Managing biological invasions: how to set priorities ?

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Abstract

The number of biological invasions increases and so are the impacts these species cause to the environment and the economy. Because resources are limited, the funds available for the management of biological invasions need to be allocated in the most efficient way. Applying a cost/benefit approach incorporating species utility, distinctiveness, robustness of species and their interactions, this paper provides with an operational optimal method for setting management priorities under a limited budget constraint.

Keywords: Prioritization, cost/benefit, optimization, diversity
JEL: Q28, Q57, Q58

1 Introduction

The number as well as the damages caused by biological invasions are tremendously increasing [Perrings et al., 2010, Vilà et al., 2011, Essl et al., 2011]. Recent studies made an important contribution to classify their impacts [Blackburn et al., 2014, Jeschke et al., 2014], providing an extensive list of environmental as well as economic damages that ought to be taken into account. One of the most worrisome feature of invasive species is their impact on biodiversity. They are an important cause of extinction, therefore being categorized as one of the major threat to biodiversity [Bax et al., 2003, Clavero and Garciaberthou, 2005, McNeely, 2001, Molnar et al., 2008, Vilà et al., 2011].

Although many biological invasions are likely to be harmful, we are left with an uncomfortable choice to make: which species should be targeted first ? How should we spend a limited budget to address the problem of invasion management ? The questioning underlying this issue is a prioritization one. Budget being limited, we unfortunately do have to set priorities in our effort to control invasions, since we do not have enough resources to manage all of them at the same time. Every single undesirable species cannot be dealt with, therefore urging for the use of a framework to help us set priorities.

Prioritization literature is mostly based on scoring approaches. Basically, scoring implies giving marks to invasive species on the basis of a set of criteria, the species with the lowest (or highest, depending on the methodology used) overall score being considered the priority. Non exhaustively, Batianoff and Butler [2002] scored expert opinions on species 'invasiveness', and then compared the obtained ranking list to impact scores [Batianoff and Butler, 2003]. Thorp and Lynch [2000] added different criteria such as potential for spread and sociological values to rank weeds. Kumschick and Nentwig [2010] and Kumschick et al. [2012] developed frameworks to prioritize action against alien species according to their impacts, incorporating experts opinions but also taking into account the diverging interests of the various stakeholders, therefore really capturing the political issue underlying prioritization.

If scoring is a practical approach in order to produce a ranking, these methods were developed outside of any formal optimization framework, and this occurred to their expense. Three important flaws can be noted: i) the costs of management are rarely explicitly taken into account, while paradoxically we observe important heterogeneity in species management costs, ii) scoring questionnaires fail to clearly present the objective of management policy making scores aggregation problematic, iii) interactions among species are, at best, superficially accounted for. This last point can be seriously misleading and for example, Zavaleta et al. [2001] showed that eluding trophic cascades reflexions while removing an invasive species could lead to major unexpected changes to other ecosystem components, potentially creating unwanted secondary impacts.

The seminal papers of Solow et al. [1993] and Weitzman [1998] are two milestones in the cost-benefit analysis of conservation policy. Weitzman's approach results in a practical methodology to prioritize conservation choices based on a rigorous optimization model. The idea is for each species, to assess benefit/cost ratios of conservation, that can next be ranked in order to set priorities. Various efforts to optimise conservation of species have developed following this work. Some of these have led to changes in allocation of conservation funding Joseph et al. [2008], McCarthy et al. [2008], and variants have been used to allocate surveillance effort over space [Hauser and McCarthy, 2009].[1] Such a methodology provides us with a formal framework to think about prioritization and could be applied to the management of invasions. As for conservation policy, biological invasion management aims at maximizing biodiversity and ecosystem services and it is to be performed in a cost-efficient way. A particularly problematic flow with regard to

[1] Other applications are quoted in Eppink and van der Bergh [2007].

Weitzman's approach is that it fails to account for species interrelations. Disruptions from invasions are mainly due to the dynamics of spread and its negative impact on native species. Interrelation network and the dynamics of reproduction is at the cornerstone of the optimization framework. Courtois et al. [2014] revisited Weitzman's optimization problem and extended his model in order to incorporate species interactions. The idea is to model species survival probability as a function of survival probabilities of other species. We use a similar approach in this paper and our principal output is to forward a general ranking formula that could be used as a rule of thumb for deciding biological invasion priorities under a limited budget constraint and accounting for species interrelations.

