# Willingness to pay for public environmental goods under uncertainty

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**Abstract:** We develop a microeconomic approach for valuing the benefits from a public environmental good under uncertainty and the possibility of insurance. Most environmental goods (and ecosystem services) are non-market-traded, and benefits from such goods are typically enjoyed under conditions of uncertainty. Uncertainty can arise from the environmental side (e.g. ecosystem or climate), or from the economics side (e.g. income). In this paper, we consider (binary) uncertainty in the provision of an environmental good. We use a constant-elasticity-of-substitution (CES) utility function, where utility depends on a market-traded consumption good and an environmental good which is exogenously provided in a fixed quantity. The CES function is nested in a constant-relative-risk-aversion form. As a benefit measure, we derive the marginal willingness to pay (WTP) for changes in (i) the probability of loss, (ii) the size of loss, and (iii) the current level of the environmental good. We also study the effect of risk aversion on willingness to pay.

**JEL-Classification:** Q51, H22, H41

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# 1 Introduction

Earth's ecosystems provide an array of goods and services many of which are non-market-traded and public in nature, e.g., air and water purification, soil fertility, waste decomposition, mitigating extreme weather events (floods and droughts), preventing soil erosion, mangrove habitat (flood protection and fish nurseries), maintaining biodiversity, etc. Two properties are often associated with these ecosystem goods and services. First, the quantity in which a particular service is available is exogenous to the individual consumer or ecosystem user (who, hence, takes the quantity as given). Second, there is uncertainty about future quantities due to variability in climatic conditions, physical, biological and other processes.

In this paper, we develop a microeconomic approach for valuing the benefits from a public environmental good under uncertainty and with heterogeneous consumption. The definition of the environmental good is kept sufficiently general so that it can range from ecosystem services to traditional public goods. The two relevant features of this good are the following: it is non-market-traded; the good is assumed to have a fixed supply and is available free of cost to the individual. The uncertainty comes in a binary form; the individual faces two states of the world – either current supply of the public environmental good is maintained or there is a loss of supply. Both marginal and non-marginal value measures will be considered and the relationship between the two measures will also be developed.

Valuation of risk has received considerable attention in the literature over the years, see, e.g., Cornes (1980), Freeman (2003, 1989), Hanley et al. (2007), Bockstael and Freeman (2005), Shogren and Crocker (1991). For example, in Freeman (1989), modeling utility as depending on a composite commodity and occurrence of an adverse event, *ex-ante* and *ex-post* values of risk reduction (decreasing the size of a loss) and risk prevention (decreasing the probability of a loss) are derived. For non-marginal changes, Freeman also derives the option price which is the maximum state-independent payment that the consumer is willing to make ex-ante. Following this literature, the amount of income change needed to keep utility constant with a small change in an environmental good provides a marginal value (willingness to pay) of the change in the environmental good. Similar marginal willingness to pay measures have also been derived for valuing the impact of environmental changes on human health; Berger et al. (1987), Bartik (1988) and Courant and Porter (1981) provide early examples of this work.

In this paper, the structure of the consumer's problem is different – utility is derived from consumption of a (composite) market good and a public environmental good. The environmental good is supplied in fixed quantity, at zero cost; the supply is also uncertain ex-ante. Under certainty, Ebert (2003) presents a theoretical model for deriving marginal willingness to pay for a public good whose quantity is exogenous to the individual consumer. However, Ebert (2003) goes on to explore how incidence of public goods' benefits changes with income. This paper attempts to provide a similar valuation framework in the presence of uncertainty. A constant elasticity of substitution (CES) utility function is used to value changes in: (i) probability of loss; (ii) amount of loss; and (iii) current supply of the environmental good. An *ex-ante* willingness to pay (or option price) is also developed for non-marginal changes. It is proven that, for small changes, the option price will converge to the marginal willingness to pay. In addition, the CES utility function is nested in a constant relative risk aversion form which enables us to study the effect of uncertainty and risk aversion on willingness to pay. Preliminary results show that income and environmental elasticity of willingness to pay are non-constant.

