Fishery stage game 0515

Conflict and cooperation in an age structured fishery

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Abstract

The literature on 'fish wars', where non-cooperative exploitation one, or several interacting, fish stock(s) is well established by now, but age and stage structured models do not seem to have been handled within this literature. In this paper we study a game where two fishing fleets compete for the same fish stock, which is divided into several age categories. The situation modelled here may be representative for many transnational fisheries, such as the North Atlantic cod fishery. The outcome of the game is compared to the optimal cooperative solution, regarding both the steady state solution and the dynamic approach path. We analyze the game under different assumptions with respect to gear selectivity, with respect to alternative model specifications, and also with respect to the information available to each fleet, both about the underlying ecological interaction and the actions of the other agent. The results differ in several respects from what is found in biomass models, and are supported by a numerical example

Key words: Fishery economics, age model, optimal exploitation, selectivity
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1.Introduction

Fisheries are frequently the source international conflicts and often characterized by suboptimal resource management. The unwillingness of fish stocks to contain themselves within national borders leads to a typical commons problem present in almost all marine fisheries to a larger or smaller extent. Even after the post war emergence of exclusive economic zones, problems still remain as regards defining a fish stock within the jurisdiction of one country alone. Fish stocks straddle across vast distances, across international borders, and are often present both in the high seas and within the exclusive economic zones of one or more countries at the same time. In addition, fish stocks are often highly migratory, travelling along coastlines and up and down rivers, spending much of their lifetime outside of the breeding grounds, thus giving rise to sequential fishing where different agents take turns in exploiting the stock. A particular aspect of this situation is that different age categories of the same stock frequently reside within the economic zones of different countries. In the latter case, different fleets do not strictly speaking aim for the same fish, but they nevertheless affect each other through the biological interaction. In fact, the same problem may occur between fleets that are distinguished not by nationality, but simply by utilizing different gear, thus aiming for different age categories of the same stock. This situation, which is not adequately handled within the existing literature on biomass models and sequential fishing, is actually quite common. Examples include the Arcto-Norwegian cod, that feeds in the Barents region, thus subject to harvest by trawlers, but where the mature fish migrates towards the Norwegian cost to spawn, there being exploited by small line fishing vessels (Suhaila, 1997, Armstrong, 1991). Other examples in the same vain include the Southern bluefin tuna that spends its immature phase along the coast of Australia, but then migrates to the high seas in the Indian Ocean. Similar descriptions apply to the Canada halibut and the North sea herring. Anadromous species such as salmon spawns in rivers, being harvested by coastal vessels and recreational fishermen, but lives most of its life in the open sea. These are some of the world’s most valuable fisheries.

The literature on 'fish wars', where agents engage in non-cooperative games of exploiting a fish stock, has grown large since the seminal contributions of Munro (1979) and Levhari and Mirman (1980). A good survey of this literature is provided by Bailey et.al. (2009). For our purpose, the literature on ‘sequential’ fishing, where agents alternate in exploiting a common stock, is of particular relevance. This literature is meant to address situations such as the ones exemplified above, where one stock migrates between several economic zones. Hannesson (1995) studies the possibility for self-enforcing agreements in such a sequential fishery, and McKelvey (1997) expands the framework to consider the possibility of side payments. Laukkanen (2001) shows that the effectiveness of trigger strategies to maintain a cooperative equilibrium is undermined when stock recruitment is subject to stochastic shocks. However, these studies all employ biomass models, implicitly assuming that the fish caught in one area is identical to the fish caught in another. Cohort models, on the other hand, are still scarce in the economic literature, as noted by Skonhoft et al, (2012). The seminal book on bioeconomic modeling by Clark (1976) treats the Beverton-Holt model to some extent, but puts main emphasis on biomass models. Seminal contributions by Reed (1980), Charles and Reed (1985) and Getz and Height (1985) subsequently enhanced the economic understanding of the exploitation of age structured stocks. In a more recent contribution, Tahvonen (2009) presents a thorough study of the optimal harvesting of age structured fish stocks, under the assumption of non-selective gear. But in general biomass models are much more used in theoretical work. This is unfortunate, as age structured models arguably give a more realistic picture of actual fish stock dynamics, as well as a more transparent view of the information available at any point in time and the timing of decisions. Moreover, they lend themselves easily to numerical applications. Very few studies address age structured stocks in a game theoretic setting, but there are two notable exceptions that both study the Arcto Norwegian cod mainly through numerical analysis. Sumaila (1997) studies the difference inprofitabiliy between a trawler fleet and a coastal fleet, and shows e.g. that the most profitable fleet in a cooperative solution may become the least profitable fleet in a non-cooperative situation where the trawler fleet utilizes its strategic advantage. Diekert et. al. (2010) assume symmetric players, i.e. two trawler fleets, that compete through mesh size and not effort. They show that a non-cooperative solution implies ‘fishing down the size categories’, and that the outcome of a non-cooperative open loop equilibrium is both far from the cooperative optimum and close to the status quo situation in terms of profit and stock size.

In this paper we study a situation not associated with any particular fishery, but where different age categories of a fish stock reside within two different economic zones. The exploitation of the stock is then modeled as a game between two countries, or fleets, that aim for different cohorts, but nevertheless affect each other's profitability through the biological interaction. The model can also be understood as a game between fleets that utilize different gear, and hence aim for different age categories of the same stock. First overall optimality is addressed, which can under certain conditions also be interpreted as a non-cooperative equilibrium with side payments. Second, we discuss the situation where both fleets are unable to organize and hence exhibit myopic behavior, and conditions for one of the fleets to be excluded from the fishery in this case. Third, The situation where one fleet is uncoordinated and the other behaves as a single entity is studied. Finally, we analyze the game between two fleets that are perfectly organized internally. It is shown that, depending on parameter values, both coexistence and exclusion is possible in all different scenarios. Also, it is not generally clear whether it pays to organize, as lack of organization acts as a credible promise of myopic behavior. Typically, the fleet that targets the old mature stock is most profitable and should take the largest quota in a cooperative situation. Nevertheless, without cooperation the fleet catching the young stock may exploit its strategic advantage and end up with the highest profit. It appears as if the asymmetric structure of the game itself is aggravating the commons problem. We consider also the situation where both fleets exhibit complete and find that the impact of non-cooperation is less severe in this case. Hence, gear selectivity is a double-edged sword when strategic interaction is taken into account. The results are subsequently illustrated with a numerical example.

The paper is organized as follows. In the next section 2, a three stage population model is formulated. In section 3 we analyze the optimal harvest regime under cooperation Section 4 presents the non-cooperative solution, both with a full Nash-Cournout solution and with myopic adjustment for one or both fleets. In section 5 some numerical examples are provided.

2. Population model and harvest

2.1 Population model

For analytical tractability, we use a population model consisting of only three cohorts: juveniles $X_{0,t}$, young mature fish $X_{1,t}$ and old mature fish $X_{2,t}$. Young and old mature fish are both harvestable, while the juveniles are not subject to fishing mortality. While recruitment is endogenous and density dependent, natural mortality is assumed fixed and density independent for all three age classes. In the single period of one year, three events take place in the following order; first, recruitment and spawning, then fishing and finally natural mortality.

The number of juveniles is governed by the recruitment function

(1) $X_{0,t} = R(X_{1,t}X_{2,t})$,

where $R(\cdot)$ is characterized by $R(0,0) = 0$ and $\partial R / \partial X_{i,t} = R_i' > 0$, together with $R_i'' < 0$ ($i = 1, 2$). As higher fertility of the old than the young mature is assumed, we also have $R_2' > R_1'$. The number of young mature fish follows next as

(2) $X_{1,t+1} = s_0 X_{0,t}$,

where s_0 is the fixed natural survival rate. Finally, the number of old mature fish is described by

(3) $X_{2,t+1} = s_1(1 - f_{1,t})X_{1,t} + s_2(1 - f_{2,t})X_{2,t}$,

where $f_{1,t}$ and $f_{2,t}$ are the fishing mortalities, or harvest rate, of the young and old mature stage, respectively, while s_1 and s_2 are the natural survival rates. When combining Eqs. (1) and (2) we have

(4) $X_{1,t+1} = s_0 R\left(X_{1,t}, X_{2,t}\right)$.

