfrac{p - p_c \mu\alpha}{p + (1-\alpha)p_c \mu} > 0$ if $p > p_c \mu\alpha$, $\dfrac{\partial \Omega}{\partial \delta} < 0$ if $p < p_c \mu\alpha$, and $\dfrac{\partial \Omega}{\partial \delta} = 0$ if $p = p_c \mu\alpha$.

Thus, if $p > p_c\mu\alpha$, then $\partial x / \partial\delta < 0$ implying that the higher is $\delta$ the lower is stand volume along the singular solution. If the stumpage price net of carbon subsidy cutback is positive, a rise in the interest rate makes it optimal to start thinning and thus the flow of stumpage revenues earlier, leading to a lower stand volume on the singular path.

If $p = p_c\mu\alpha$, then $\partial x / \partial\delta = 0$ implying that the level of $\delta$ will not have any effect on the stand volume along the singular solution. The net revenues from harvesting will be zero, and the income will only consist of the subsidies that are paid according to the stand growth. Thus it is optimal to reach and maintain the stand level that maximizes stand growth, irrespective of the interest rate.

However, if $p < p_c\mu\alpha$, then $\partial x / \partial\delta > 0$ implying that the higher is $\delta$ the higher is stand volume along the singular solution. If stumpage price net of carbon subsidy cutback is negative, it is worthwhile to utilize the high-growth early years of the stand exclusively for carbon sequestering and to begin thinning only when the high volume of the stand starts to become an impediment to its growth. A rise in the interest rate amplifies the preference for current net subsidies versus future gains through enhanced growth, thus postponing the start of thinning and leading to a higher stand volume on the singular path.

*Numerical examples of singular paths*

We apply a growth function specification that is roughly in line with Bollandsås et al. (2008) and assume

$$g(t)f[x(t)] = \left(\frac{1.6}{1+0.04t^{1.2}}+1\right)0.065[x(t)+8]\left[1-\frac{x(t)+8}{378}\right], x_0 = 0 . \qquad (15)$$

In addition, we set $\mu = 0.7$ $tCO_2$ $m^{-3}$ (Niinimäki et al. 2013) and we assume that harvesting a timber volume unit releases half of its carbon content ($\alpha = 0.5$). Singular stand volume paths with three different carbon prices are obtained as shown in Figure 1 (dotted line and dashed lines), given a stumpage price of €40 $m^{-3}$ and an annual interest rate of 0.03 . Assuming the specification (15), the differential equation $dx/dt = g(t)f[x(t)],\ \ x_0 = 0$ can be solved analytically, and the solution is the light green solid line. An interception point of a singular path and the undisturbed path is denoted by $t_1$, which is the switching moment and the start of optimal thinning.

The lowest stand volume on the singular path follows with zero carbon price (Figure 1). With a positive carbon price thinning starts later and the stand volume on the singular path is higher. If the stumpage price is larger than the carbon subsidy cutback, the stand volume will lie below the volume maximizing level and will decrease on the singular path. However, if the carbon price is sufficiently large to render the term $p - p_c\mu\alpha$ negative, the stand volume will be above the growth-maximizing level and keep on increasing while the stand is thinned.

The dependence of stand growth rate on stand age and volume is presented in three-dimensional space in Figure 2. Growth is at its highest when the stand is young and tapers down as the stand ages. Given any stand age, the growth rate increases as the stand volume approaches 181 $m^3$ $ha^{-1}$ from below or above. The solid green line depicts the development of an undisturbed stand: when the stand is 31 years old, it reaches the growth-maximizing volume, and as it approaches the carrying capacity steady state volume of 370 $m^3$ $ha^{-1}$, growth diminishes to zero. As the dotted line and the dashed line show, setting the carbon price to €25 $tCO_2^{-1}$ increases stand volume and growth rate on the singular path compared to the solution with zero carbon price.

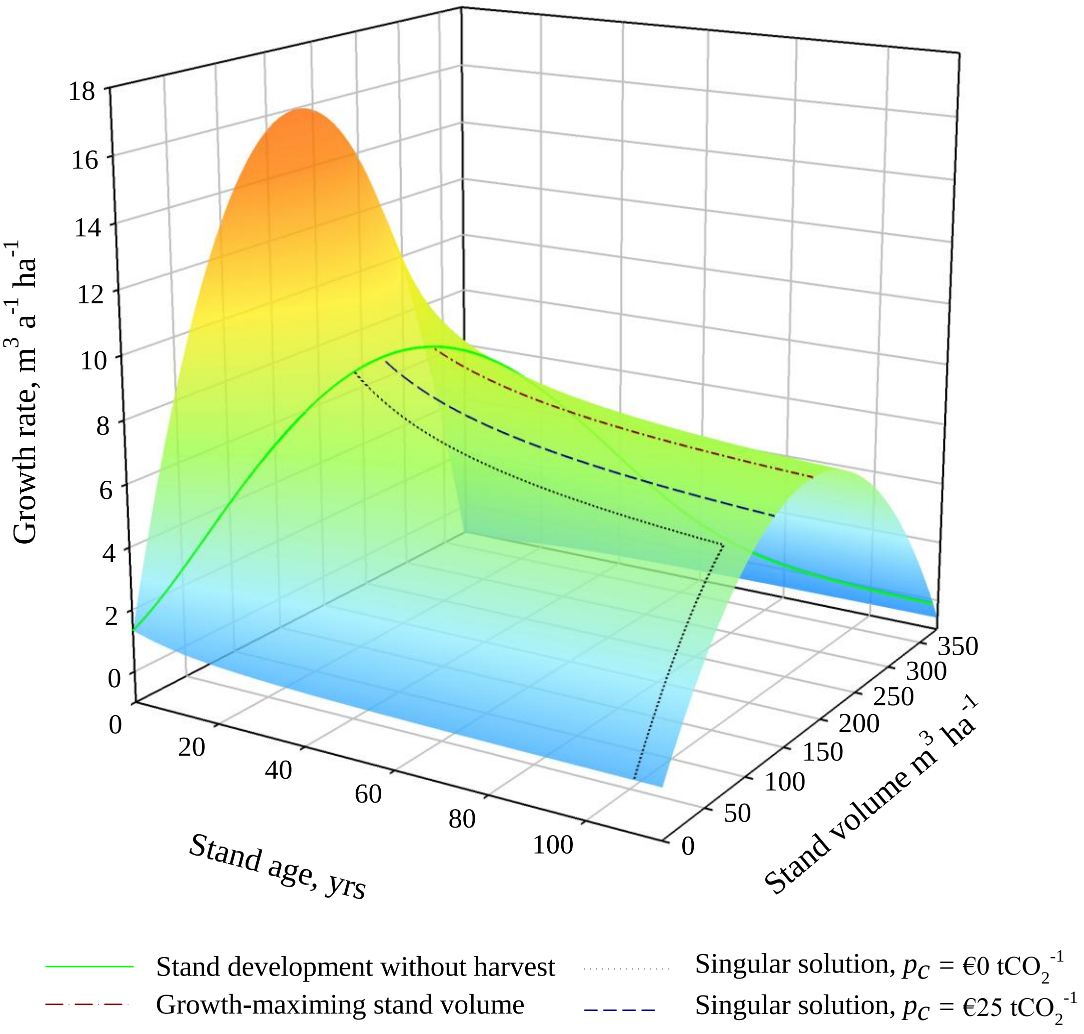


Figure 2. The dependence of stand growth on stand age and volume.
Note: $p$ = €40 $m^{-3}$, $\delta$ = 0.03, $\alpha$ = 0.5, $\mu$ = 0.7.

*Optimal rotation age*

The question of whether a clearcut regime is economically preferable to continuous cover management is ultimately a question of optimal rotation age. Finite optimal rotation means clearcuts while infinite rotation means continuous forest cover. The effect of subsidized carbon sequestration on this decision is one of the central questions of this study.