The sketch of the paper proceeds as follow. In section 2, we consider a simple stylized model of prioritization with two native and two invasive species. We define the optimization framework assuming specific class of utility and diversity functions and analyse the budget allocation choice of a manager aiming at minimize the disruptions due to biological invasions. We proceed in section 3 with a generalization of this optimization framework by considering any number of species and any class of utility and diversity functions. Section 4 concludes on the use of the resulting prioritization criterion for applications.

2 A stylized model

Consider an hypothetical ecosystem composed of four interacting species $i = \{1, 2, 3, 4\}$. Among these species, two are invasive species we denote with subscript k, $k = \{1, 2\}$, and two are native species we denote with subscript l, $l = \{3, 4\}$. We distinguish two types of impact of invasive species to the ecosystem: ecological impacts among which impacts to the native ecosystem through species interactions, resources competition, etc, and economic impact, like eutrophication or obstruction of canalizations.[2] Impacts although often negative can be positive and for example an invasive species may well affect positively some stakeholders exploiting it (*e.g.* fish, crayfish) as it can affect positively several native species of the ecosystem for example through predation or mutualism.

Imagine that a manager in charge of this ecosystem is to efficiently limit the negative impacts due to invasive species. Given his limited resources, he has to efficiently allocate his budget in order to minimize net losses given the relative

[2]e.g. the brown tree snake (*Boiga irregularis*) introduced in the snake-free Guam forest after World War II because of military equipment being moved onto Guam [Fritts and Rodda, 1995, Pimentel et al., 2005] participated in the extinction of 10 native forest birds [Rodda et al., 1997]. One of the many examples of disutility produced by an invasive species is the case of *Dreissena polymorpha*, also known as the European zebra mussels, invading and cloging water pipes, filtration systems, and electric generating plants; it is estimated that they cause 1 billion USD/year in damages and associated control costs per year [Vilà et al., 2011].

marginal costs of controlling species k. This translates into a maximization problem of an objective function under a monetary constraint.

Because impacts from invasive species are twofold, the objective function of this manager is made of two components. The first is an ecological component. The manager wants the expected diversity of the ecosystem to be as high as possible. In our two native species ecosystem it means that given species dissimilarity and survival probabilities of these natives, the manager aims at controlling the negative impact of invasives on expected diversity. We denote this function $W(\{P_l\}_{l=3}^4)$, with P_l the survival probability of species l. We assume that $P_l \in [0, 1]$ is an index value, with $P_l = 0$ meaning extinction and $P_l = 1$ meaning profusion of species l. We consider purposefully that invasive species do not participate to the diversity of this ecosystem. They contribute to diversity but of their own native system.

Several competing expected diversity function can be considered and choosing one functional form versus another is an important choice as it reflects a philosophy of conservation.[3] Two expected diversity functions are particularly relevant for the current paper, Rao's quadratic entropy [Rao, 1986] and Weitzman' expected diversity [Weitzman, 1992, 1998]. In the current section, we consider Weitzman [1998] expected diversity function, a generalization of our approach to any functional form being proposed in next section. Weitzman considered that each species could be seen as a *library* containing a certain number of *books*. The value of a set of libraries is made of the collection of different books available, but also of the different libraries themselves because they can be considered has having an intrinsic value (for instance, the Trinity College Library in Dublin would be considered a wonder even if the book of Kells were not there). Biologically, libraries being species would mean that books would be genes, or phenotypic characteristics, or even something else. To keep it simple, we consider diversity in terms of different genes, like Weitzman did. Assume that species 3 contains E_3 genes and that species 4 contains E_4 genes. Furthermore, although the model could accommodate for gene sharing, we further assume in this stylized model that species 1 and 2 do not have any gene in common. The expected diversity function reads as:

$$
\begin{aligned}
W(\{P_l\}_{l=3}^4) &= P_3P_4(E_3 + E_4) + P_3(1 - P_4)E_3 \\
&\quad + (1 - P_3)P_4E_4 + (1 - P_3)(1 - P_4)0 \\
&= E_3P_3 + E_4P_4 = \sum_{l=3}^{4} E_l * P_l
\end{aligned} \tag{1}
$$

The second component of the objective function is the utility derived from each species i. Utility of both native and invasive species may well range from positive