The paper is organized as follows. Section 2 describes the assumptions of the model, the nature of the uncertainty and a complete statement of the consumer's problem. The underlying theoretical foundation is presented in section 3 and section 4 gives the main results. Section 5 concludes the paper.

# 2 Model

There is an individual who consumes two goods, a private market-traded good and a public non-market-traded environmental good. The amount consumed of the market-traded good, $X$, is certain and can be chosen by the consumer. In contrast, the environmental good is exogenously provided in a fixed quantity which is uncertain ex-ante. Thus, the amount of the environmental good is a random variable; its actually realized amount, $E$, is drawn from a binary distribution:

$$E = \begin{cases} E_0 - L & \text{with probability } r \\ E_0 & \text{with probability } 1-r \end{cases} . \tag{1}$$

Thus, the consumer enjoys a reference level of environmental-good provision of $E_0 > 0$ and faces a potential loss of $L$ (with $0 \leq L < E_0$) in one state of nature that occurs with probability $r$, and no loss in the other state of nature that occurs with probability $1 - r$ (where $0 \leq r \leq 1$).

The consumer's preferences over the two goods are represented by von-Neumann-Morgenstern expected utility $\mathcal{E}[U]$,[1] with a Bernoulli utility function $U(X, E)$ that is continuous, twice differentiable, strictly increasing and quasi-concave in both goods; that is,[2] $U_X > 0$, $U_E > 0$, and $U_X^2 U_{EE} - 2U_X U_E U_{XE} + U_E^2 U_{XX} < 0$. More specifically, we assume the following functional form of $U(X, E)$, which captures and parameterizes two important and relevant characteristics of preferences – (limited) *substitutability* between the private market-traded consumption good and the public non-market-traded environmental good, and *risk-aversion* with respect to the uncertain environmental-good provision:[3]

$$U(X, E) = \frac{1}{1-\rho} \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha) E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}} \tag{2}$$

$$\text{with} \quad 0 < \alpha < 1, \quad 0 \leq \sigma < +\infty, \quad \rho \geq 0 .$$

[1] $\mathcal{E}$ denotes the expectation operator.

[2] Subscripts denote partial derivatives: $U_X(X, E) := \partial U(X, E)/\partial X$ etc.

[3] For $\sigma = 1$ or $\rho = 1$, the right-hand side of Equation (2) is not defined. For $\sigma = 1$ or $\rho = 1$, we therefore define $U(X, E)$ as the continuous extension of the expression on the right-hand side of Equation (2) for $\sigma \to 1$ or $\rho \to 1$, respectively. These are well-known to be

$$\lim_{\rho \to 1} \frac{1}{1-\rho} \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha) E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}} = \log \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha) E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} ,$$

$$\lim_{\sigma \to 1} \frac{1}{1-\rho} \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha) E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}} = \frac{1}{1-\rho} \left[X^\alpha E^{1-\alpha}\right]^{1-\rho} .$$

Note that the log-limit is only true if one amends $-1$ in Equation (2).

Here, a constant-elasticity-of-substitution (CES) function has been nested in a constant-relative-risk-aversion (CRRA) function. The parameter $\sigma$ is the elasticity of substitution between the two goods $X$ and $E$. As special cases, the two goods may be perfect substitutes ($\sigma \to +\infty$), perfect complements ($\sigma = 0$), or Cobb-Douglas goods ($\sigma = 1$). Examples of nested CES utility functions are found in many fields of economics, especially in the climate change literature; see e.g., Heal (2009), Hoel and Sterner (2007), Kucheryavyy (2012) and Sterner and Persson (2007). The parameter $\rho$ measures relative risk aversion with respect to aggregate consumption $C$.[4]

Besides environmental-good provision also income may be uncertain. Thus, income is a random variable, too. Its actually realized amount $Y$ is drawn from a distribution with support $[Y_{\min}, Y_{\max}]$ where $Y_{\min} \geq 0$ and $Y_{\max} < +\infty$. Expected income is $\mathcal{E}[Y]$ and the variance of income is $\mathrm{var}[Y]$.[5] The minimum realized income in any state of nature is the lower bound of the support of the income distribution, $Y_{\min}$.