Eqs. (3) and (4) represent a reduced form model in two age-classes, where both equations are first order difference equations. The population equilibrium for *fixed* fishing mortalities $f_{i,t} = f_i$ is defined by $X_{i,t+1} = X_{i,t} = X_i$ $(i = 1, 2)$ such that Eqs. (3) holds as

(3’) $X_2 = s_1(1 - f_1)X_1 + s_2(1 - f_2)X_2$

and Eq. (4) as

(4’) $$X_1 = s_0 R\left(X_1, X_2\right).$$

(3’) is notified as the *spawning constraint* while (4’) is the *recruitment constraint*. An interior equilibrium holds for $0 \leq f_1 < 1$ only; that is, not all the young mature fish can be harvested. Thus, with $f_1 = 1$ and $0 < f_2 < 1$ we find $X_2 = 0$ together with $X_1 > 0$. An interior equilibrium is shown in Figure 1. Higher fishing mortalities will shift up the spawning constraint (3’) and hence lead to smaller stocks, while higher natural survival rates work in the opposite direction. The ratio of old to young mature fish is given by the slope of the spawning constraint, $X_2 / X_1 = s_1(1 - f_1)/(1 - s_2(1 - f_2))$. Therefore, neither the scaling nor the shape parameters in the recruitment function influence the equilibrium fish ratio. It is seen that lower fishing mortalities increases the proportion of old mature fish. The reason why lower f_1 increases the proportion of X_2 is that the increased population of the young mature age class spills over to an even larger increase in the old mature population.

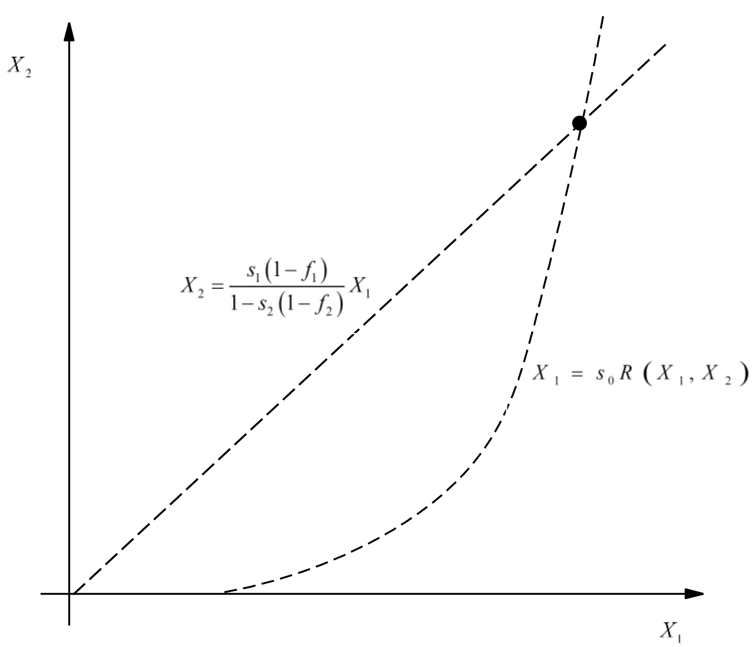


Figure 1. Biological equilibrium with fixed fishing mortalities.

2.2 Harvest

The fish stock is exploited by two fishing fleets, and each fleet is targeting a particular age class of the fish. As explained in the introduction, this harvesting scenario fits reality in many instances, either because of differences in gear selection or because the two age classes reside in different economic zones. To a certain extent, the fleets might be able to influence their catch composition. For example, the mesh size may be increased, or other gears may be adopted to leave the younger and smaller fish less exploited (see, e.g., Beverton and Holt 1957 and Clark 1990, and the recent Singh and Weninger 2009). However, in most instances the catches are composed of species from different cohorts and there is hence 'bycatch'. By convention, it is assumed that fleet one targets the young mature fish (stock one) while agent two targets the old mature fish (stock two). Even though our results do not depend on the specific formulation of the production functions, we choose a specific function form, the so-called Spence function, to ease the analytical exposition:

(5) $$H_{i,t} = X_{i,t}\left(1-e^{-q_i E_{i,t}}\right),\ i=1,2.$$

Where $H_{i,t}$ is the harvest of fleet i at time t, $E_{i,t}$ is the fishing effort, interpreted as, e.g., the number of standardized fishing vessels, and q_i is a productivity, or 'cathchability', parameter (1/effort).

The bycatch is described as

(6) $$B_{j,t} = X_{j,t}\left(1-e^{-\tilde{q}_i E_{i,t}}\right),$$

such that $H_{i,t}$ describes the catch of the targeted stock and $B_{i,t}$ the bycatch. The catchability coefficients q_i and $\tilde{q}_i$ $(i=1,2)$ thus determine the intended and unintended catch per unit of effort, respectively.

With the total mortality rate defined as $f_{i,t} = (H_{i,t} / X_{i,t} + B_{i,t} / X_{i,t}) = h_{i,t} + b_{i,t}$, the mature age class growth Eq. (3) becomes:

$$(7) \qquad X_{2+1,t} = s_1\left(e^{-q_1E_{1,t}} + e^{-\tilde{q}_2E_{2,t}} - 1\right)X_{1,t} + s_2\left(e^{-q_2E_{2,t}} + e^{-\tilde{q}_1E_{1,t}} - 1\right)X_{2,t},$$

which with no bycatch reduces to:

$$(8) \qquad X_{2+1,t} = s_1e^{-q_1E_{1,t}}X_{1,t} + s_2e^{-q_2E_{2,t}}\ X_{2,t}.$$

While $e^{-q_1E_{i,t}}$ is interpreted as the escapement rate of the stock after harvesting, $(1-e^{-q_1E_{i,t}})$ represents the fishing mortality, or harvest rate. Notice that with the Spence harvesting function, the fishing mortalities can never reach one for a finite amount of effort, and the extinction of the population is hence not possible. In the following two sections we assume perfect selectivity and no bycatch, while non-selectivity and bycatch is treated in section 5.

3. Exploitation I: Cooperation

3.1 The optimal program

We start by looking at the cooperative solution where the maximum present-value profit of both fleets is determined jointly. As we wish to focus on biological interaction, we assume that the fleets do not interfere with each other through market mechanisms and hence assume fixed prices. With p_1 and p_2 as the fish prices (Euro/fish), assumed to be fixed over time and not influenced by the size of the catch, and c_i as the unit effort cost (Euro/effort) ($i = 1, 2$), also assumed to be fixed, $\pi_t = p_1X_{1,t}\left(1-e^{-q_1E_{1,t}}\right) - c_1E_{1,t} + p_2X_{2,t}\left(1-e^{-q_2E_{2,t}}\right) - c_2\,E_{2,t}$ describes the current total profit with perfect selectivity and no bycatch. The constraints of this problem are the biological equations (4) and (8). In addition, the initial stock sizes, $X_{i,0}$, are assumed known.