[3] Interested readers may refer to Courtois et al. [2015]

to negative values. We assume that the marginal utility of each species is constant at rate u_i and write:

$$U(\{P_i\}_{i=1}^4) = \sum_{i=1}^{4} u_i * P_i \tag{2}$$

Now that the objective of the manager is defined, let us focus on his constraints. First the manager is to account for species interrelations. We consider as in Courtois et al. [2014] that each species i has an autonomous surviving probability q_i which is the survival probability of species i in an ecosystem absent of species interactions and, of manager. Because of species interaction, each species i surviving probability depends also on all the other species surviving probabilities, through interrelation parameters $r_{i,l \neq i}$. Finally, the manager can decide to impact the surviving probabilities of the invasive species present in the ecosystem. His managing effort on species k is denoted x_k. The resulting survival probabilities of species i in our stylized two-natives two-invasive species ecosystem read as:

$$\begin{cases} P_l = q_l + \sum_{i \neq l} r_{li}\ P_i\ , \quad q_l \in [0,1[\\ P_k = q_k - x_k + \sum_{i \neq k} r_{ki}\ P_i\ , \quad q_k \in [0,1[\ , \quad x_k \in [0, \overline{x}_k]\,. \end{cases} \tag{3}$$

Second, the manager is to account for complying with his budget which is assumed to be limited. Let c_k be the marginal cost of the effort to control invasive species k. Denote by B the overall budget he can exhaust, an additional constraint is the budget constraint:

$$\sum_k c_k * x_k \leq B\ , \tag{4}$$

We now can establish the manager's optimization problem in terms of managing efforts x_k:

$$\max_{\{x_k\}_{k=1}^2 \in\ \times_{k=1}^2 [0,\overline{x}_k]} \sum_{l=3}^{4} E_l * P_l + \sum_{i=1}^{4} u_i * P_i \quad \textit{subject to}\ \ (3)\ \textit{and}\ (4). \tag{5}$$

Solving the system of survival probabilities described by (3), we obtain a system of equations that links survival probability P_i to control effort values x_k:

$$\begin{cases} P_1 &= \frac{\alpha_1}{\delta} x_1 + \frac{\theta_1}{\delta} x_2 + \frac{\gamma_1}{\delta} \\ P_2 &= \frac{\alpha_2}{\delta} x_1 + \frac{\theta_2}{\delta} x_2 + \frac{\gamma_2}{\delta} \\ P_3 &= \frac{\alpha_3}{\delta'} x_1 + \frac{\theta_3}{\delta'} x_2 + \frac{\gamma_3}{\delta'} \\ P_4 &= \frac{\alpha_4}{\delta'} x_1 + \frac{\theta_4}{\delta'} x_2 + \frac{\gamma_4}{\delta'} \end{cases} \tag{6}$$

with $\delta' = \delta(1-r_{12}r_{21})$ and δ, α_i, θ_i are coefficient that only depend on the matrix of species interdependence r_{ih}, γ_i coefficients that depend on species interdependences and on autonomous surviving probabilities q_i.

Plugging (6) in the objective function (5), the maximization shrinks to the trivial problem:

$$\max_{x_1,x_2} \ ax_1 + bx_2 + cste \tag{7}$$
$$s.t \ c_1x_1 + c_2x_2 \leq B \ .$$

where a and b are coefficients that only depend on the matrix of species interdependence r_{ih}, the vector q_i, the distinctiveness parameters E_l and the marginal utility u_i.

As the objective function is linear in efforts, the solution of the maximization program (7) is extreme, *i.e.* in the usual case where $a > 0$ and $b > 0$, effort is devoted in priority to the control of a single invasive species and if resources are more than sufficient to fulfill the control of this species, to the other. For simplicity and without loss of generality we consider budget B is limited enough to be completely exhausted with the control of a single species. We have then three extreme solutions, $(0, 0)$, $(\overline{x_1}, 0)$ and $(0, \overline{x_2})$, with $\overline{x_1}$ and $\overline{x_2}$ standing for the maximal admissible ranges of efforts determined by economical and biological constraints.