Following the singular path to the end of rotation satisfies (8) as an equality, i.e. $\varphi(T) = p - p_c\mu\alpha$. Thus (9) can be given as

$$y(T) \equiv (p - p_c\mu\alpha)h(T) + p_c\mu g(T) f(x(T)) + (p - p_c\mu\alpha)\left[g(t) f(x(T)) - h(T)\right] - \delta\left[(p - p_c\mu\alpha)x(T) + V\right] = 0$$

implying

$$y(T) = \left[p + (1-\alpha)p_c\mu\right]g(T) f(x(T)) - \delta\left[(p - p_c\mu\alpha)x(T) + V\right] = 0. \tag{16}$$

Note that

$$\mathrm{V} = \frac{-w + \int_0^{t_1} p_c\mu g(t) f(x)e^{-\delta t}dt + \int_{t_1}^{T}\left[(p - p_c\mu\alpha)h(t) + p_c\mu g(t) f(x(t))\right]e^{-\delta t}dt + (p - p_c\mu\alpha)x(T)e^{-\delta T}}{1 - e^{-\delta T}}$$

where $h(t),\ t_1 \le t \le T$ is determined by the singular solution.

Assume that $y(t_1) > 0$. If $y'(T) < 0$, the optimal finite $T$ is unique and if $y(T) \ge 0$ when $T \to \infty$, there cannot exist any finite $T$ satisfying (16). Differentiating *y* with respect to *T* yields

$$y'(T)_{|y(T)=0} = \left[p + (1-\alpha)p_c\mu\right]\left[g'(T) f(x(T)) + g(T) f'(x)\dot{x}\right] - \delta\left[(p - p_c\mu\alpha)\dot{x} + V'\right],$$

$$y'(T)_{|y(T)=0} = \left[p + (1-\alpha)p_c\mu\right](g'f + gf'\dot{x}) - \delta(p - p_c\mu\alpha)\dot{x}.$$

Rearranging (10) into $\delta(p - p_c\mu\alpha) = g(t) f'(x)[p + (1-\alpha)p_c\mu]$, we can write

$$y'(T)_{|y(T)=0} = \left[p + (1-\alpha)p_c\mu\right](g'f + gf'\dot{x} - gf'\dot{x}) = \left[p + (1-\alpha)p_c\mu\right]g'f < 0. \tag{17}$$

Because $p + (1-\alpha)p_c\mu$ is positive and $g'f$ negative, *y* slopes monotonically downwards at $T$. Thus the necessary conditions are sufficient for optimal rotation age and the optimal rotation is unique.

If equation (16) is solved by a finite *T*, a clearcut is optimal at *T*. If not, the optimal rotation is infinite and one should carry on thinning forever without clearcuts. If $\lim_{t\to\infty} g(t) = \tilde{g}$ is very low and $(p - p_c\mu\alpha)x(T) + V > 0$, then *y(T)* will be negative when *T* is sufficiently large. Thus the optimal rotation will be finite if the long term yield from a continuous cover forest is low enough and the sum of clearcut net revenues and bare land value is positive. However,

$$\left[p+(1-\alpha)p_c\mu\right]g(T)f\left(x(T)\right)-\delta\left[\left(p-p_c\mu\alpha\right)x(T)\right]>0$$

by (10) and the concavity of $f$.[3] This implies that if the bare land value is sufficiently small (e.g. negative), then $y(T)>0$ for $T\left[0,\infty\right)$, in which case the optimal rotation will be infinite. Additionally, if $p<p_c\mu\alpha$, then the interest cost on the bare land value is the only potentially negative element in $y(T)$, and infinite rotation follows if

$$\left[p+(1-\alpha)p_c\mu\right]g(T)f\left(x(T)\right)-\delta\left(p-p_c\mu\alpha\right)x(T)>\delta V$$

for $T\left[0,\infty\right)$.

The interpretation of (16) can be facilitated by utilizing (2) and writing

$$\left[p+(1-\alpha)p_c\mu\right]\left[h(T)+\dot{x}(T)\right]=\delta\left[\left(p-p_c\mu\alpha\right)x(T)+V\right]. \tag{18}$$

This implies that at the moment of the clearcut, the rate of timber and carbon revenues net of their decrease equals the interest on the clearcut net revenues and the value of bare land. Conversely, an infinite rotation is optimal if

$$\left[p+(1-\alpha)p_c\mu\right]\left[h(T)+\dot{x}(T)\right]-\delta\left[\left(p-p_c\mu\alpha\right)x(T)+V\right]>0 \tag{19}$$

for all $T\in\left[0,\infty\right)$. By (A2), (2) and (12),

$$\lim_{t\to\infty}h(t)=\tilde{g}f(\tilde{x})=\tilde{h} \tag{20}$$

and $\lim_{t\to\infty}\dot{x}(t)=0$. (21)

Thus when $T\to\infty$, the inequality (19) can be given as

$$\frac{\left[p+(1-\alpha)p_c\mu\right]\tilde{h}}{\delta}>\left(p-p_c\mu\alpha\right)\tilde{x}+V. \tag{22}$$

This together with the uniqueness result (17) implies that it is optimal to never clearcut the stand, if the present value of harvesting and net carbon subsidy revenues over the infinite horizon exceeds the sum of clearcut net revenues and bare land value.

*Comparative statics of optimal rotation age*

To study the effect of Pigouvian carbon subsidies on the relative competitiveness of continuous cover forestry and clearcuts, we start by considering whether an increase in carbon price lengthens

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[3] By (10), $\left[p+(1-\alpha)p_c\mu\right]g(T)f\left(x(T)\right)-\delta\left(p-p_c\mu\alpha\right)x(T)=\left[p+(1-\alpha)p_c\mu\right]g(T)\left[f\left(x(T)\right)-f'\left(x(T)\right)x(T)\right]$.
The concavity of $f$ implies that $f(x(T))/x(T)-f'\left(x(T)\right)>0$ and thus $f\left(x(T)\right)-f'\left(x(T)\right)x(T)>0$.

or shortens the optimal rotation. Taking into account that the solution satisfies the singular condition (10), the derivative of (16) with respect to carbon price can be given in the form

$$\frac{\partial y(T)}{\partial p_c} = (1-\alpha)\mu g(T) f(x(T)) + \delta\mu\alpha x(T) - \delta \frac{\partial V}{\partial p_c}, \tag{23}$$

where the first two terms are positive and the third negative. The first term represents an increase in the net carbon subsidy revenues at $T$ as the carbon price increases. The second term represents a decrease in the interest cost on the stand value right before the clearcut, because a higher carbon price translates to a larger subtraction of subsidies when the stand is clearcut. On the other hand, the third term reflects an increase in interest cost through the increased bare land value. Since carbon subsidies are an additional source of income to the forest owner, they can only increase the bare land value. The sign of the partial derivative is determined by the relative magnitudes of these three terms. Thus, a higher carbon price may lengthen or shorten the optimal rotation age.

If $\alpha = 0$, the partial derivative reduces to

$$\frac{\partial y(T)}{\partial p_c} = \mu g(T) f(x(T)) - \delta \frac{\partial V}{\partial p_c}. \tag{24}$$

Under a gross subsidy system, the carbon subsidy subtractions are not present and the sign of the partial derivative depends on whether the increase in carbon subsidy revenues at $T$ dominates the increase in interest cost on the bare land value.

If $\alpha = 1$, the partial derivative takes the form

$$\frac{\partial y(T)}{\partial p_c} = \delta \left[ \mu x(T) - \frac{\partial V}{\partial p_c} \right]. \tag{25}$$

In the case of immediate carbon release after harvest, a higher carbon price implies a longer rotation if the increase in the subsidies subtracted at clearcut is greater than the increase in the bare land value.

Hence the effect of carbon price on optimal rotation age and whether the rotation is finite or infinite will depend on the function specifications and parameter values. Given the growth function specification (15), we obtain optimal rotation lengths shown in Figures 3 and 4.