We assume the existence of formal or informal insurance or credit-and-saving-markets, so that the consumer can hedge against income risk (but not against environmental risk). The extent to which the consumer will hedge against income risk depends on the real costs of such hedging. If income insurance or credit-and saving was available at actuarially fair conditions, that is at real costs of zero, the consumer would fully hedge income risk and, as a result, could dispose of the expected income $\mathcal{E}[Y]$ for certain. If hedging was possible only at prohibitively high real costs, or was not possible at all, the consumer would not at all hedge against income risk. In this case, the maximum income that is available for certain to the consumer is the minimum income $Y_{\min}$.[6] If the real costs of hedging were in between zero and "prohibitively high", the consumer would seek partial hedging against income risk and, as a result, the maximum certain income

[4] Re-write the nested utility function as $U(C) = \frac{C^{1-\rho}}{1-\rho}$, where $C = \left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha) E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$.

[5] In slight misuse of notation, we also denote the random variable income by $Y$. This should not cause any confusion, as this is the only place where we do this. Normally, $Y$ denotes the actual realization of random income.

[6] As income is uncertain and the consumer is not hedging against income risk, actually realized income may be higher than $Y_{\min}$, but this is not certain.

would be in between the minimum income $Y_{\min}$ and the expected income $\mathcal{E}[Y]$. Let $\phi$ with $0 \leq \phi \leq 1$ denote the degree of perfection of insurance and credit-and-savings markets, where $\phi = 0$ means that such markets do not exist or income insurance or credit-and-saving is available only at prohibitively high costs, and $\phi = 1$ means that markets are perfect so that income insurance or credit-and-saving is available at zero real costs. Then the consumer's maximum certain income is given by:[7]

$$Y_C := \phi\mathcal{E}[Y] + (1-\phi)Y_{\min} \ . \tag{3}$$

It is this maximum certain income $Y_C$ (Equation 3) that constrains the consumer's choice of the amount $X$ of the market-traded consumption good, as this choice has to be made ex-ante, i.e. before resolution of income uncertainty.

Under the assumption that the income distribution is such that, *ceteris paribus*, a smaller $Y_{\min}$ is equivalent to a larger variance of the distribution, $\text{var}[Y]$, it follows from Equation (3) that the consumer's maximum certain income has the following properties:[8]

$$Y_{\min} \leq Y_C \leq \mathcal{E}[Y] \quad \text{where the first (second) equality holds if and only if } \phi = 0 \ \ (\phi = 1) \tag{4}$$

$$\frac{dY_C}{d\mathcal{E}[Y]} \geq 0 \quad \text{where equality holds if and only if } \phi = 0 \ , \tag{5}$$

$$\frac{dY_C}{d\text{var}[Y]} \leq 0 \quad \text{where equality holds if and only if } \phi = 1 \ , \tag{6}$$

$$\frac{dY_C}{d\phi} > 0 \quad . \tag{7}$$

For simplicity, we assume that the two uncertainties, of environmental-good provision and of income, are uncorrelated, so that the two distributions of $E$ and of $Y$ are statistically independent.

The consumer chooses the amount $X$ of the market-traded consumption good before the resolution of (environmental and income) uncertainty, with the aim of maximizing

[7] Again, as income is uncertain, actually realized income may be higher than $Y_C$ for $\phi < 1$, as the consumer will not fully hedge against income risk if such hedging comes at positive real costs.