Proposition 1 *In our stylized model, optimal management plan proceeds as follows:*

- *if $a \leq 0$ and $b \leq 0$, no effort should be made to control the invasions because effort is not desirable. The solution to the maximization program is $(0, 0)$;*
- *if $a \leq 0$ and $b > 0$, effort is granted to species 2. The solution to the maximization program is $(0, \overline{x_2})$;*
- *if $a > 0$ and $b \leq 0$, effort is granted to species 1. The solution to the maximization program is $(\overline{x_1}, 0)$;*
- *if $a > 0$ and $b > 0$, effort is granted to species 2 when $\frac{c_1}{c_2} > \frac{a}{b}$ and to species 1 else. The solution to the maximization programme is either $(\overline{x_1}, 0)$ or $(0, \overline{x_2})$. In the very specific case where $\frac{c_1}{c_2} = \frac{a}{b}$ any combination of efforts is applicable.*

Coefficients a and b depend on species interactions parameters $r_{i,h\neq i}$, the vector q_i, the distinctiveness parameters E_l and the marginal utility u_i. Key message of the proposition is that when both invasive species disrupts the ecosystem, effort

is made toward the most cost-efficient plan. The idea is thus to limit ecosystem disruption at the lower cost.
This stylized model is useful in that it provides us with a formal framework to think about the optimal management of biological invasions. However, it is not sufficient as such to make any clear-cut generalization about budget allocation and priorities in a more complex world. First, a model with more species is to be considered as more species translates in more interrelations and could make the problem untractable. Second, simple linear expected diversity and utility functions are restrictive assumptions. Other diversity functions among which the Rao general entropy concept [Rao, 1986], the Allan diversity function (Allan 1993) or even Weitzman's expected diversity with species sharing common genes, exhibit local convexities. Utility functions admit also often concavities or convexities and our prioritization model is to deal with any of these types of situations. Third, more than an optimization framework, we wish to develop a an easy-to-use tool that a manager could use in any socio-ecosystem configuration. In the following section we address these three points and provide with a criterion for a manager to set priorities in any socio-ecosystem configuration.

3 A prioritization criterion

Consider now an ecosystem made of $N = [\![1; n]\!]$ distinct species, k of them being invasive and $n-k$ being native. Invasive species are indexed $\forall i \in [\![1; k]\!]$ and native species are indexed $\forall i \in [\![k+1, n]\!]$. Again, we ask how a manager should allocate his budget in order to limit the negative impacts associated with invasions.

The problem is more general as we consider many species and therefore many more interactions. We also wish the model to apply with several formulations of expected diversity functions. Weitzman [1998] diversity concept is indeed one among many others and although the concept is appropriate for various management projects, it is not for others. In order for our results to remain as general as possible, we consider the expected diversity function W and the utility function U pertain to the class of $\mathcal{C}^2$ functions, *i.e* whose first and second order derivative both exist and are continuous.

As in the stylized model, we consider a manager is to choose a vector of effort X that maximizes an objective function, given species interdependence, and under the constraint of resources. We assume:

$$(8) \qquad P_i = q_i - x_i + \sum_{h \neq i} r_{ih} \; P_h \; , \quad q_i \in [0, 1[\;\; , \quad x_i \in [0, \overline{x}_i] \, .$$

with $P_i \in \Pi_i = \left[\underline{\Pi}_i, \overline{\Pi}_i\right] \sqsubseteq [0, 1]$, $\forall i$, $\forall \; x_i \in [0, \overline{x}_i]$, $\overline{x}_i$ the admissible range of control, and q_i the probability of survival of species i in the absence of control and without species interactions. Note that for native species, we consider $x_i = 0$, meaning that effort of control is only toward invasive species. As previously considered, the budget constraint is linear in efforts and we have:

$$\sum_{i=1}^{k} c_i * x_i \leq B \ . \tag{9}$$

where B is the total budget to be allocated to invasive species control and c_i is the cost per unit of effort to control species i.

The maximization programme of the manager is:

$$\begin{aligned} &\max_{\{x_i\}_{i=1}^k \in \ \times_{i=1}^k [0,\overline{x}_i]} \quad W\left(\{P_i\}_{i=k+1}^n\right) + U\left(\{P_i\}_{i=1}^n\right) \\ &\textit{subject to} \ \ (8) \textit{ and } (9). \end{aligned} \tag{10}$$

Remark that we follow the exact same assumptions than in the stylized model and consider that only native species contribute to the diversity of their ecosystem while utility of all species are considered in the objective. It goes without saying that the utility of a species might be negative.

It is convenient subsequently to work with matrix expressions, written in bold characters. For any matrix $\mathbf{M}$, let $\mathbf{M}^\top$ denote its transpose. Further, $\mathbf{I}^n$ is the $(n \times n)$ identity matrix, ι^n is the n dimensional column vector whose elements are all 1.