Optimal solutions under the three different subsidy policies are presented in Figure 3, given an interest rate of 3 %, a stumpage price of €40 $m^{-3}$ and a regeneration cost of €1000 $ha^{-1}$. The net subsidy system ($\alpha = 1$) assumes that all of the carbon stored in a wood volume unit is released to the atmosphere immediately at the time of its harvest, while the product adjusted net subsidy system ($\alpha = 0.5$) implies that half of the carbon content stays stored in wood products indefinitely. These cases correspond to the partial derivatives (25) and (23), respectively. In the gross subsidy system, all of the carbon is assumed to stay stored in wood products ($\alpha = 0$). Consequently harvesting is not punished, as seen in the partial derivative (24). Harvesting paths computed for the cases in Figures 3 and 4 confirm that harvest rates remain positive even when the carbon price is high enough to lead to an infinite rotation.

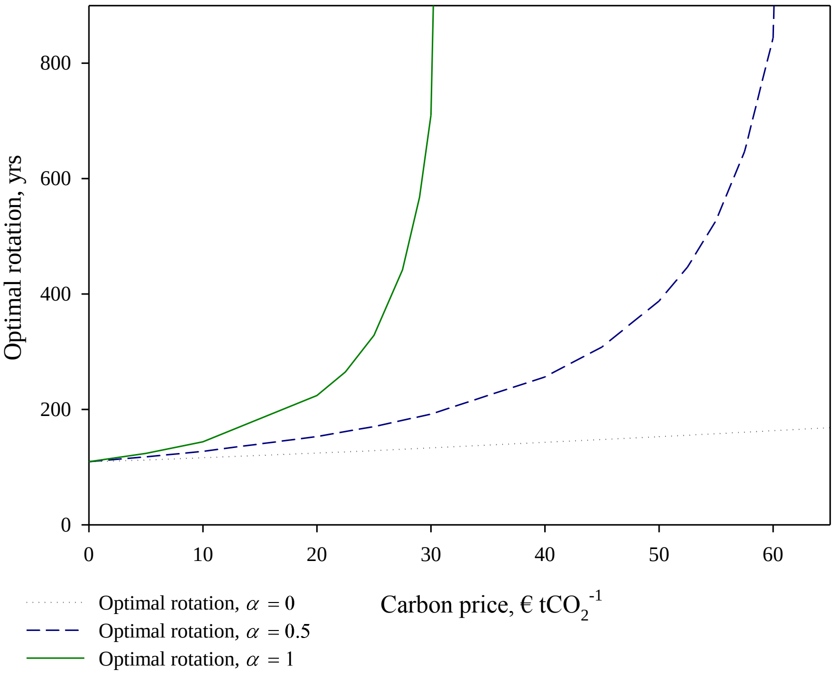


Figure 3. The effect of carbon price on optimal rotation.
Note: $p$ = €40 m$^{-3}$, $\mu$ = 0.7, $\delta$ = 0.03, $w$ = 1000.

Optimal rotation increases with carbon price regardless of subsidy system, but the strength of the effect varies dramatically. Given the net subsidy system ($\alpha = 1$), a carbon price little above €30 $tCO_2^{-1}$ leads to an infinite rotation. The strong rotation lengthening effect of carbon pricing stems from the considerable subsidy subtractions from harvests, diminishing the net carbon revenues and thus limiting the increase in bare land value (cf. Pihlainen et al. 2014). Hence the subsidy subtraction at clearcut is sufficient to overshadow the increase in the bare land value (see partial derivative (25)). Given a positive interest rate, carbon sequestration increases net present value and changes the clearcutting decision even if the carbon content of a wood volume unit is completely released at the time of its harvest. This is because discounting takes into account that the revenues from carbon sequestering precede the subsidy subtractions from harvests (especially clearcuts), i.e. under discounting it is optimal to shift net emissions forward in time. Under the product adjusted net subsidy system ($\alpha = 0.5$), carbon pricing clearly lengthens the rotation, but continuous cover management becomes optimal only when the carbon price is higher than €60 $tCO_2^{-1}$. Compared to the net subsidy policy, harvesting is punished less. Correspondingly, the incentive to postpone clearcutting (a decrease in interest cost on the stand value) is somewhat weaker, and the incentive to advance the clearcut (an increase in interest cost on the bare land value) is stronger. Under the gross subsidy system ($\alpha = 0$), the increase in rotation age is very minor. Because harvesting causes no subsidy subtractions, carbon pricing does not reduce clearcut net revenues at all. Moreover, carbon pricing increases bare land value more than under the two other subsidy systems, and the increase in carbon subsidy revenues at the moment before the clearcut barely offsets this effect. In short, the choice of subsidy system significantly affects how much an increase in the carbon price lengthens optimal rotation and the competitiveness of continuous cover forestry.

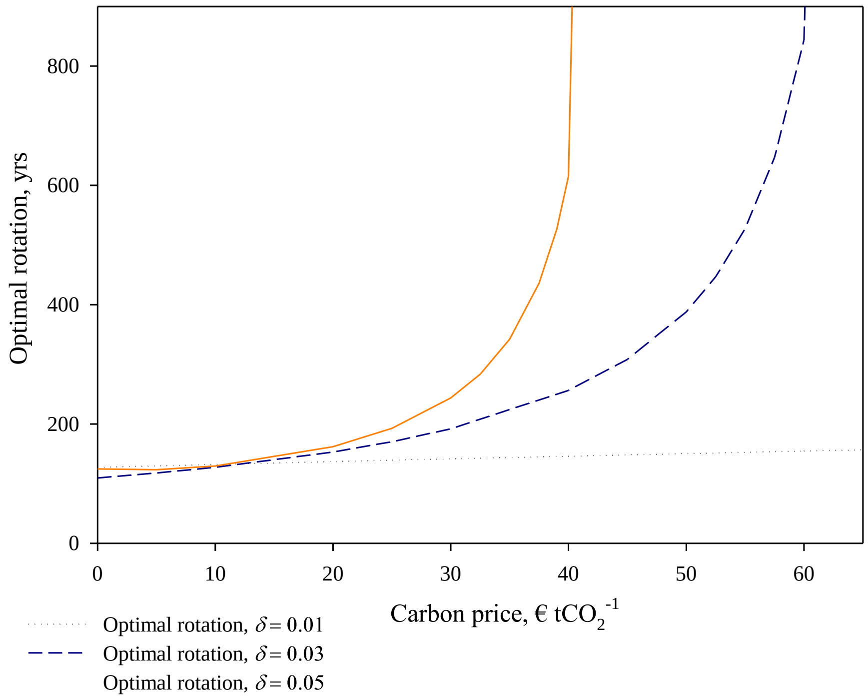


Figure 4. The dependence of optimal rotation on the carbon price, with various interest rates.
Note: $p$ = €40 $m^{-3}$, $\alpha = 0.5$, $\mu = 0.7$, $w$ = 1000.

The effect of carbon pricing on optimal rotation with various interest rates is shown in Figure 4. Note that all three cases portray a product adjusted net subsidy system ($\alpha = 0.5$). Thus the middle curve is equal to the middle curve in Figure 3. Given a 5 % interest rate, carbon price above €40 $tCO_2^{-1}$ makes continuous cover management optimal. Under a 3 % interest rate, continuous cover management becomes optimal when the carbon price exceeds €60 $tCO_2^{-1}$. Under a low interest rate (1%), the introduction of carbon pricing hardly increases the optimal rotation. Thus the rotation lengthening effect of a higher carbon price is the stronger the higher the interest rate. This observation can be interpreted as follows: under a product adjusted net subsidy system, a higher carbon price works in the favor of a longer rotation by increasing revenues just before the clearcut, and by diminishing clearcut net revenues and thus the interest cost of postponing the clearcut. The latter effect is amplified if the interest rate is higher. On the other hand, additional income from carbon subsidization also increases the cost of postponing the future rotations. However, the significance of these future revenues is reduced by discounting – the more the higher the interest rate.