[8] See Appendix A.1 for an elementary model of income uncertainty and insurance that yields these properties.

expected utility, thereby taking as given the uncertain amount $E$ of environmental-good provision (Equation 1), the price $P > 0$ of the market-traded consumption good, and the maximum certain income $Y_C$ (Equation 3):[9]

$$\begin{aligned} \max_{X} \quad & \mathcal{E}[U] = rU(X, E_0 - L) + (1 - r)U(X, E_0) \\ \text{s.t.} \quad & PX \leq Y_C \ . \end{aligned} \tag{8}$$

From now on we treat $Y_C$ as an elementary parameter of the model. That is we express all formula and results in terms of $Y_C$. Because, $Y_C$ is the amount of income that constraints consumer's choices.

# 3 Conceptual foundation

The theoretical foundation of the model is presented in this section assuming income is certain (denoted by $Y$). The approach is first developed and explained under conditions of environmental certainty in section 3.1. It is established that the income saved by the individual when an additional unit of the environmental good is provided can be regarded as a virtual (or pseudo) price for the environmental good. The existence and justification of using the virtual price is based on Ebert (2003). Section 3.2, then, presents an alternative framework when environmental good supply is uncertain (as defined by equation 1). Three marginal willingness to pay expressions are derived and explained in section 3.2. In addition, the *ex-ante* willingness to pay is discussed for non-marginal changes.

[9] This choice problem with exogenously given level of the environmental good has some resemblance to the consumption choice problem under rationing constraints ("conditional choice problem"; see, e.g., Neary and Roberts 1980, Latham 1980, Ebert 1998, 2001, Pollak 1969). However, in that literature the rationed good carries a price, so that its consumption diminishes income available for the un-rationed good. This is different from the problem studied here, where the rationed good is not market-traded, so that its consumption does not diminish income available for the un-rationed good.

## 3.1 WTP under certainty

Environmental certainty implies $r = 0$, that is, environmental supply $E$ is realized in either state. Then, following Ebert (2003), consumer faces the problem of maximizing utility by choosing market good consumption under the conditions that income is constrained at $Y_C$ and environmental good supply is fixed at $E$:

$$\begin{aligned} \max_{X} \quad & U(X, E) \\ \text{s.t.} \quad & PX \leq Y_C \quad \text{and} \quad E \text{ fixed} \end{aligned} \tag{9}$$

where, under strict quasi-concavity (implying non-satiation) the solution to the above is $X^* \equiv X^*(P, Y_C, E)$. Because the environmental good is supplied exogenously and at no cost to the consumer, the consumer spends all of maximum certain income on buying market good[10]. This means that the demand for $X$, in this model, will be independent of the quantity of environmental good supplied. Specifically, the consumer's Marshallian demand function for the market good is $X^* = X^*(Y_C, P) = \frac{Y_C}{P}$, which means $\frac{dX^*}{dE} = 0$[11]. Intuitively, this is similar to conditional Marshallian demand functions under rationing constraints; a conditional function occurs when the consumer faces exogenous (or predetermined) quantity for, at least, one commodity in his/her choice problem (Pollak 1969, Ebert 1998). Substituting in the objective function yields the (conditional) indirect utility function $V(P, Y_C, E) \equiv U(X^*(P, Y_C), E)$.

Alternatively, and hypothetically, assuming that the environmental good is traded in the market (at a virtual price of $P_E$), we can define the consumer's pseudo (or virtual) choice problem. The concept of *virtual* price, introduced in Rothbarth (1941), has been around in the literature on behavior under rationing, see e.g., Neary and Roberts (1980), Lee and Pitt (1986), Kuuluvainen (1990), Flores and Carson (1997), Hanemann

[10]Use of the Lagrange technique is trivial since the consumer spends entire income on purchasing $X$. Of course, if the consumer were to be *satiated*, the Lagrange function becomes relevant.

[11]In general, the sign of $\frac{dX^*}{dE}$ will depend on the particular utility function being used. Using the Cobb-Douglas utility function $U(X, Y, E) = 2(X + E)^{0.5} Y^{0.5}$, one can derive the Marshallian demand functions $X = \frac{M}{2P_X} - \frac{E}{2}$ and $Y = \frac{M}{2P_Y} + \frac{EP_X}{2P_Y}$ for the two market goods $X$ and $Y$, where, $\frac{\partial X^*}{\partial E} = -\frac{1}{2} < 0$ and $\frac{\partial Y^*}{\partial E} = \frac{P_X}{2P_Y} > 0$ (Ebert 1997).