We define:

$$\mathbf{Q} \equiv \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ \\ q_n \end{bmatrix}, \ \mathbf{R} \equiv \begin{bmatrix} 0 & r_{12} & \dots & r_{1n} \\ r_{21} & 0 & \dots & r_{2n} \\ \dots & \dots & \ddots & \vdots \\ \\ r_{n1} & r_{n2} & \dots & 0 \end{bmatrix}, \ \mathbf{P} \equiv \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}, \ \mathbf{c} \equiv \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\overline{\mathbf{P}} \equiv \begin{bmatrix} \overline{P}_1 \\ \overline{P}_2 \\ \vdots \\ \overline{P}_n \end{bmatrix}, \quad \underline{\mathbf{P}} \equiv \begin{bmatrix} \underline{P}_1 \\ \underline{P}_2 \\ \vdots \\ \underline{P}_n \end{bmatrix}, \quad \mathbf{X} \equiv \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \overline{\mathbf{X}} \equiv \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \vdots \\ \overline{x}_k \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

In matrix form, the system (8) reads as:

$$\mathbf{P} = \mathbf{Q} - \mathbf{X} + \mathbf{R} * \mathbf{P}. \tag{11}$$

Under the *weak* assumption that matrix $\mathbf{I}^n - \mathbf{R}$ is invertible , the system (11) is solvable and the solution of this system reads as:

$$\mathbf{P} = \mathbf{\Lambda} * (\mathbf{Q} - \mathbf{X}), \tag{12}$$

where $\mathbf{\Lambda} \equiv [\mathbf{I}^n - \mathbf{R}]^{-1}$.

Let $\mathcal{P}(\mathbf{X}) \equiv \mathbf{\Lambda} * (\mathbf{Q} - \mathbf{X})$ refer to the affine mapping from efforts to probabilities.

We plug (12) into (10) to get rid of probabilities, and express our management of invasive species only in terms of efforts. Define the two *composite* functions, which here are mappings from the values taken by function $\mathcal{P}(\mathbf{X})$ to the set of real numbers:

$$\begin{aligned} W \circ \mathcal{P}(\mathbf{X}) &\equiv W(\mathcal{P}(\mathbf{X})), \\ U \circ \mathcal{P}(\mathbf{X}) &\equiv U(\mathcal{P}(\mathbf{X})). \end{aligned}$$

To each vector $\mathbf{X}$ corresponds a unique vector $\mathbf{P} = \mathcal{P}(\mathbf{X})$. Therefore the invasive species management problem becomes the constrained maximization of a function of management efforts $\mathbf{X}$:

$$\max_{\mathbf{X}} W \circ \mathcal{P}(\mathbf{X}) + U \circ \mathcal{P}(\mathbf{X}), \tag{13}$$

subject to:

$$\mathbf{c}^\top * \mathbf{X} \leq B, \tag{14}$$

$$0 * \iota^{\mathbf{n}} \leqq \mathbf{X} \leqq \overline{\mathbf{X}}. \tag{15}$$

Finding the vector $\mathbf{X}$ solution to the optimization problem above is strictly equivalent to finding the optimal set of management efforts $x_j, \forall j \in [\![1; k]\!]$. Should the budget be dispersed to deal with many different invasive species or should it be concentrated on a subset of few invasive species ? This second option could be seen as an *extreme policy* as in our stylized model where budget was allocated prioritarily to one species. Given the budget constraint is assumed linear, answering this question translates in discussing the gradient of the objective function, that is the gradient of functions W and U.

3.0.1 Case 1. Objective is not concave

This is a well know result that the maximization under a linear constraint of a function that is non-negative semi definitive, i.e. which is not concave, admits an extreme solution. Considering the case where $W(\mathbf{P}) + U(\mathbf{P})$ is non-negative semi definite, we can easily see that because $\mathcal{P}(\mathbf{X})$ is an affine mapping, $W \circ \mathcal{P}(\mathbf{X}) + U \circ \mathcal{P}(\mathbf{X})$ is also non-negative semi definite.
When $\sum_{i=1}^{n} P_i \neq 1$, most diversity function $W(\mathbf{P})$ pertains to the class of non-negative semi definite functions (proofs upon request for Rao and Weitzman diversity functions). We can then easily prove that $W(\mathbf{P}) + U(\mathbf{P})$ is non-negative semi definite when $U(\mathbf{P})$ is positive semi-definite, i.e. linear and convex functional form. Else, there exists conditions on the gradient of $U(\mathbf{P})$ for $W(\mathbf{P}) + U(\mathbf{P})$ to be non-negative semi-definite.