Next, we will approach the problem from another direction by studying the comparative statics of interest rate. We differentiate (16) with respect to interest rate and obtain

$$\frac{\partial y(T)}{\partial \delta} = \left[p + (1-\alpha)p_c\mu\right]\frac{\partial g(T)f(x(T))}{\partial \delta} - (p - p_c\mu\alpha)x(T) - \delta(p - p_c\mu\alpha)\frac{\partial x(T)}{\partial \delta} - V - \delta\frac{\partial V}{\partial \delta},$$

which can by (10) be given as $\frac{\partial y(T)}{\partial \delta} = -(p - p_c\mu\alpha)x(T) - V - \delta\frac{\partial V}{\partial \delta}$. (26)

The first term relates to the interest cost on the stand value at the moment of the clearcut, and it is negative (positive) if stumpage price is higher (lower) than the carbon subsidy cutback. The second term is negative if the bare land value is positive. The third term is positive given that an increase in interest rate decreases the bare land value. The sign of the partial derivative remains *a priori* indeterminate, and hence it is well possible that a higher interest rate lengthens optimal rotation. This result is in stark contrast to the classical Faustmann case, where rotation invariably decreases with interest rate because a higher interest rate leads to a higher interest cost on the stand value at the moment of the clearcut (cf. Tahvonen 2014).

With carbon price set to zero, the optimal rotation age decreases slightly as the interest rate increases to 4 % and starts to increase thereafter, rising sharply after 5 % (Figure 5). This is the case portrayed in Tahvonen (2014). When the interest rate is sufficiently high, it is optimal to avoid the investment in artificial regeneration as natural regeneration keeps the stand growing for free, albeit at a modest pace. Under a product adjusted net subsidy system $(\alpha = 0.5)$ and a moderate carbon price ( $p_c =$ €25 $tCO_2^{-1}$), the optimal rotation age increases with interest rate but becomes infinite only when the interest rate is above 7 %. If the carbon price is set higher, as in the third case ( $p_c =$ €50 $tCO_2^{-1}$), the optimal rotation age strongly increases already as the interest rate increases to 2 %, and continuous cover management is optimal when interest rate is higher than 3%. A higher interest rate affects the optimal thinning path by advancing the switching moment and decreasing the stand volume (as long as $p > p_c\mu\alpha$ ), which affects value growth just prior to the clearcut as well as the interest cost on the stand value (cf. Tahvonen 2014). Additionally, a higher interest implies lower bare land value. This complexity accounts for the fact that subsidized carbon sequestration may amplify as well as moderate the effect of a higher interest rate on rotation age. In short, even low interest rates yield continuous cover solutions if the carbon price is sufficiently high; with lower carbon prices the optimality of an infinite rotation requires notably high interest rates.

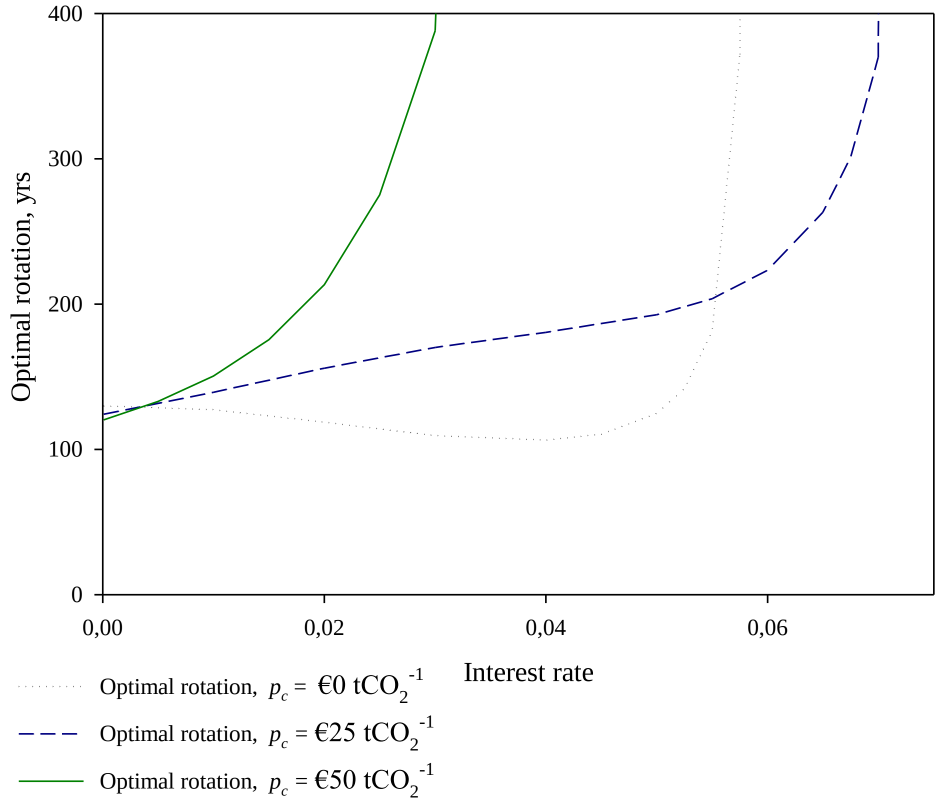


Figure 5. The dependence of optimal rotation on the interest rate.
Note: $p$ = €40 $m^{-3}$, $\alpha = 0.5$, $\mu = 0.7$, $w = 1000$.

The optimality of continuous cover *versus* clearcut forest management under three different carbon prices is depicted in Figure 6. Along the break-even curves the two management regimes are equally profitable. In general, high interest rate and high regeneration cost favor continuous cover forestry. A carbon price of €50 $tCO_2^{-1}$ widens the optimal application area of continuous cover management considerably compared to the case without carbon pricing. However, with a moderate carbon price (€25 $tCO_2^{-1}$) the optimality of continuous cover management improves only if the regeneration cost is moderate to high and reduces if it is low. A lower regeneration cost implies a higher bare land value. Given $p_c$ = €25 $tCO_2^{-1}$ carbon subsidization increases that bare land value to an extent that overrides the subsidy's rotation lengthening effects. Thus the interplay between the effects of carbon price, interest rate and other economic parameters is not straightforward. Carbon subsidization can affect the optimal choice between clearcuts and continuous cover forestry considerably, but the direction and extent of the change may vary according to circumstances.

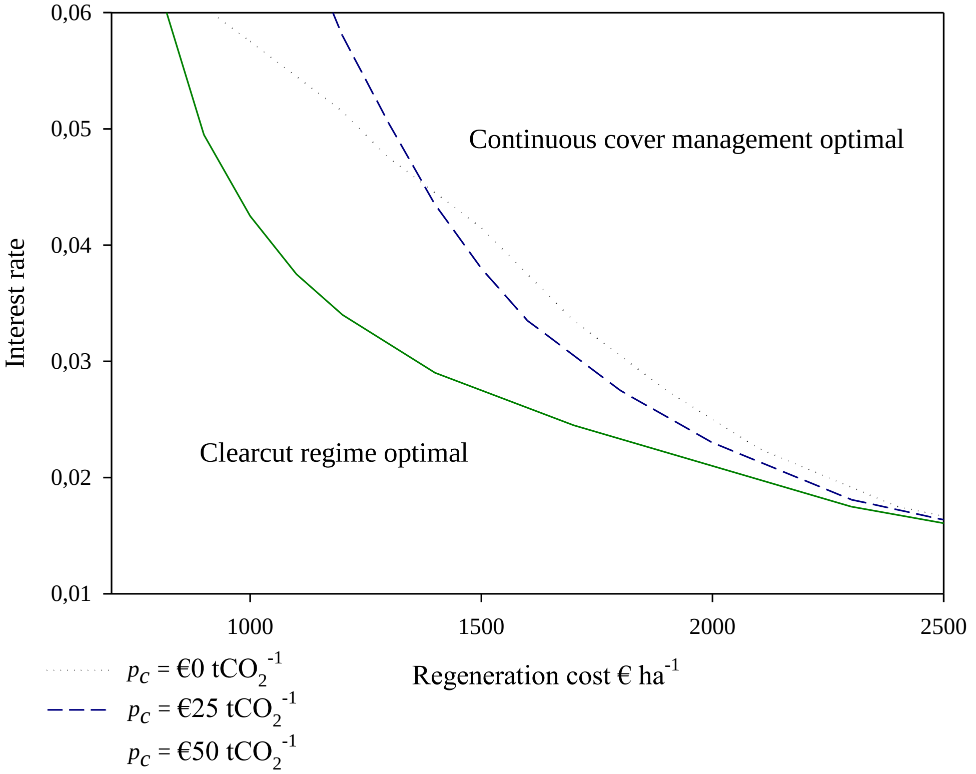


Figure 6. The optimality of continuous cover forestry versus clearcuts.
Note: $p$ = €40 m$^{-3}$, $\alpha = 0.5$, $\mu = 0.7$.