(1991), Bettendorf and Buyst (1997), Ray (1989), Besley (1988). Virtual price can be defined as the price that elicits unrationed behavior from an individual in the presence of rationing constraints. Unrationed, in this context, is associated with the consumer's freely optimizing choices. The pseudo choice problem is[12]:

$$\begin{aligned} \max_{X,E} \quad & U(X, E) \\ \text{s.t.} \quad & PX + EP_E = \hat{Y}_C \end{aligned} \tag{10}$$

where, $\hat{Y}_C$ is the consumer's pseudo (or virtual) maximum certain income. Denoting $\lambda$ as the Lagrange multiplier, first order conditions are:

$$\mathcal{L}_X : U_X - \lambda P = 0, \tag{11}$$

$$\mathcal{L}_E : U_E - \lambda P_E = 0, \tag{12}$$

$$\mathcal{L}_\lambda : \hat{Y}_C - PX - EP_E = 0, \tag{13}$$

which follow directly from consumer theory. Solving the first order conditions yields the Marshallian demand system, $\hat{X} = \hat{X}(P, P_E, \hat{Y}_C)$ and $\hat{E} = \hat{E}(P, P_E, \hat{Y}_C)$, which can be used to obtain the indirect utility function $\hat{V} \equiv U[\hat{X}(\cdot), \hat{E}(\cdot)]$ for the pseudo choice problem[13]. Appendix A.2 derives the Marshallian demand system for the pseudo choice problem using the nested-CES utility function. The pseudo choice problem is important, where, it is assumed that the consumer is willing to pay $P_E$ for another unit of the environmental good under market conditions.

Recall the conditional indirect utility function given by $V(P, Y_C, E)$. Taking total differential of this function yields (holding $V$, and $P$ constant, i.e., $dP = dV = 0$):

$$\frac{\partial V}{\partial E} dE + \frac{\partial V}{\partial Y_C} dY_C = 0,$$

$$\left.\frac{dY_C}{dE}\right|_{dV=0} = -\frac{\frac{\partial V}{\partial E}}{\frac{\partial V}{\partial Y_C}}, \tag{14}$$

[12]Ebert (1998) presents both the conditional and the pseudo choice problems in a more general model with $m$ market goods and $n$ non-market goods.

[13]A hat will be used for functions associated with the pseudo problem.

where, $\frac{dY_C}{dE}$ is the amount of (maximum certain) income saved when an additional unit of the environmental good is provided (evaluated at the optimum). Willingness to pay for the environmental good ($WTP_E$) is defined as the negative of the above ratio (Ebert 1993, King 1986, Cornes 1992):

$$WTP_E \equiv -\frac{dY_C}{dE}\bigg|_{dV=0} = \frac{\frac{\partial V}{\partial E}}{\frac{\partial V}{\partial Y_C}}. \tag{15}$$

It is straightforward to see that the $WTP_E$ will be positive. Note that:

$$\frac{\partial V}{\partial E} = \left(\frac{\partial U}{\partial X^*} \cdot \frac{\partial X^*}{\partial E}\right) + \frac{\partial U}{\partial E} \qquad \text{and} \qquad \frac{\partial V}{\partial Y_C} = \frac{\partial U}{\partial X^*} \cdot \frac{\partial X^*}{\partial Y_C},$$

where, from model assumptions, $\frac{\partial U}{\partial X^*}$, $\frac{\partial U}{\partial E}$ and $\frac{\partial X^*}{\partial Y_C}$ are all positive. In addition, since $E$ is provided free of cost, $\frac{\partial X^*}{\partial E} = 0$ (change in environmental supply does not affect the demand for market good). This means that the ratio in equation 15 will be strictly positive.