We deduce that in a large majority of cases, the objective of the manager pertains to this class of function and the solution to the maximization problem lies on the boundary of the efforts set. The boundary involves corners, e.g. $x_i = 0$ or $x_i = \overline{x}_i$, and possibly a segment between two corners, therefore with $x_i \in [0, \overline{x}_i]$ for at most one species.

As the objective function of the manager is usually not an affine mapping as it is the case in our stylized model, finding the solution to this problem is not trivial. Following [Weitzman, 1998] and [Courtois et al., 2014], we resort to a linear approximation of the objective function in order to find this solution.[4]

Let us denote:

$$D_i \equiv \left.\frac{\partial W}{\partial P_i}\right|_{\mathbf{P}=\underline{\mathbf{P}}}, \qquad U_i \equiv \left.\frac{\partial U}{\partial P_i}\right|_{\mathbf{P}=\underline{\mathbf{P}}},$$

and define the two matrices:

$$\mathbf{A} \equiv \begin{bmatrix} D_1 + U_1 \\ D_2 + U_2 \\ \vdots \\ D_n + U_n \end{bmatrix}, \qquad \mathbf{\Upsilon} \equiv \mathbf{A}^\top * \mathbf{\Lambda}.$$

The linearized problem in matrix form turns out to be:

$$\max_{\mathbf{X}} \ \mathbf{\Upsilon} * \mathbf{X} + \text{ constant terms,} \tag{16}$$

subject to (14) and (15).

[4]We suggest the interested reader to refer to [Courtois et al., 2014] for a discussion over the legitimacy of this approximation in this class of problems.

The matrix $\mathbf{\Lambda} = [\mathbf{I}^n - \mathbf{R}]^{-1}$ permits the transformation of the information about ecological interactions conveyed by matrix $\mathbf{R}$ into operational data. The computation of the matrix $\mathbf{\Lambda}$ is easily made and if Λ_{ij} denotes a typical element of $\mathbf{\Lambda}$, then $\mathbf{\Upsilon}$ is a n-dimensional line vector of the type:

$$\mathbf{\Upsilon} = [\alpha_1 \ , \ \ \alpha_2 \ , \ \ ..., \ \alpha_n] \ ,$$

where

$$\alpha_i \equiv \sum_{h=1}^{n} (D_h + U_h)\,\Lambda_{hi} \ .$$

We can now define the "benefit"-cost ratios $\overline{R}^i \equiv \alpha_i / c_i$, or with explicit reference to relevant information:

$$\overline{R}^i \equiv \frac{1}{c_i} \sum_{h=1}^{n} (D_h + U_h)\,\Lambda_{hi} \ , \qquad i = 1, ..., k. \tag{17}$$

Assume invasive species $i, i \in [\![1; k]\!]$ is assigned with the highest value of $\overline{R}^i$. Then, if this value is superior to zero, species i should be targeted first and control efforts should focus on this species until efforts reach a maximum, *i.e* until $x_i = \overline{x_i}$. Then, if $c_i \overline{x_i} < B$, the invasive species with the second highest R_i>0 should be the next target and this iterative process would go on until budget B is fully exhausted.

As we can appreciate, the R^i score of invasive species i does not depend merely on its own impacts but actually on the overall impacts generated by this species on other species, $\sum_{h=1}^{n} (D_h + U_h)\,\Lambda_{hi}$, via ecological interactions. Therefore, a species with a strong disutility can be overridden by another, endowed with a lower disutility, but whose importance is enhanced because of its ecological role on other species.