*Supply of carbon storage and wood*

One possibility to describe the supply of carbon storage is to specify the average amount of carbon stored in the stand as a function of carbon price. When clearcutting is optimal, the average carbon stock over a rotation is

$$\frac{\int_0^{t_1(p_c)} \mu x(t)\,dt + \int_{t_1(p_c)}^{T(p_c)} \mu x(t, p_c)\,dt}{T(p_c)}. \tag{27}$$

In the case of continuous cover solutions, i.e. infinitely long rotations, we are looking for the optimal long-term $CO_2$ stock. As seen from (A2), (20) and (21), the assumptions on the aging function $g$ imply that the stand approaches a steady state as $t$ approaches infinity. Thus we approximate the steady state carbon storage by evaluating the amount of carbon present in the stand, $\mu x(t, p_c)$, at $t = 1000$. Figure 7 shows the effect of carbon pricing on the average $CO_2$ stock in the stand with three interest rates.

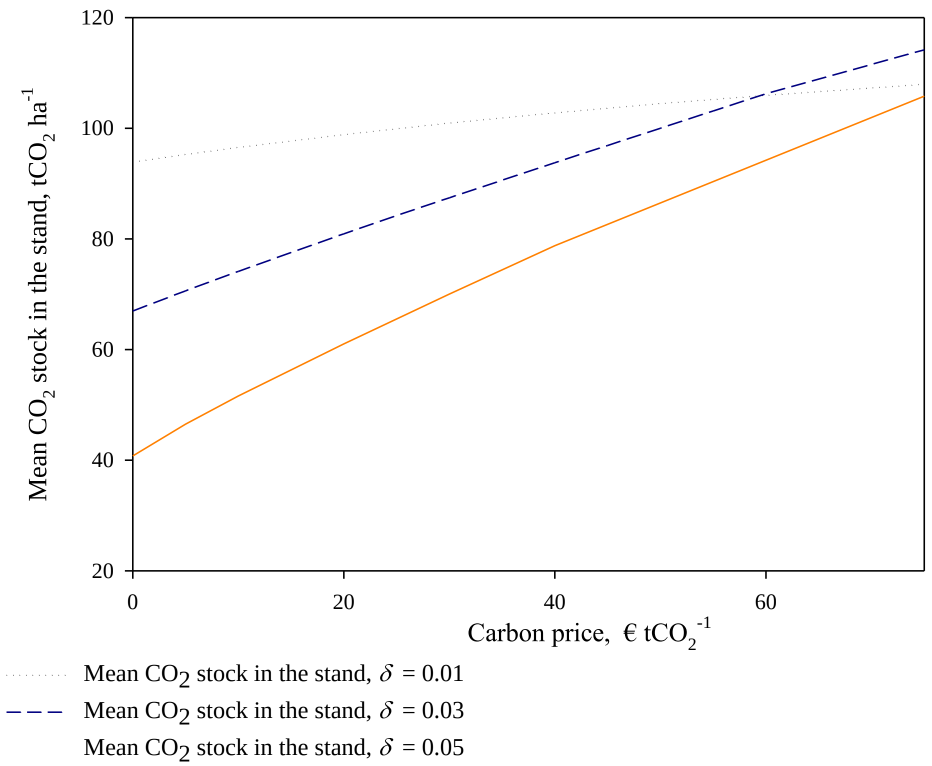


Figure 7. Mean $CO_2$ stock in the stand as a function of carbon price.
Note: $p$ = €40 m$^{-3}$, $\mu = 0.7$, $w = 1000$.

Given a 1 % interest rate and zero carbon price, the mean $CO_2$ stock is 94 $tCO_2$ ha$^{-1}$. The low interest rate makes it optimal to keep a high level of capital in the stand, i.e. to maintain a high stand volume which implies a high $CO_2$ stock. Increasing the carbon price to €60 $tCO_2^{-1}$ increases the mean $CO_2$ stock only slightly. Given a 3 % interest rate, the mean $CO_2$ stock is below 70 $tCO_2$ ha$^{-1}$ with a zero carbon price but clearly increases with carbon price; with a carbon price above €60 $tCO_2^{-1}$, the mean $CO_2$ stock is actually higher than in the case of low interest rate. Given a 5 % interest rate, the effect of carbon pricing on the mean $CO_2$ stock is even stronger: a carbon price of €40 $tCO_2^{-1}$ almost doubles the optimal storage in the stand compared to zero carbon price. Note that the optimal rotation is infinite with a carbon price higher than €68 $tCO_2^{-1}$ (€45 $tCO_2^{-1}$) given a 3 % (5 %) interest rate. The higher the interest rate, the more incentive there is to shift net emissions forward in time by storing carbon in the stand – by increasing stand volume on the singular path and by lengthening the rotation. As seen in Figure 5, carbon pricing increases the optimal rotation age the more the higher the interest rate. These results suggest that the impact of carbon subsidization on forest $CO_2$ stocks may be remarkable, particularly if the interest rate applied by the forest owners is high.

Under carbon subsidization, the goal of a forest owner is to maximize the combined net present revenues from timber production and carbon sequestration. We have shown that carbon pricing changes the optimal thinning solution and rotation age, which in turn influences wood production. To study the impacts of carbon subsidies on annual wood supply from a stand, we write the mean amount of wood harvested over a rotation as a function of carbon price:

$$\frac{\int_{t_1(p_c)}^{T(p_c)} h(t, p_c)\,dt + x\big(T(p_c), p_c\big)}{T(p_c)}. \tag{28}$$

In the case of continuous cover solutions, we approximate the steady state harvest rate by evaluating $h(t, p_c)$ at $t = 1000$.

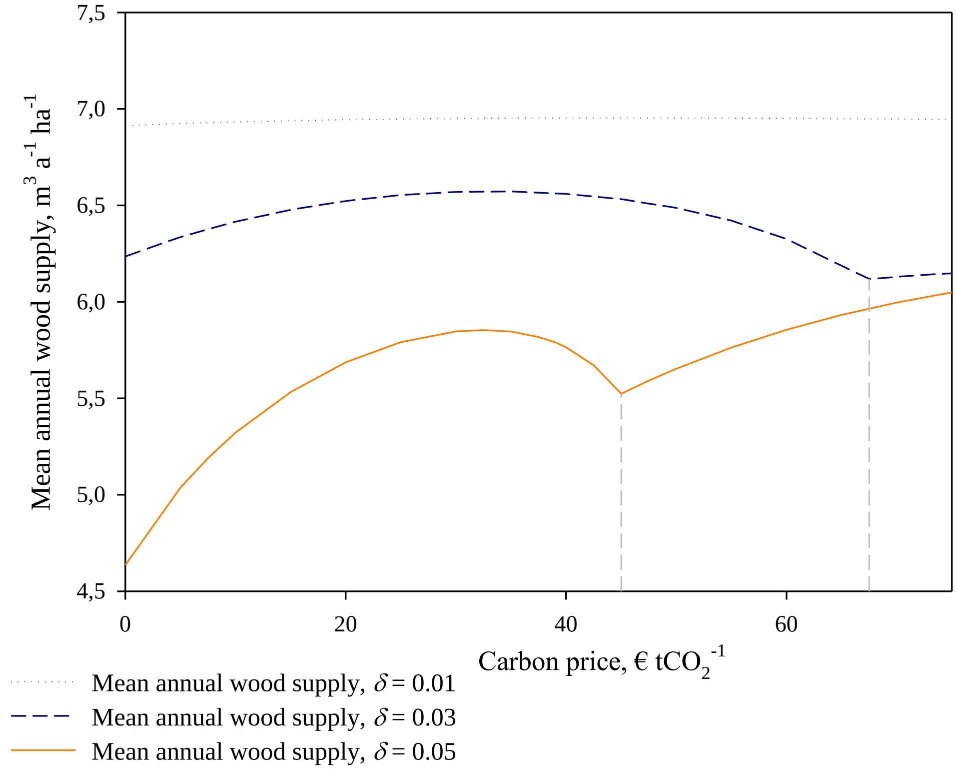


Figure 8. Mean annual wood supply as a function of carbon price.
Note: $p$ = €40 m$^{-3}$, $\mu$ = 0.7, $w$ = 1000.