The lagrangian of this present-value maximizing problem may be written as

$$L=\sum_{t=0}^{\infty}\rho^{t}\{p_1X_{1,t}\left(1-e^{-q_1E_{1,t}}\right)-c_1E_{1,t}+p_2X_{2,t}\left(1-e^{-q_2E_{2,t}}\right)-c_2\,E_{2,t}$$
$$-\rho\lambda_{t+1}\left[X_{1,t+1}-s_0R\left(X_{1,t}X_{2,t}\right)\right]-\rho\mu_{t+1}\left[X_{2,t+1}-s_1e^{-q_1E_{1,t}}\;X_{1,t}-s_2e^{-q_2E_{2,t}}\;X_{2,t}\right]\},$$

where $\lambda_t>0$ and $\mu_t>0$ are the shadow prices of the biological constraints (4) and (8), respectively, and $\rho\in[0,1]$ is a discount factor. Following the Kuhn-Tucker theorem the first order necessary conditions (with $X_{i,t}>0$, $i=1,2$) are:

(9) $\partial L/\partial E_{1,t}=p_1q_1X_{1,t}e^{-q_1E_{1,t}}-c_1-\rho\mu_{t+1}s_1q_1X_{1,t}e^{-q_1E_{1,t}}\le 0$; $E_{1,t}\ge 0$, $t=0,1,2,\ldots$,

(10) $\partial L/\partial E_{2,t}=p_2q_2X_{2,t}e^{-q_2E_{2,t}}-c_2-\rho\mu_{t+1}s_2X_{2,t}e^{-q_2E_{2,t}}\le 0$, $E_{2,t}\ge 0$, $t=0,1,2,\ldots$,

(11) $\partial L/\partial X_{1,t}=p_1\left(1-e^{-q_1E_{1,t}}\right)-\lambda_t+\rho\lambda_{t+1}s_0R_1'+\rho\mu_{t+1}s_1e^{-q_1E_{1,t}}=0$, $t=1,2,3,\ldots$,

and

(12) $\partial L/\partial X_{2,t}=p_2\left(1-e^{-q_2E_{2,t}}\right)+\rho\lambda_{t+1}s_0R_2'-\mu_t+\rho\mu_{t+1}s_2e^{-q_2E_{2,t}}=0$, $t=1,2,3,\ldots$,.

The interpretation of the control conditions (9) and (10) are straightforward. Condition (9) indicates that the fishing effort of fleet 1 should take place up to the point where the marginal profit is equal to, or below, the economically, ρ, and biologically, s_1, discounted marginal biomass loss of the immature stage, as evaluated by the shadow price of the biological constraint (8). Condition (10) is analogous for the old mature stock. Eqs. (11) and (12) steer the shadow price values. Rewriting Eq. (11) as $\lambda_t=p_1\left(1-e^{-q_1E_{1,t}}\right)+\rho\lambda_{t+1}s_0R_1'+\rho\mu_{t+1}s_1e^{-q_1E_{1,t}}$, it is seen that the number of young mature fish should be maintained such that its shadow price equalizes the marginal harvest value plus its growth contribution to recruitment and the old mature stage, as evaluated at their shadow prices with biological and economic discounting taking into account. Eq. (12) can be given a similar interpretation.

Rewriting the control conditions (9) and (10) as:

(9') $\dfrac{p_1}{s_1}\left(\dfrac{X_{1,t}e^{-q_1E_{1,t}}-c_1/p_1q_1}{X_{1,t}e^{-q_1E_{1,t}}}\right)\le\rho\mu_{t+1}$; $E_{1,t}\ge 0$, $t=0,1,2,\ldots$

and

$$(10') \quad \frac{p_2}{s_2}\left(\frac{X_{2,t}e^{-q_2E_{2,t}} - c_2 / p_2q_2}{X_{2,t}e^{-q_2E_{2,t}}}\right) \le \rho\mu_{t+1}\,; E_{2,t} \ge 0\,,\; t = 0,1,2,...,$$

reveals that the survival rates $s_i, i = 1,2$ and the economic parameters p_i, q_i and c_i alone determine the optimal harvesting priority. Fertility plays no direct role. Assuming an interior solution, the optimality condition for each stock can be rewritten in terms of the optimal escapement $X_{i,t}e^{-q_iX_{i,t}}$ as a function of the economic parameters and the shadow price of stock 2 as

$$(11) \qquad X_{i,t}e^{-q_iE_{i,t}} = \frac{c_i / q_i}{p_i - \rho s_i\mu_{t+1}}, i = 1,2\,.$$

With $\rho s_i\mu_{t+1} = 0$, that is, when either the discount factor, the survival rate of stock i or the shadow value of stock 2 is zero, myopic adjustment ensues, where stock i is harvested down to its zero marginal profit level c_i / p_iq_i each year.

Therefore, although the recruitment function certainly impacts on the optimal harvest of the two stocks, its properties are not observed directly in the optimal harvesting policy. This result is similar to what is obtained by Reed (1980) in a model where the maximum sustainable yield (MSY) is maximized and no economic parameters are included. Altogether, when the possibility of no harvesting at all is ignored, the optimal harvest policy comprises the following three possibilities; Case i) with $E_{1,t} > 0$ and $E_{2,t} > 0$, Case ii) with $E_{1,t} > 0$ and $E_{2,t} = 0$ and Case iii) with $E_{1,t} = 0$ and $E_{2,t} > 0$. Case i) is the interior solution and is in contrast to Reed (1980) and Skonhoft et al. (2012) a possible option in our model because the lagrangian is jointly concave in the control and state variables.

Combining (9') and (10') gives the condition

$$\frac{p_1}{s_1}\left(\frac{X_{1,t}e^{-q_1E_{1,t}} - c_1 / p_1q_1}{X_{1,t}e^{-q_1E_{1,t}}}\right) = \frac{p_2}{s_2}\left(\frac{X_{2,t}e^{-q_2E_{2,t}} - c_2 / p_2q_2}{X_{2,t}e^{-q_2E_{2,t}}}\right),$$

which states that share of the escapement of each stock above its zero marginal profit level c_i / p_iq_i is equal across the two stocks, when weighted by the price-to-survival ratio p_i / s_i. The stock that has the highest price-to-survival ratio will have the smallest escapement share above its zero marginal profit level, and can thus be said to be harvested more aggressively. With similar survival rates, and when the market price is higher for stock 2, stock 2 should be harvested more intensively than stock 1, a result in accordance with previous studies (i.e. Diekert et. al., 2010). In the special case where $c_1 / p_1 q_1 = c_2 / p_2 q_2$, the escapement in terms of tonnes is simply higher for the stock with the lower price-to-survival ratio. Also note that, while negative marginal profit will not be optimal for any of the two stocks, as they are controlled separately, zero marginal profit may be optimal under myopic adjustment but then for both stocks at the same time. It is thus not optimal to harvest only one of the two stocks myopically, while leaving the other stock at a level where marginal profit is positive. Rewriting this condition as

$$\frac{1}{s_1}\left(p_1 - \frac{c_1}{q_1 X_1 e^{-q_1 E_{1,t}}}\right) = \frac{1}{s_2}\left(p_2 - \frac{c_2}{q_2 X_{2,t} e^{-q_2 E_{2,t}}}\right)$$

allows the interpretation that the survival adjusted marginal profit at the end of the harvesting period is equal for the two stocks.

3.2 Steady state analysis

In a steady state the biological constraints read (4’), and

(3’’) $X_2 = s_1 e^{-q_1 E_1} X_1 + s_2 e^{-q_2 E_2} X_2$,

such that the escapement rates $e^{-q_i E_i}$, or fishing mortalities $f_i = (1 - e^{-q_i E_i})$ ($i = 1, 2$), are constant through time. As already explained, the slope of the spawning constraint indicates the fishing pressure. It is difficult to draw general conclusions about the differences of the slope of the spawning constraint with our harvest options Case i) – Case iii). Therefore, harvest option Case i) can be either more aggressive or less aggressive than Case ii), and so on. However, rewriting the spawning constraint as $X_2 = \frac{s_1 e^{-q_1 E_1}}{1 - s_2 e^{-q_2 E_2}} X_1$ reveals that increased effort of both fleets contributes to decreasing the slope of the spawning constraint. As the spawning constraint cuts the recruitment constraint from above, this corresponds to a higher ratio of stock 2 compared to stock 1 in biological equilibrium, see figure 1. Thus, a more intense harvesting regime generally leads to stock 2 being reduced compared to stock 1, and thus a higher relative profitability in the harvest of the young mature stock. A more aggressive harvesting regime in general thus implies a shift towards relatively more intensive harvesting of the young mature stock.