Following [Courtois et al., 2014], we are able to make a ranking criterion operational to decide whether or not to spend money on the management of an invasive species:

Proposition 2 *In our optimization problem with ecological interactions, defined by (16), (14) and (15), there exists a cutoff value $\overline{R}^*$ such that:*

- *if $\overline{R}^i > \overline{R}^* \Longrightarrow x_i = \overline{x}_i$, (species i is granted full management),*
- *if $\overline{R}^i < \overline{R}^* \Longrightarrow x_i = 0$, (species i is granted zero management).*

This myopic ranking criterion is a transparent measure to set management priorities and decide whether or not to allocate part of a budget toward the management of an invasive species. Note that the criterion being the result of an approximation of the objective, there exists an induced error that can be estimated on the basis of the gradient of the objective. The biggest the curvature of the function, the higher the error.[5]

3.0.2 Case 2. Objective is concave

If a manager conveys informations on species interdependencies, species utilities, species contribution to expected diversity and cost of control of invasive species, the criterion proposed in proposition (2) is a simple rule for allocating budget. However, this rule can only be used as such if the objective of the manager is non-negative semi definite, that is if the objective is not concave. Unfortunately, although many classes of biological invasion management problems are not concave, some are. In particular, the utility function from controlling harmful invasive species may be negative semi definite and there exists conditions for the objective function $W \circ \mathcal{P}(\mathbf{X}) + U \circ \mathcal{P}(\mathbf{X})$ to be negative semi definite. In such a case, this is a well known result that the maximisation under a linear constraint of a negative semi definite function admits an interior solution, that is an optimal effort allocation vector.

Gradient method is useful in estimating this optimal effort vector. Linearizing the objective function and minimizing the distance between the gradient of the objective and of the constraint allows for gradually approximate the value of this optimal policy. However, we aim here at defining a simple rule of thumb that can be used by a manager and this approach is inappropriate in that it is rather complex to handle. Albeit not fully satisfying, another solution is for the manager to use an iterative algorithm in order to allocate his budget using myopic criterion presented in (2).

Let divide budget B in s shares, such that $b = B/s$. If s is sufficiently big then b is small allowing for considering that allocation of budget b to the maximization problem lies on the boundary of the efforts set. This boundary involves corners, e.g. $x_i = 0$ or $x_i = \overline{x}_i$, and possibly a segment between two corners, therefore with $x_i \in [0, \overline{x}_i]$ for at most one species. The following iterative procedure gives the interior solution of our maximisation problem using the simple myopic rule previoulsy defined:

Algorithm 3 *In our optimization problem with ecological interactions, the following iterative procedure is an approximation of the interior solution:*

1. *Compute the increased proportion ΔP_h for any invasive species h given b is spent on the control of h only*

[5] We suggest interested readers to refer to [Courtois et al., 2014] for an evaluation of this error.

2. *Compute R^h score for all invasive species h using ranking criterion formula (17)*

3. *Allocate the share of budget b to the control of the invasive species with the highest score*

4. *Update the proportion P_i given this allocation*

5. *Allocate the next share, until all shares are allocated.*

This algorithm is technically demanding as many computations are made necessary in order to approximate the solution of our maximization problem. It constitutes however an easy-to-use tool in order to set priorities in invasive species management.

4 Conclusion

Echoeing the work of Witting and Loeschcke [1995] whom stated that the optimization of biodiversity conservation should be a minimization of the future loss of biodiversity, we demonstrated in this paper that an optimization framework is relevant to tackle the issue of prioritizing invasive species management projects. Such a framework is able to take into account pragmatic limitations, such as a budget constraint, as well as more unusual constraints such as ecological ones. This model is to our knowledge the first prioritization tool that effectively take into account relative management costs and impact cascades in choosing which invasions to control in priority.

A key output of the paper is the design of a myopic rule a non expert manager could use in order to efficiently allocate his budget to limit ecosystem disruptions. Recall that similar rules were used in order to set conservation priorities in New Zealand [Joseph et al., 2008]. An straightforward continuation of the present work is to provide with an application of our decision criterion.

Several additional assumptions are required in order to perform this application. First, an appropriate diversity function concept is to be picked. Among the several available concepts in the literature, Weitzman expected diversity and Rao quadratic entropy are a priori the best candidates but correspond to two distinct philosophy of diversity that needs to be further discussed. Second, the measure of species distinctiveness is to be elicited. Genetic dissimilarity might not be the best information to measure diversity. Third, our model can either work with P_i standing for the survival probability of species i or for P_i standing for the relative abundance of species i within the ecosystem. According to the species selected in the application, one variable or the other is to be used but if the second option is selected the model needs to be modified at the margin in order to incorporate the additional constraint $\sum_{i=1}^{n} P_i = 1$. Finally, the crux of the framework is to account for species interdependences. The robustness of the ranking rule is fully dependent on the quality of the inderdependence informations (i.e. R_{ih} matrix). Generally, a specific focus is to be put on availability of the data required for making use of this ranking criterion.

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