Proving the definition of $WTP_E$ proceeds as follows. We ask the question that under what conditions does the consumer buy the bundle $(X^*, E)$ optimally, as a result of utility maximizing behavior, when there is a market for the environmental good (instead of $\hat{E}$). And, this is where the pseudo price ($P_E$) and the pseudo income ($\hat{Y}_C$) become important in linking the conditional and the pseudo maximization problems. If the consumer's pseudo income were to be adjusted as $(\hat{Y}_C + WTP_E \cdot E)$, then, the consumer would buy the bundle $(X^*, E)$ optimally (when there is a market for $E$). The Marshallian demand functions will be:

$$X^*(P, Y_C, E) = \hat{X}(P, P_E, (\hat{Y}_C + E \cdot P_E)), \tag{16}$$

$$E = \hat{E}(P, P_E, (\hat{Y}_C + E \cdot P_E)). \tag{17}$$

This implies that the budget constraint for the pseudo problem can be written as:

$$PX + E \cdot P_E = \hat{Y}_C = Y + WTP_E(P, Y_C, E) \cdot E \tag{18}$$

where, the two budget constraints will be equal if and only if $P_E = WTP_E$. Only under this condition, the consumer will buy the bundle $(X^*, E)$ in both the conditional (when supply of $E$ is fixed) and the pseudo choice (when there is a market for $E$) problem. This proves that $WTP_E(P, Y_C, E)$ is the virtual price of the environmental good.

## 3.2 WTP under environmental uncertainty

With no income uncertainty, the utility maximization problem in equation 8 becomes:

$$\begin{aligned} \max_{X} \quad & \mathcal{E}[U(X,E)] = rU(X, E_0 - L) + (1-r)U(X, E_0), \\ \text{s.t.} \quad & PX \leq Y_C. \end{aligned} \tag{19}$$

Intuitively, the solution to the above problem is similar to the certainty case. Given that environmental good is provided free of cost and at fixed quantity (either $E_0$ or $E_0 - L$), the Marshallian demand function for the market good, $X^*(P, Y_C) = \frac{Y_C}{P}$. The *ex-post* indirect utility function under uncertainty is therefore:

$$\mathcal{E}[U^*] = rV(P, Y_C, E_0 - L) + (1-r)V(P, Y_C, E_0) \tag{20}$$

where, $V(\cdot)$ is the indirect utility function in a particular state.

From valuation of risk literature, the change in income due to a small change in environmental good supply, while holding expected utility constant, gives the marginal value of the change in supply (Cornes 1980, Freeman 1989, 2003, Hanley et al. 2007, Shogren and Crocker 1991, Berger et al. 1987). Following the standard technique from this literature, take total derivative of the indirect utility function (for small changes in $r$, $Y_C$, $E_0$ and $L$) and set it equal to zero:

$$\begin{aligned} d\mathcal{E}[U^*] = & [V(Y_C, E_0 - L) - V(Y_C, E_0)]dr \\ & + \left[ r\frac{\partial V(Y_C, E_0 - L)}{\partial Y_C} + (1-r)\frac{\partial V(Y_C, E_0)}{\partial Y_C} \right] dY_C \\ & + \left[ r\frac{\partial V(Y_C, E_0 - L)}{\partial (E_0 - L)}\frac{\partial (E_0 - L)}{\partial E_0} + (1-r)\frac{\partial V(Y_C, E_0)}{\partial E_0} \right] dE_0 \\ & + r\frac{\partial V(Y_C, E_0 - L)}{\partial (E_0 - L)}\frac{\partial (E_0 - L)}{\partial L} dL = 0, \end{aligned} \tag{21}$$

where, the price ($P$) argument has been suppressed for brevity. Three *ex-ante* marginal willingness to pay measures can be derived from the above.