In the optimal steady state, the portfolio conditions (11) and (12) can be used to eliminate λ. We then obtain

(12) $$\mu = \frac{p_1\left(1-e^{-q_1E_1}\right)+\psi p_2\left(1-e^{-q_2E_2}\right)}{\psi-\rho s_1 e^{-q_1E_1}-\psi\rho s_2 e^{-q_2E_2}} \quad ,$$

where $\psi = \frac{1-\rho s_0 R_1}{\rho s_0 R_2}$ is the relative contribution of the two stocks to the recruitment of next year's young mature stock, bionomically discounted by the factor ρs_0. For $\rho = 1$, that is, when economic discounting is abstracted from, this can be recognized as the slope of the recruitment constraint (8) in Figure 1. The numerator in expression (12) is the contribution of the two stocks to profit. The denominator is the bionomically discounted sum of the contributions of the two stocks to next year's spawning stock. As expected, we see that as the shadow price must be nonnegative. Conditions (9) and (10) together with (12) together determine the two efforts as functions of the stocks in Case i). The expressions are messy and we do not incude them here.

In Case ii) the optimal escapement of stock 2 when inserting for μ is given as

$$X_1 e^{-q_1E_1} = \frac{c_1 / p_1q_1}{1-\sigma_1},$$

where $\sigma_1 = \frac{X_1 - c_1 / p_1q_1}{X_1}\frac{\rho s_1 / \psi}{1-\rho s_2}$ depends on the relative contribution to recruitment ψ, the discount factor, survival rates and the share of stock 1 that can be harvested with a profit. If $\sigma_1 = 0$

, myopic harvesting occurs. For case iii) we have $X_2 e^{-q_2 E_2} = \frac{c_2 / p_2 q_2}{1-\sigma_2}$ with $\sigma_2 = \frac{X_2 - c_2 / q_2 p_2}{X_2} \frac{\rho s_2}{1-\rho s_1 / \psi}$, and with a similar interpretation.

The comparative static effects of changes in the economic parameters on the optimal harvesting effort are straightforward. Exploitation of each stock increases with own price and catchability, and decreases with unit costs. On the other hand, a decrease in the discount factor contributes to increasing the harvesting pressure of both stocks. Finally, the effect of changes in the survival rates are less straight forward. Increased survival of one stock all others equal will reduce the harvesting effort of that stock. But the effect on the exploitation of the other stock is difficult to identify.

3.3. Dynamic properties

Above the steady state with a constant number of fish through time was analyzed. To ensure stability of the system, we require that the spawning constraint intersects the recruitment from above, and hence that $R' < 1$, see the appendix. Under this assumption, the system will be locally asymptotically stable for all constant harvesting effort levels. As the profit function is nonlinear in the controls, theory suggests that fishing should be optimally adjusted to lead the population gradually to steady state; that is, some kind of saddle-path dynamics, but with some degree of under- or overshooting due to the discrete time formulation.

4. Exploitation II: Non-cooperation

4.1 The setting

We now consider the situation where the two fleets are owned and managed by separate agents that exploit the fish stocks in a non-cooperative manner. Both agents are assumed to have full information about each other's profit functions, and we also assume that both agents know the biological equations and can monitor and assess the size of both stocks perfectly. These are standard assumptions as used by e.g. Mesterton-Gibbons (1996), and Fischer and Mirman (1992). See also Pereau et al. (2013) and the references therein. In this full information game each agent maximizes its own profit only, subject to both growth constraints.

A fully dynamic, or analysis of the game studied here requires some assumption about the updating of information and strategy choices made by the two fleets. A natural candidate for a closed loop, or feedback, equilibrium in the present situation would be to assume that information is updated once a year. However, due to the sequential nature of the game, the standard backwards induction solution procedure employed in the literature (e.g. Levhari and Mirman, 1980) leads to the uninteresting result that completely myopic behavior is used by both fleets in this case. This is because, at every stage and for both fleets, the stock size available at the beginning of the season is determined by the escapement of the other stock at the previous stage, which is completely in the hands of the other fleet when complete fishing selectivity is assumed,. Hence, there is no incentive for either fleet to take the future into account when forming its own harvesting policy. Alternative assumptions could be to let exploitation policies be updated more seldom, such as every other year. However, this feels somewhat arbitrary; a more probable case is that the other fleet's effort can be monitored more often than once a year, something that would normally lead to more agressive behaviour. In this section we thus choose to focus on an open loop equilibrium solution, where polices are formed once and for all at the beginning of the game, and compare this solution with the situation where one or both fleets are myopic, in addition to the optimal cooperative solution.

4.2 Myopic equilibrium

We first consider a myopic solution, where both agents maximize their respective current profit while taking the behavior of the other agent as given. This corresponds to the open access situation, where each fleet contains infinitely many individual vessel owners, but may be realistic also with a finite amount of agents. Indeed, as shown by Clark (1980), open access behaviour may occur even with only two agents, in a continuous time setting. For fleet 1, we then find

(13)| $\partial L / \partial E_{1,t} = p_1 q_1 X_{1,t} e^{-q_1 E_{1,t}} - c_1 \leq 0;\ \ E_{1,t} \geq 0,$

while

(14) $\partial \pi_{2,t} / \partial E_{2,t} = p_2 q_2 X_{2,t} e^{-q_2 E_{2,t}} - c_2 \leq 0;\ \ E_{2,t} \geq 0$

is for fleet 2. These two conditions together with the constraints (3) and (4) determine the effort use, stock sizes and the dynamic interaction among our two fleets.

Harvest is profitable if and only if marginal profit exceeds marginal cost for zero effort; that is,

$p_i q_i X_{i,t} - c_i > 0$. We then have $p_i q_i X_{i,t} e^{-q_i E_{i,t}} = c_i$ with $E_{i,t} > 0$, so that escapement equals the zero marginal profit stock level $c_i / p_i q_i$. If this holds for both agents we have Case i). For Fleet 2 this implies that the condition

$$X_{2,t} = s_1 \frac{c_1}{p_1 q_1} + s_2 \frac{c_2}{p_2 q_2} > \frac{c_2}{p_2 q_2} \Rightarrow$$
$$\frac{c_2}{p_2 q_2} < \frac{s_1}{1 - s_2} \frac{c_1}{p_1 q_1}$$

must hold. This is the case for sufficiently high survival rates, for instance with $s_1, s_2 > 1/2$ when $c_1 / p_1 q_1 = c_2 / p_2 q_2$. With $p_2 > p_1$ this condition is more likely to be satisfied. Inserting this condition into the spawning constraint (8) yields now $X_{2,t+1} = s_1 c_1 / p_1 q_1 + s_2 c_2 / p_2 q_2$ which is a static equation and states that the steady state number of old mature fish will be equal to the sum of the survival adjusted zero marginal profit stock levels. The corresponding condition for fleets 1 to be in operation is that X_1, as implicitly defined from the expression $X_1 = s_0 R\left(X_1 + \alpha c_2 / p_2 q_2\right)$, exceeds $c_1 / p_1 q_1$. If either the former or latter condition is not satisfied, we will be in case Case ii) or Case iii), respectively. In Case ii) where Fleet 2 is unprofitable, the spawning constraint reads $X_{2,t+1} = s_1 c_1 / p_1 q_1 + s_2 X_{2,t}$. This describes a linear difference equation of stock 2 that is stable because $s_2 < 1$. In Case iii) with unprofitable harvest of Fleet 1, we find that the spawning constraint reads $X_{2,t+1} = s_1 X_{1,t} + s_2 c_2 / p_2 q_2$. Therefore, in this case we have the dynamic system: $X_{1,t+1} = s_0 R(X_{1,t}, X_{2,t})$ and $X_{2,t+1} = s_1 X_{1,t} + s_2 c_2 / p_2 q_2$, which is stable because $R' < 1$ and $s_1 < 1$.