Figure 8 shows the effect of carbon price on the mean annual wood supply with three interest rates. Given zero carbon price and a 1 % interest rate, the mean annual wood supply is as high as 6.9 m$^{-3}$. The low interest rate makes it optimal to reach and maintain a high stand volume, and thus high thinning yields (recall that if $p > p_c \mu\alpha$, then $f'(x) > 0$). A higher interest rate, in turn, favors advancing net revenues at the cost of yield maximization, implying lower mean annual wood supply. As explained above, carbon subsidization affects optimal thinning and rotation the more the higher the interest rate. Hence the impact of carbon pricing on the mean annual wood supply is negligible given an interest rate of 1 % but clearly visible given a 3 % interest rate and quite dramatic given a 5 % interest rate. Given a moderate to high interest rate, carbon subsidization affects the mean wood supply over a rotation in two opposing ways: on one hand it increases optimal stand volume and thus the yield from thinning, but on the other hand it lengthens the optimal rotation. As seen from the 3 % and 5 % curves, the mean annual wood supply first increases with carbon price and begins to decrease when the rotation lengthening effect starts to dominate the stand volume increasing effect. However, the former effect disappears after the optimal management regime switches from clearcuts to continuous cover. The regime shift is visible as a point of non-differentiability at $p_c =$ €68 $tCO_2^{-1}$ (€45 $tCO_2^{-1}$) given a 3 % (5 %) interest rate. Thereafter the mean annual wood supply increases with carbon price. Based on our results, carbon subsidization is likely to increase wood supply from a stand, especially with high interest rates.

## 3. Summary and discussion

In this study we have analyzed the optimality of clearcuts vs. continuous cover forestry in a setting where the society applies Pigouvian carbon subsidies to internalize carbon sequestration benefits of forestry. The subsidy takes into account that carbon is stored not only in the living trees but also in wood products, from which it is gradually released over time. Thus a fraction of subsidies is subtracted for each harvested wood unit. However, we also consider two further subsidy policies: Under the gross subsidy system, timber harvesting is assumed to cause no release of carbon. While the assumption that carbon is permanently stored in wood products is hardly realistic, the gross subsidy policy may be valid if wood is used as a substitute for a carbon-intensive input in manufacturing or energy production. The opposite is the net subsidy system that only acknowledges carbon present in the living trees.

We show analytically and numerically and present numerical examples where subsidized carbon sequestration postpones the start of thinning and increases the optimal stand volume. If the carbon price is very high relative to stumpage price, it is optimal to begin thinning when the stand volume has already surpassed the growth-maximizing level. In this case the shadow value of stand volume and the direct net revenues from thinning are negative. However, the stand is harvested to maintain optimal stand growth and carbon sequestration revenues.

According to our analytical results, subsidized carbon sequestration increases value growth at the moment of clearcut and decreases the interest cost on clearcut net revenues. This implies an incentive to postpone clearcutting (possibly *ad infinitum*). On the other hand, carbon subsidies increase bare land value and thus give incentive to advance the clearcut. The relative magnitudes of these opposite effects determine whether carbon subsidies lengthen or shorten the optimal rotation – and further, whether carbon subsidies favor continuous cover management or clearcuts. Thus the impact of carbon subsidies on the choice of management system may vary from case to case. Numerical examples, computed with realistic economic parameter values and a growth function specification based on empirically estimated growth model (Bollandsås et al. 2008) suggest that rotation age tends to increase with carbon price. Under a product adjusted net subsidy system and given an interest rate of 3% (5%), continuous cover management becomes optimal when the carbon price is €60 $tCO_2^{-1}$ (€40 $tCO_2^{-1}$) or above. However, under a gross subsidy policy carbon pricing has minor effect on rotation age. In other words, the optimal choice between the two regimes is found to be sensitive to the level of timber product decay and subsequent cuts to the subsidy payments.

Unlike in the case of the generic Faustmann formulation, a higher interest rate can lead to a longer or shorter rotation. Numerical examples suggest that rotation age increases with interest rate when carbon sequestration is subsidized. High interest rate makes it optimal to postpone or avoid the investment in artificial regeneration and to rely on natural regeneration. In a more general sense the optimal choice between clearcuts and continuous cover management depends on an intricate combination of economic and ecological parameters. Continuous cover management is the more competitive the more expensive is artificial regeneration.

Additionally, we show that the effect of carbon subsidization on the supply of wood and $CO_2$ storage in the stand is the stronger the higher the interest rate. Given low to moderate carbon prices and interest rates ranging from 1% to 5 %, carbon subsidization increases mean annual wood supply from a stand.

Our model differs from that of van Kooten et al. (1995) and his numerous followers since we include thinning and the possibility of continuous cover forestry. According to the numerical results of van Kooten et al. (1995), carbon subsidies increase rotation age, but with plausible carbon prices the effect is usually moderate. Our numerical results confirm that rotation age tends to increase with carbon price. However, our findings suggest that when thinning is taken into account, carbon prices within a realistic range may lengthen the rotation period considerably and lead to infinite rotations, i.e. to a switch from plantation type of forestry to continuous cover forestry.

Pihlainen et al. (2014) and Niinimäki et al. (2013) study timber production and carbon storage in artificially regenerated even-aged stands of Scots pine and Norway spruce. Both use numerical optimization based on very detailed ecological models and take into account carbon stored in the stand and in timber products. Pihlainen et al. (2014) additionally consider the whole aboveground biomass as well as dead trees. The authors optimize rotation length, initial stand (planting) density and the number, intensity, timing and type of thinnings. According to the results in Pihlainen et al. (2014), carbon subsidization increases initial stand density, i.e. it is optimal to plant more seedlings. In general, stand volume throughout the rotation is the larger the higher is the carbon price, which is in line with our results. The authors also find that carbon pricing lengthens the optimal rotation. Pihlainen et al. (2014) observe distinct solutions for net subsidy (which corresponds to our product adjusted net subsidy) systems and gross subsidy systems, implying that optimal management is strongly affected by the product decay rate. This supports our finding that the rate of carbon release at harvest may have a significant impact on optimal thinning and rotation age.

According to Niinimäki et al. (2013), a "higher $CO_2$ price tends to postpone harvests and increase basal area and standing volume", which matches our results. However, in Niinimäki et al. (2013) the stand is allowed to reach a much higher volume than in our solutions. Norway spruce is a shade tolerant species and thus under even-aged management and artificial regeneration thinnings can start substantially later than in our case, where the optimal utilization of natural regeneration requires controlling the stand volume from quite early on. This results in somewhat lower average carbon storage in the stand than that reported in Niinimäki et al. (2013). When it comes to the importance of carbon release specification, our results differ considerably from those presented in Niinimäki et al. (2013). The authors state that varying the policy from their default (product adjusted net) subsidy system to a net subsidy system or to a gross subsidy system has only minor effects on optimal solutions. This difference can also be explained by the later commencement of thinning: the postponement of harvesting implies that the value of carbon released from wood products is discounted over a longer time interval and thus diminishes in importance (cf. Pihlainen et al. 2014). In the case of the high fertility site ($H_{100}$=30), the authors observe two tendencies that are prevalent also in our solutions: higher interest rate increases optimal rotation length, and carbon pricing amplifies this effect. According to our results, rotation age increases monotonically with interest rate under moderate carbon prices but only from 4 % onwards without carbon pricing (Figure 5).