Figure 2 demonstrates the equilibrium situations resulting from the different harvesting regimes that can arise in the myopic situation. The dashed lines depict the recruitment constraint and the 'natural' spawning constraint when $E_1 = E_2 = 0$. Both fleets are in operation if the lines defined by $X_1 = c_1 / p_1 q_1$ and $X_2 = c_2 / p_2 q_2$ intersect in the area above the recruitment constraint and below the natural spawning constraint. The actual spawning constraint, identified as a solid line, is defined piecewise from the different harvesting regimes that correspond to the three cases. An example of Case i) is given in Figure 2a) , where the zero marginal profit lines intersect in the point

P . The resulting equilibrium is where the spawning constraint $X_2 = s_1c_1 / p_1q_1 + s_2c_2 / p_2q_2$ intersects the recruitment constraint, resulting in the equilibrium Q .

In Case ii) with $(1-s_2)c_2 / p_2q_2 > s_1c_1 / p_1q_1$, only fleet 1 is in operation. As seen in Figure 2b), the point P is above the natural spawning constraint. The equilibrium point Q is found where the line $X_2 = \left[s_1 / (1-s_2)\right](c_1 / p_1q_1)$ intersects the recruitment constraint. Figure 2c) shows the situation where only fleet 2 is in operation, as the point P lies below the recruitment constraint. The spawning constraint reads $X_2 = s_1X_{1,t} + s_2c_2 / p_2q_2$ and Q shows the resulting equilibrium.

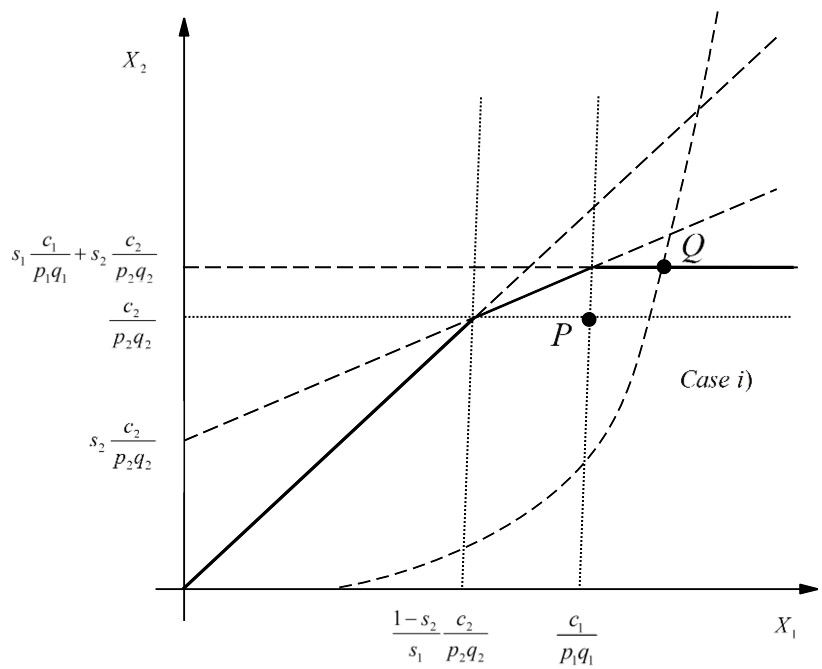


Figure 2a

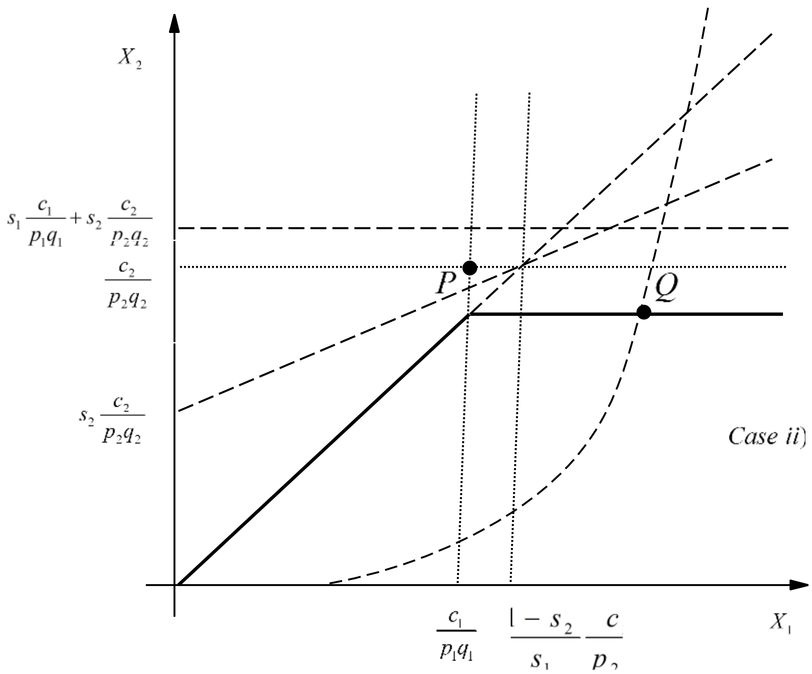


Figure 2b)

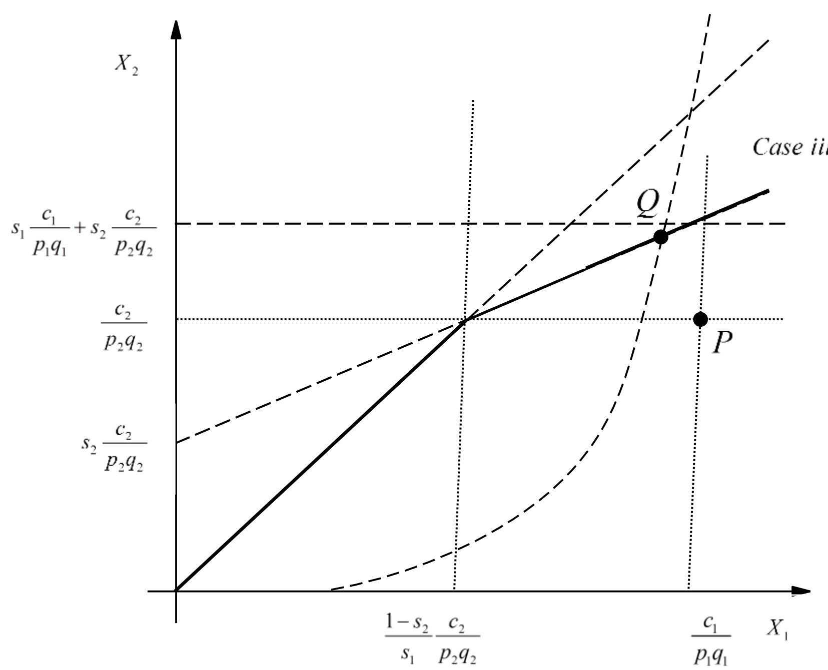


Figure 2c)

4.3 One of the fleets is myopic

In this subsection it is assumed that one of the two fleets is myopic and maximizes profit each year without taking the future into account. At least for fleet 2 we consider this to be a rather realistic case, as the coastal fishery can be view as consisting of many small vessel owners that are not sufficiently organized to behave strategically so as to affect the harvest decision of fleet 1.

We thus choose to focus mainly on the case where fleet 2 is the myopic player in this section. As all strategic considerations then belong to fleet 1, and although we assume simultaneous moves, the model can be considered as a Stackelberg game with fleet 1 as the dominant player. Fleet 2 thus adjusts passively to the behaviour of fleet 1 while fleet 1 takes fleet 2's optimal adjustment into account before forming its own harvest decision.

The game is solved by backwards induction and we first solve the problem of fleet 2 in stage two. Fleet 2 maximizes current profit $\pi_{2,t} = p_2 X_{2,t}\left(1-e^{-q_2 E_{2,t}}\right) - c_2\, E_{2,t}$ while taking the stock size $X_{2,t}$ as given. Therefore, if harvest is profitable for fleet 2, i.e. $X_{2.t} > c_2 / p_2 q_2$, we have $p_2 q_2 X_{2.t} e^{-q_2 E_{2,t}} - c_2 = 0$. On stage 1, fleet 1 faces three different potential optimization problems, depending on whether fleet 2 is in operation or not, and, in the latter case whether the absence of fleet 2 is due to strategic overfishing by fleet 1. When fleet 2 operates, (14) holds with equality and the spawning constraint changes to $X_{2,t+1} = s_1 X_{1,t} e^{-q_1 E_1} + s_2 c_2 / p_2 q_2$. The Lagrangian of this problem is

$$
\begin{aligned}
L_1 = \sum_{t=0}^{\infty} \rho^t \{ & p_1 X_{1,t}\left(1-e^{-q_1 E_{1,t}}\right) - c_1 E_{1,t} \\
& - \rho\lambda_{t+1}\left[X_{1,t+1} - s_0 R\left(X_{1,t} X_{2,t}\right)\right] - \rho\mu_{t+1}\left[X_{2,t+1} - s_1 e^{-q_1 E_{1,t}}\ X_{1,t} - s_2 (c_2 / p_2 q_2)\right]\}
\end{aligned}
$$

Note that the fishing effort of fleet 2 is not included here. This gives the necessary conditions for maximum for fleet 1 as:

(15) $\partial L_1 / \partial E_{1,t} = p_1 q_1 X_{1.t} e^{-q_1 E_{1,t}} - c_1 - \rho \mu_{1,t+1} s_1 q_1 X_{1.t} e^{-q_1 E_{1,t}} \leq 0$; $E_{1,t} \geq 0$, $t = 0,1,2,\ldots$,

(16) $\partial L_1 / \partial X_{1,t} = p_1 \left(1 - e^{-q_1 E_{1,t}}\right) - \lambda_{1,t} + \rho \lambda_{1,t+1} s_0 R_1 ' + \rho \mu_{1,t+1} s_1 e^{-q_1 E_{1,t}} = 0$, $t = 1,2,3,\ldots$,

and

(17) $\partial L_1 / \partial X_{2,t} = \rho \lambda_{1,t+1} s_0 R_2 ' - \mu_{1,t} = 0$, $t = 1,2,3,\ldots$.

The shadow price of stock 2 in the steady state, as seen from the perspective of fleet 1, can now be solved for as

(19)
$$\mu = \frac{p_1 \left(1 - e^{q_1 E_1}\right)}{\psi - \rho s_1 e^{-q_1 E_1}} ,$$

whereby the optimal steady harvest policy for fleet 1 can then be found explicitly in terms of the optimal escapement of stock 1 by using (11):

(20)
$$X_1 e^{-q_1 E_1} = \frac{\psi c_1 / p_1 q_1}{\psi - \rho s_1 + c_1 / p_1 q_1 X_1}$$

This condition can be rewritten in the form of a 'Modified Golden Rule' (MDG; Clark, 1990), which is a familiar optimality condition for the exploitation of renewable resources in discrete time.

(20')
$$\frac{p_1 - c_1 / \left(q_1 X_1\right)}{p_1 - c_1 / \left(q_1 X_1 e^{-q_1 E_1}\right)} = \frac{\psi}{\rho s_1} .$$

The other case to consider is when fleet 2 is not operating even though fleet does not make any effort to keep fleet 2 out of business. Fleet 1 then enjoys a natural monopoly and optimizes as if being a sole owner. The recruitment constraint is then $X_{2,t+1} = s_1 X_{1,t} e^{-q_1 E_{1,t}} + s_2 X_{2,t}$. Going through the same optimization procedure as above leads to the shadow price of stock 2 as

(21)
$$\mu = \frac{p_1 \left(1 - e^{q_1 E_1}\right)}{\psi \left(1 - \rho s_2\right) - \rho s_1 e^{-q_1 E_1}} ,$$

and the optimal escapement of stock 2 as

(22)
$$X_1 e^{-q_1 E_1} = \frac{\left(1 - \rho s_2\right) c_1 / p_1 q_1}{\psi \left(1 - \rho s_2\right) - \rho s_1 + c_1 / p_1 q_1 X_1} .$$

The MDG version of this condition is

(22')
$$\frac{p_1 - c_1/\left(q_1 X_1\right)}{p_1 - c_1/\left(q_1 X_1 e^{-q_1 E_1}\right)} = \psi \frac{1-\rho s_2}{\rho s_1}.$$

In the last situation, fleet 1 finds it profitable to overfish to deter fleet 2 from operating. This leads to the condition $X_1 e^{-q_1 E_1} = \left[\left(1-s_2\right)/s_1\right] c_2 / p_2 q_2$. Together with the condition $X_1 = s_0 R\left(X_{1,} c_2 / p_2 q_2\right)$, this determines both the effort of fleet 1 and the stock size in steady state. It is a numerical task to determine whether this solution is more profitable for fleet 1 than coexistence. We briefly state the corresponding solutions for fleet 2 when fleet 1 is myopic as

$$\frac{p_2 - c_2/\left(q_2 X_2\right)}{p_2 - c_2\left(q_2 X_2 e^{-q_2 E_2}\right)} = \frac{1}{\rho s_2},$$

when fleet 1 is in operation, and

$$\frac{p_2 - c_2/\left(q_2 X_2\right)}{p_2 - c_2\left(q_2 X_2 e^{-q_2 E_2}\right)} = \frac{\psi - \rho s_1}{\psi \rho s_2}$$

when fleet 1 does no operate. To deter fleet 1 from fishing, the condition ….must be met

It is not clear which situation is most preferable to each fleet: being myopic or being the optimizer. If both fleets prefer being myopic, no matter what the other fleet does, the only Nash equilibrium is that both fleets are myopic, if they are able to coexist. But if each fleet prefers the myopic solution if and only if the other fleet optimizes, there are two asymmetric pure Nash equilibria, in addition to one in mixed strategies. In this case, we have a chicken game where each fleet has an incentive to stay disorganized to commit itself to a myopic policy, in order to induce the other fleet to optimize. Lastly, if both prefer a solution where both fleets optimize, but would rather be myopic if the other fleet is so, a prisoner dilemma game ensues where a cooperative solution can in pprinciple be sustained by trigger strategies. This question is analyzed in the numerical section.

Open loop equilibrium

Now we consider the situation where each fleet behaves strategically and optimize fully with respect to the biological constraints, taking the behavior of the other fleet as given, We confine ourselves to an open loop Nash-Cournout equilibrium, as explained above, assuming that the fleets do not update their strategies during the course of the game. Both fleets maximize own profit taking the behaviour of the other fleet as exogenous, and obtains, the necessary conditions

(25) $\partial L/\partial E_{1,t} = p_1 q_1 X_{1,t} e^{-q_1 E_{1,t}} - c_1 - \rho\mu_{1,t+1} s_1 q_1 X_{1,t} e^{-q_1 E_{1,t}} = 0$, $t = 0,1,2,\dots$,

(26) $\partial L/\partial X_{1,t} = p_1\left(1 - e^{-q_1 E_{1,t}}\right) - \lambda_{1,t} + \rho\lambda_{1,t+1} s_0 R_1{}' + \rho\mu_{1,t+1} s_1 e^{-q_1 E_{1,t}} = 0$, $t = 1,2,3,\dots$,

and

(27) $\partial L/\partial X_{2,t} = \rho\lambda_{1,t+1} s_0 R_2{}' - \mu_{1,t} + \rho\mu_{1,t+1} s_2 e^{-q_2 E_{2,t}} = 0$, $t = 1,2,3,\dots$,.