Goetz et al. (2010) seems to be the first paper to study the economics of uneven-aged forestry with carbon sequestration. The authors state that their framework could be extended to naturally reproducing stands, but their "theoretical and empirical analysis is framed within the context of a completely managed forest" where new trees are planted in pursuance of thinning. According to their numerical results, an increase in carbon price leads to an increase in the number of trees through increased planting and decreased logging. The effect on the stand volume is not reported. Based on a sensitivity analysis, Goetz et al. (2010) find that the decay of carbon from wood products does not affect optimal solution. This is in contrast with our results as well as those presented in Pihlainen et al. (2014). The option of shifting to clearcut management is not considered in the paper.

Pukkala et al. (2011) compare uneven- and even-aged management systems numerically in terms of timber, carbon and bilberry services. However, their approach differs from ours because of their use of the problematic “investment efficient” (IE) approach. The authors report the net present values (NPV) and physical yields from optimized uneven- and even-aged management as well as those resulting from currently recommended even-aged management. The effect of carbon pricing on optimal uneven-aged management is not explicitly addressed. For Norway spruce, uneven-aged management is more profitable than even-aged management regardless of carbon price, but the difference decreases when the carbon price is higher. For Scots pine, increasing the carbon price from €15 to €30 $tCO_2^{-1}$ actually turns the situation around, making even-aged management by far more profitable. Pukkala et al. (2011) state that “the superiority of uneven-aged management increases with increasing discount rate”; and on the other hand, “increasing the discount rate shortenes the optimal rotation lengths in even-aged management”. This finding is contradictory from the perspective of our setup, where an infinite rotation age implies optimality of continuous cover management, and probably results from the applied IE modeling.

Also Buongiorno et al. (2012) apply the IE approach for numerically optimizing the steady-state uneven-aged management for carbon sequestration and timber production. The study concentrates on comparing the IE objective and stand characteristics given a cutting cycle of 5 or 20 years. When maximizing the IE objective including combined carbon storage and timber production, stand basal area and volume increase with carbon price. This is, despite of the divergent optimization method, in line with the analytical and numerical results of the present study. The authors also state that maximizing the net present value from combined carbon storage and timber production shows complementarity for up to NOK 300 $tCO_2^{-1}$ ($\approx$ €36 $tCO_2^{-1}$) and lower timber production at higher carbon prices. At first sight, this closely resembles our result that wood supply increases with carbon price given a low carbon price but may decrease when the price increases further (Figure 9). However, while our result follows from the opposing effects of increased stand volume and lengthened rotation, the latter effect cannot explain Buongiorno et al. (2012)’s result because their optimization framework does not include the option of clearcutting. Hence the observed conflict between carbon storage and wood production must, in their case, follow from exceeding the growth-maximizing stand volume.

Parajuli and Chang (2012) apply the “generalized Faustmann formula”, a variation of the IE approach, to study uneven-aged management of loblolly pine stands with carbon sequestration. Because of the aforementioned optimization method and other problematic features, the results of Parajuli and Chang (2012) should be interpreted with caution. To determine the optimal cutting cycle and residual growing stock for joint production of timber and carbon sequestration, the authors try out different combinations of cutting cycles within a time interval that equals the least common multiple of each potential cutting cycle and the optimal timber-only cutting cycle of 10 years. The value of the cutting cycles that succeed the said time interval is claimed to be captured by a “future land value”, which is varied in sensitivity analysis. According to the authors, including carbon sequestration benefits did not alter the optimum management regimes significantly. As far as we can tell, Parajuli and Chang (2012) compute the carbon benefits by comparing the joint production solution to the timber-only solution, whereas our study takes into account the positive externality of carbon sequestration in its entirety.

In this paper, we have presented a way to study economically optimal forestry under carbon subsidization by determining the optimal management system endogenously: Unlike previous studies, we do not limit the analysis to even-aged or, alternatively, uneven-aged forestry, or to a comparison of the two. Instead, we use a coherent optimization framework that allows us to show how changes in various economic parameters may bring about management regime shifts. We demonstrate that using a continuous biomass model, the effects of carbon subsidies on optimal clearcut and continuous cover forest management can be studied analytically as well as numerically. The results produced by this approach are economically intuitive and can be compared to results from existing numerical studies, many of which they support. However, the analytical insight reached by this method comes with a price, because the biomass model represents a simplification of the complex dynamics of an uneven-aged forest stand. The next step, then, will be to numerically apply a size-structured model to the same economic problem and to study the effects of carbon storage in more detail.

# References


Adams, D.M. and Ek, A.R. 1974. Optimizing the management of uneven-aged forest stands. Canadian Journal of Forest Research 4(3): 274–287.

Adams, D.M. 1976. A Note on the Inderdependence of Stand Structure and Best Stocking in a Selection Forest. Forest Science 22(2): 180–184.

Adams, T. and Turner, J.A. 2012. An investigation into the effects of an emissions trading scheme on forest management and land use in New Zealand. Forest Policy and Economics 15: 78–90.

Bollandsås, O.M., Buongiorno, J. and Gobakken, T. 2008. Predicting the growth of stands of trees of mixed species and size: A matrix model for Norway. Scandinavian Journal of Forest Research 23: 167–178.

Buongiorno, J., Halvorsen, E.A., Bollandsås, O.M., Gobakken, T. and Hofstad, O. 2012. Optimizing management regimes for carbon storage and other benefits in uneven-aged stands dominated by Norway spruce, with a derivation of the economic supply of carbon storage. Scandinavian Journal of Forest Research 27(5): 460–473.

Calladine, J., Bray, J., Broome, A. and Fuller, R.J. 2015. Comparison of breeding bird assemblages in conifer plantations managed by continuous cover forestry and clearfelling. Forest Ecology and Management 344: 20–29.

Canadell, J.G. and Raupach, M.R. 2008. Managing Forests for Climate Change Mitigation. Science 320: 1456–1457.

Cedergren, J. 2008. Kontinuitetsskogar och hyggesfritt skogsbruk. Skogsstyrelsen, Jönköping.

Clark, C.W. 1976. *Mathematical Bioeconomics. The optimal management of renewable resources*. Wiley.

Dahlberg, A. 2011. Kontinuitetsskogar och hyggesfritt skogsbruk. Slutrapport för delprojekt naturvärden. Skogsstyrelsen, Jönköping.

Edenhofer, O. 2014. Reforming emissions trading. Nature Climate Change 4: 663–664.

FAO. 2006. *Global Forest Resources Assessment 2005. Progress towards sustainable forest management.* FAO Forestry paper 147. Food and Agriculture Organization of the United Nations, Rome.

Faustmann, M. 1849. Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen. Allgemeine Forst- und Jagd-Zeitung 25: 441–455.

Gauthier, S., Vaillancourt, M-A., Kneeshaw, D., Drapeau, P., De Grandpré, L., Claveau, Y., and Paré, D. 2009. Forest Ecosystem Management: Origins and Foundations. In: Gauthier, S., Vaillancourt, M-A., Leduc, A., De Grandpré, L., Kneeshaw, D., Morin, H., Drapeau, P., and Bergeron, Y. 2009. *Ecosystem Management in the Boreal Forest*. Presses de l'Université du Québec. p. 13–37.

Getz, W. and Haight, R.G. 1989. *Population Harvesting: Demographic Models for Fish, Forest and Animal Resources*. Princeton University Press, New Jersey.

Goetz, R-U., Hritonenko, N., Mur, R.J., Xabadia, A. and Yatsenko, Y. 2010. Forest Management and Carbon Sequestration in Size-Structured Forests: The Case of Pinus sylvestris in Spain. Forest Science 56: 242–256.

Haight, R.G. 1985. Comparison of Dynamic and Static Economic Models of Uneven-aged Stand Management. Forest Science 31: 957–974.

Haight, R.G. and Getz, W.M. 1987. Fixed and equilibrium endpoint problems in uneven-aged stand management. Forest Science 33: 903–931.

Haight, R.G. and Monserud, R. 1990. Optimizing any-aged management of mixed-species stands: II. effects of decision criteria. Forest Science 36(1): 125–144.