Note that the shadow prices $\mu_{i,t}$ and $\lambda_{i,t}$ will now generally differ between the two fleets, hence the subscripts. The resulting harvesting rule is still given by condition (13) above, but the steady state shadow price of the old mature stock, as seen from fleet 1's perspective is now given as

(28) $$\mu_1 = \frac{p_1\left(1 - e^{-q_1 E_1}\right)}{\psi\left(1 - \rho s_2 e^{-q_2 E_2}\right) - \rho s_1 e^{-q_1 E_1}}.$$

Using this expression in (13) gives the optimal harvesting rule

$$X_1 e^{-q_1 E_1} = \frac{c_1 / p_1 q_1}{1 - \tilde{X}_1 \dfrac{\rho s_1}{\psi\left(1 - \rho s_2 e^{-q_2 E_2}\right)}},$$

where $\tilde{X}_1 = \dfrac{X_1 - c_1 / q_1 p_1}{X_1}$ is the share of X_1 that is above the zero profit level.or, in MDG form,

$$\frac{p_1 - c_1 / \left(q_1 X_1\right)}{p_1 - c_1 / \left(q_1 X_1 e^{-q_1 E_1}\right)} = \psi \frac{1 - \rho s_2 e^{-q_2 E_2}}{\rho s_1}.$$

Similarly, the same procedure applied to fleet 2 gives

(29) $$\mu_2 = \frac{\psi p_2\left(1 - e^{-q_2 E_2}\right)}{\psi\left(1 - \rho s_2 e^{-q_2 E_2}\right) - \rho s_1 e^{-q_1 E_1}},$$

which gives

$$X_2 e^{-q_2 E_2} = \frac{c_2 / p_2 q_2}{1 - \tilde{X}_2 \dfrac{\psi\rho s_2}{\psi - \rho s_1 e^{-q_1 E_1}}}$$

where $\tilde{X}_2 = \frac{X_2 - c_2 / q_2 p_2}{X_2}$, and

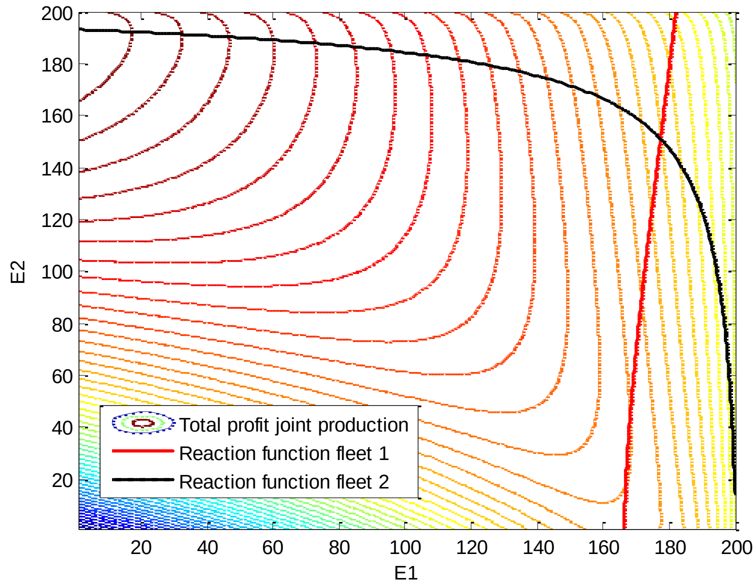
$$\frac{p_2 - c_2 / \left(q_2 X_2\right)}{p_2 - c_2 / \left(q_2 X_2 e^{-q_2 E_2}\right)} = \frac{\psi - \rho s_1 e^{-q_1 E_1}}{\rho s_2} \quad .$$

It is not clear whether the non-myopic fleet will harvest more or less depending on whether the other fleet operates. In a static Cournout model, aggressive behavior from one agent tends to induce the other agent to less aggressive behaviour. However, in this case the opposite may also be conceivable. With aggressive behaviour from fleet 2 for instance, fleet 1 will have a smaller incentive to harvest conservatively, as its natural capital in the form of a large standing stock will be captured by the other fleet. On the other hand, profitability goes down for fleet 1 when fleet 2 operates aggressively, something that would a priori lead to less effort by fleet 1. The overall effect is ambiguous.

5. Numerical illustrations

To illustrate the model numerically we use the following parameter values taken from Skonhoft et. al. (2012): s0=0.6, s1=s2=0.7, p1 =2, p2 =3, q1=q2=0.5, c1=c2=10. They are not meant to represent any particular fishery, but only to illustrate the workings of the model. The recruitment function is specified as the Beverton-Holt function $R\left(X_{1,t} X_{2,t}\right) = \frac{a(X_{1,t} + \alpha X_{2,t})}{b + (X_{1,t} + \alpha X_{2,t})}$ with a=1,500 as the scaling parameter and b =500 as the shape parameter, and the parameter α =1.5 indicating higher fertilility of the old than the young mature age class.

The non cooperative situation is illustrated with reaction functions for the two fleets. The contour lines indicate total profit in the cooperative solution, which implies harvesting of stock 2 only for the baseline parameter values. Note that the reaction function for fleet 2 slopes upwards, indicating that better profitability for fleet 1, which leads to an outwartds shift, induces also fleet 2 to devote more effort. This is due to the strategic advantage of fleet 1 because stock 1 is able to reproduce.



It is also evident from the figure that the Nash-Cournout equilibrium solution implies signifigcantly lower total profit in the fishery.

6. Extensions

6.1 Non-selectivity issues

In this subsection we relax the assumption of perfect selectivity. First we treat the case where there is no selectivity at all, so that the two fleets do not discriminate between the young and mature stock. Then we discuss the intermediate case, where selectivity is imperfect. When the fleets have no way to discriminate between age classes, the population equation for stock 2 is modified by setting $q_1 = \tilde{q}_1$ and $q_2 = \tilde{q}_2$, so that $X_{2+1,t} = s_1\left(e^{q_1E_{1,t}} + e^{q_2E_{2,t}} - 1\right)X_{1,t} + s_2\left(e^{q_2E_{2,t}} + e^{q_1E_{1,t}} - 1\right)X_{2,t}$

The two fleets are now identical, except for possibly different catchability and unit cost parameter values.