Hartman, R.G. 1976. The Harvesting Decision When a Standing Forest Has Value. Economic Inquiry 14(1): 52–58.

IPCC. 2000. *Land Use, Land-Use Change and Forestry*. Eds. Watson, R., Noble, I., Bolin, B., Ravindranath, N., Verardo, D. and Dokken, D. Cambridge University Press, UK.

IPCC. 2014. Summary for Policymakers. In: Core Writing Team, Pachauri, R.K. and Meyer, L.A. (eds.) *Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*. IPCC, Switzerland.

Kilkki, P. and Väisänen, U.1969. Determination of the optimal policy for forest stands by means of dynamic programming. Acta Forestalia Fennica 102: 100–112.

Kotilainen, J. and Rytteri, T. 2011. Transformation of forest policy regimes in Finland since the 19th century. Journal of Historical Geography 37: 429–439.

Kuuluvainen, T., Tahvonen, O. and Aakala, T. 2012. Even-aged and uneven-aged forest management in boreal Fennoscandia: a review. Ambio 41(7): 720–737.

Lundmark, H., Josefsson, T., and Östlund, L. 2013. The history of clear-cutting in northern Sweden. Driving forces and myths in boreal silviculture. Forest Ecology and Management 307: 112–122.

Lämås, T., and Fries, C. 1995. Emergence of a Biodiversity Concept in Swedish Forest Policy. In: Apps, M.J., Price, D.T., and Wisniewski, J. (eds.). *Boreal Forests and Global Change*. Springer. p. 57–66.

MAF, Finnish Ministry of Agriculture and Forestry. 2014. Forest Act (1093/1996; amendments up to 1085/2013 included). Unofficial translation, accessed 10 May 2015. http://www.mmm.fi/attachments/metsat/lainsaadanto/xYu2E9F6x/ForestAct.pdf

Manley, B. and Maclaren, P. 2010. Harvested Wood Products in the ETS. What would be the Impact? NZ Journal of Forestry 55(3): 20–26.

Mason, W. 2015. Implementing Continuous Cover Forestry in Planted Forests: Experience with Sitka Spruce (Picea Sitchensis) in the British Isles. Forests 6: 879–902.

New Zealand Ministry for Primary Industries. 2014 a. “Post-1990 Forest”. Last updated 18 December 2014, accessed 22 May 2015. http://archive.mpi.govt.nz/forestry/forestry-in-the-ets/post-1989-forest

New Zealand Ministry for Primary Industries. 2014 b. “Pre-1990 Forest”. Last updated 22 December 2014, accessed 22 May 2015. http://archive.mpi.govt.nz/forestry/forestry-in-the-ets/pre-1990-forest

Niinimäki, S., Tahvonen, O., Mäkelä, A. and Linkosalo, T. 2013. On the economics of Norway spruce stands and carbon storage. Canadian Journal of Forest Research 43(7): 637–648.

Nordhaus, W. 2014. Estimates of the Social Cost of Carbon: Concepts and Results from the DICE-2013R Model and Alternative Approaches. Journal of the Association of Environmental and Resource Economists 1(1): 273–312.

Pan, Y., Birdsey, R., Fang, Y., Houghton, R., Kauppi, P., Kurz, W., Phillips, O., Shvidenko, A., Lewis, S., Canadell, J., Ciais, P., Jackson, R., Pacala, S., McGuire, McGuire, A.D., Piao, S., Rautiainen, A., Sitch, S., Hayes, D. 2011. A Large and Persistent Carbon Sink in the World’s Forests. Science 333: 988–993.

Parajuli, R. and Chang, S.J. 2012. Carbon sequestration and uneven-aged management of loblolly pine stands in the Southern USA: A joint optimization approach. Forest Policy and Economics 22: 65–71.

Pihlainen, S., Tahvonen, O. and Niinimäki, S. 2014. The economics of timber and bioenergy production and carbon storage in Scots pine stands. Canadian Journal of Forest Research 44(9): 1091–1102.

Pindyck, R. 2013. Pricing Carbon When We Don’t Know the Right Price. Regulation 36(2): 43–46.

Puettmann, K.J., Coates, K.D., and Messier, C. 2009. *A critique of silviculture: managing for complexity*. Washington (DC): Island Press.

Pukkala, T., Lähde, E., Laiho, O., Salo, K. and Hotanen, J-P. 2011. A multifunctional comparison of even-aged and uneven-aged forest management in a boreal region. Canadian Journal of Forest Research 41(4): 851–862.

Ray, D., Morison, J. and Broadmeadow, M. 2010. Climate change: impacts and adaptation in England’s woodlands. Forest Research Note 201.

Rämö, J. and Tahvonen, O. 2014. Economics of harvesting uneven-aged forest stands in Fennoscandia. Scandinavian Journal of Forest Research 29(8): 777–792.

Schaefer, M.B. 1957. Some Considerations of Population Dynamics and Economics in Relation to the Management of the Commercial Marine Fisheries. Journal of the Fisheries Research Board of Canada 14(5): 669–681.

Schütz, J., Pukkala, T., Donoso, P. and von Gadow, K. 2012. Chapter 1: Historical Emergence and Current Application of CCF. In: Pukkala, T. and von Gadow, K. (eds.) *Managing Forest Ecosystems 23: Continuous Cover Forestry*. Springer Netherlands.

Seierstad, A. and Sydsæter, K. 1987. *Optimal control theory with economic applications*. Elsevier, Amsterdam.

Sydsæter, K., Hammond, P., Seierstad, A. and Strom, A. 2008. *Further Mathematics for Economic Analysis*. Prentice Hall.

Siiskonen, H. 2007. The conflict between traditional and scientific forest management in 20th century Finland. Forest Ecology and Management 249: 125–133.

Stern, N. and Dietz, S. 2014. Endogenous growth, convexity of damages and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. Centre for Climate Change Economics and Policy Working Paper No. 180.

Tahvonen, O. 1995. Net national emissions, $CO_2$ taxation and the role of forestry. Resource and Energy Economics 17(4): 307–315.

Tahvonen, O. 2009. Optimal choice between even- and uneven-aged forestry. Natural Resource Modeling 22(2): 289–321.

Tahvonen, O. 2011. Optimal structure and development of uneven-aged Norway spruce forests. Canadian Journal of Forest Research 41: 2389–2402.

Tahvonen, O. 2014. Economics of rotation and thinning revisited: the optimality of clearcuts versus continuous cover forestry. Submitted manuscript.

Tahvonen, O. 2015. Economics of naturally regenerating heterogeneous forests. Journal of the Association of Environmental and Resource Economists, in print.

Tahvonen, O. and Viitala, E.J. 2006. Does Faustmann rotation apply to fully regulated forests? Forest Science 52(1): 23–30.

Tahvonen, O., Pukkala, T., Laiho, O., Lähde, E., and Niinimäki, S. 2010. Optimal management of uneven-aged Norway spruce stands. Forest Ecology and Management 260: 106–115.

Tee, J., Scarpa, R., Marsh, D. and Guthrie, G. 2014. Forest Valuation under the New Zealand Emissions Trading Scheme: A Real Options Binomial Tree with Stochastic Carbon and Timber Prices. Land Economics, 90(1): 44–60.

Thompson, I., Mackey, B., McNulty, S. and Mosseler, A. 2009. Forest Resilience, Biodiversity, and Climate Change. A synthesis of the biodiversity/resilience/stability relationship in forest ecosystems. Technical Series no. 43. Secretariat of the Convention on Biological Diversity, Montreal.

Tol, R. 2005. The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. Energy Policy 33: 2064–2074.

Valkeapää, A. and Karppinen, H. 2013. Citizens' view of legitimacy in the context of Finnish forest policy. Forest Policy and Economics 28: 52–59.

van Kooten, G., Binkley, C. and Delcourt, G. 1995. Effect of Carbon Taxes and Subsidies on Optimal Forest Rotation Age and Supply of Carbon Services. American Journal of Agricultural Economics 77(2): 365–374.

Weitzman, M. 2014. Fat Tails and the Social Cost of Carbon. The American Economic Review 104 (5): 544–546.