Performing the same optimization procedure as in the previous sections leads to the following first order conditions

() $\quad \partial L / \partial E_{1,t} = p_1q_1X_{1,t}e^{-q_1E_{1,t}} - c_1 - \rho\mu_{t+1}s_1q_1X_{1,t}e^{-q_1E_{1,t}} = 0\,,\ t = 0,1,2,\ldots,$

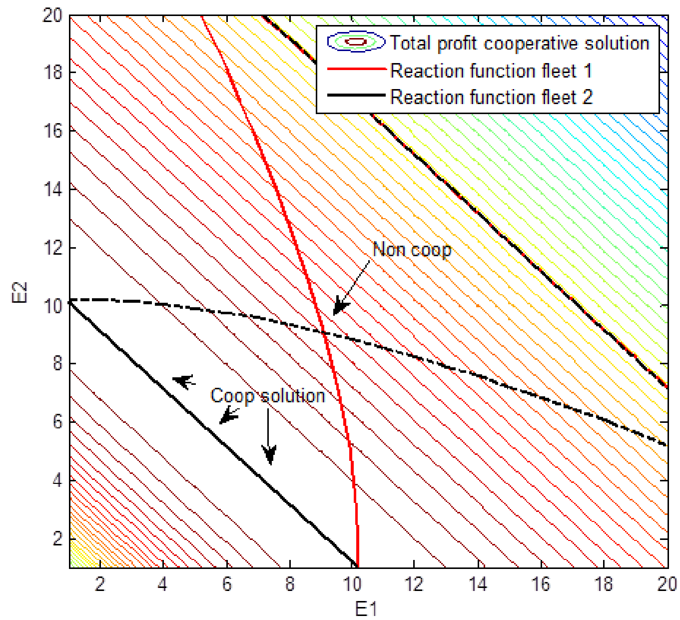
() $\quad \partial L / \partial E_{2,t} = p_2q_2X_{2,t}e^{-q_2E_{2,t}} - c_2 - \rho\mu_{t+1}s_2X_{2,t}e^{-q_2E_{2,t}} = 0\,,\ t = 0,1,2,\ldots,$

() $\quad \partial L / \partial X_{1,t} = p_1\left(1 - e^{-q_1E_{1,t}}\right) - \lambda_t + \rho\lambda_{t+1}s_0R_1{}' + \rho\mu_{t+1}s_1\left(e^{q_1E_{1,t}} + e^{q_2E_{2,t}} - 1\right) = 0\,,\ t = 1,2,3,\ldots,$

and

() $\quad \partial L / \partial X_{2,t} = p_2\left(1 - e^{-q_2E_{2,t}}\right) + \rho\lambda_{t+1}s_0R_2{}' - \mu_t + \rho\mu_{t+1}s_2\left(e^{q_2E_{2,t}} + e^{q_1E_{1,t}} - 1\right) = 0\,,\ t = 1,2,3,\ldots,.$

The reaction functions along with the cooperative solution are illustrated in Figure 3. As the fleets are identical, the optimal cooperative solution is represented by a straight line on which every combination of the two fleets give the same total profit.



As the figure indicates, the Nash Cournout solution implies lower total profit also in this case. However, the difference between Nash and cooperative solution is smaller than in the situation with full gear selectivity. Perhaps surprisingly, gear selectivity, while being beneficial in a situation with full cooperation, may contribute to more aggressive harvesting effort in a competitive fishery.

7. Concluding remarks

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Appendix

Stability

The general system is given as

$$\begin{aligned} X_{1,t+1} &= R\left(X_{1,t} + \alpha X_{2,t}\right) \\ X_{2,t+1} &= s_1 X_{1,t} e^{-q_1 E_{1,t}} + s_2 X_{2,t} e^{-q_2 E_{2,t}} \end{aligned} .$$

The Jacobian matrix $J = \begin{vmatrix} R' & \alpha R' \\ s_1 e^{-q_1 E_{1,t}} & s_2 e^{-q_2 E_{2,t}} \end{vmatrix}$ gives the expression

$$\begin{aligned} \lambda &= \frac{1}{2}\left[R' + s_2 e^{-q_2 E_{2,t}} \pm \sqrt{\left(R' + s_2 e^{-q_2 E_{2,t}}\right)^2 - 4R'\left(s_2 e^{-q_2 E_{2,t}} - \alpha s_1 e^{-q_1 E_{1,t}}\right)} \right] \\ &= \frac{1}{2}\left[R' + s_2 e^{-q_2 E_{2,t}} \pm \sqrt{\left(R' - s_2 e^{-q_2 E_{2,t}}\right)^2 + 4R'\alpha s_1 e^{-q_1 E_{1,t}}} \right] \end{aligned}$$

to determine the characteristic roots of the system. Both roots are required to have modulus less than unity for asymptotic stability. As the trace is postive and both roots are real, stability is ensured iff $R' + s_2 e^{-q_2 E_{2,t}} + \sqrt{\left(R' - s_2 e^{-q_2 E_{2,t}}\right)^2 + 4R'\alpha s_1 e^{-q_1 E_{1,t}}} < 2$.

Define

$$\begin{aligned} a_1 &= -R' - s_2 e^{-q_2 E_2} \\ a_2 &= R'\left(s_2 e^{-q_2 E_{2,t}} - \alpha s_1 e^{-q_1 E_{1,t}}\right) \end{aligned}$$

The necessary and sufficient condition for stability are (Gandolfo, 1997, ch. 5)

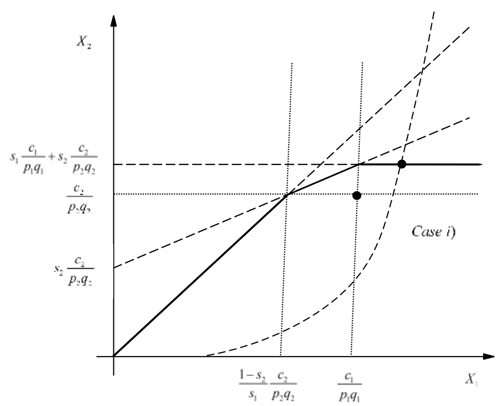
$$\begin{aligned} 1 + a_1 + a_2 &> 0 \\ 1 - a_2 &> 0 \\ 1 - a_1 + a_2 &> 0 \end{aligned} .$$

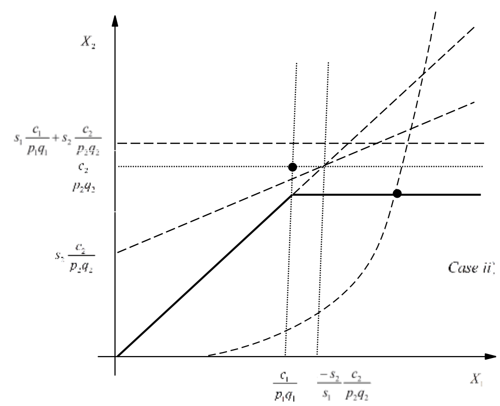
These conditions can be written as

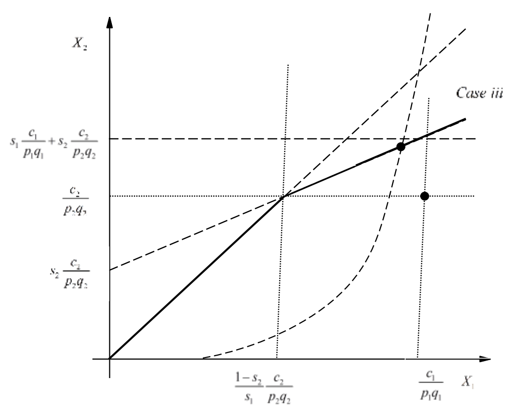
$$\begin{aligned} \frac{1-R'}{\alpha R'} &> \frac{s_1 e^{-q_1 E_1}}{1 - s_2 e^{q_2 E_2}} \\ \frac{1-R'}{\alpha R'} &> \frac{1}{\alpha} s_2 e^{-q_2 E_2} - s_1 e^{-q_1 E_1} - 1 \\ \frac{1-R'}{\alpha R'} &> -\frac{\left(1+R'\right)/\alpha + s_1 e^{-q_1 E_1}}{s_2 e^{-q_2 E_2}} \end{aligned} .$$

The first inequality states that the slope of the recruitment constraint must be less than the slope of the spawning constraint at the equilibrium, and holds only if $R' < 1$. Once this is satisfied, the other inequalities are satisfied as well. Hence, the condition that the spawning constraint cuts the recruitment constraint from above is a necessary and sufficient condition for stability. Moreover, this condition holds for all $E_1, E_2 \geq 0$, so that there are no constant levels of fishing effort that can destabilize the system.

Figures







Upper left: Both fleets operate.

Upper right: fleet 1 only.

Lower left: fleet 2 only.