First, assuming $dE_0 = dL = 0$, the willingness to pay for a marginal reduction in the probability of loss (risk prevention) is:

$$MWTP_r = \frac{dY_C}{dr} = \frac{V(Y_C, E_0) - V(Y_C, E_0 - L)}{rV_Y^L + (1-r)V_Y^0} = \frac{\Delta V}{\frac{\partial \mathcal{E}[U^*]}{\partial Y_C}} > 0 \tag{22}$$

where, the denominator is the expected marginal utility of income; $V_Y^L \equiv \frac{\partial V(Y_C, E_0 - L)}{\partial Y_C}$ denotes marginal utility of income when there is environmental loss and $V_Y^0 \equiv \frac{\partial V(Y_C, E_0)}{\partial Y}$ when there is no such loss. By definition, $\frac{\partial \mathcal{E}[U^*]}{\partial Y_C} > 0$, because, $V_Y^L, V_Y^0 > 0$. The numerator is the gain in total utility enjoyed because of the reduction of loss probability. By assumption of non-satiation in $E_0$ and $L > 0$, $V(Y_C, E_0) - V(Y_C, E_0 - L) > 0$. Therefore, for a marginal reduction in the probability of loss, equation 22 gives a positive $MWTP_r$.

Second, assuming $dr = dE_0 = 0$ in equation 21, the willingness to pay for a marginal reduction in the size of the loss (risk reduction) is:

$$MWTP_L = \frac{dY_C}{dL} = -\frac{rV_L}{rV_Y^L + (1-r)V_Y^0} = -\frac{\frac{\partial \mathcal{E}[U^*]}{\partial L}}{\frac{\partial \mathcal{E}[U^*]}{\partial Y_C}} > 0 \tag{23}$$

where, $V_L \equiv \frac{\partial V(Y_C, E_0 - L)}{\partial (E_0 - L)} \cdot \frac{\partial (E_0 - L)}{\partial L} < 0$ is the expected marginal disutility of environmental loss. Divided by the expected marginal utility of income, equation 23 gives the positive $MWTP_L$ in monetary terms.

Third, assuming $dr = dL = 0$, the willingness to pay for a marginal increase in the realized level of environmental good in either state is derived:

$$MWTP_E = -\frac{dY_C}{dE_0} = \frac{rV^L + (1-r)V_E^0}{rV_Y^L + (1-r)V_Y^0} = \frac{\frac{\partial \mathcal{E}[U^*]}{\partial E_0}}{\frac{\partial \mathcal{E}[U^*]}{\partial Y_C}} > 0 \tag{24}$$

where, $V_E^L \equiv \frac{\partial V(Y_C, E_0 - L)}{\partial (E_0 - L)} \cdot \frac{\partial (E_0 - L)}{\partial E_0} > 0$, $V_E^0 \equiv \frac{\partial V(Y_C, E_0)}{\partial E_0} > 0$ which means that the numerator is the expected marginal utility of the environmental good. By definition, $\frac{\partial \mathcal{E}[U^*]}{\partial E_0} > 0$, because, $V^L, V_E^0 > 0$. Dividing by the expected marginal utility of income gives a positive $MWTP_E$ in monetary terms. Compared to the certainty case (see equation 15), the above expression is based on derivatives of the indirect expected utility function.

Now consider a non-marginal change in environmental supply where reference level $E_0$ increases to $E_1$, but, probability ($r$) and amount of loss ($L$) remain unchanged. Imagine the individual faces the following alternative distribution for the realized level of the environmental good:

$$E + \Delta E = \begin{cases} E_1 - L & \text{with probability } r \\ E_1 & \text{with probability } 1 - r \end{cases}, \tag{25}$$

where, denote $E_1 \equiv E_0 + \Delta E$. Thus, $\Delta E$ is the change in the reference level of supply which is also the change in the realized level of environmental supply. Following standard definition, see, e.g., Freeman (2003), Kolstad (2000), *ex-ante* willingness to pay (or option price) is the maximum certain payment that the individual is willing to make in either state such that:

$$rV(Y_C - WTP_E, E_1 - L) + (1 - r)V(Y_C - WTP_E, E_1) = \mathcal{E}_0[U] \tag{26}$$

where, $\mathcal{E}_0[U]$ is existing indirect utility as defined in equation 20. Equation 26 means that the individual is indifferent (in terms of indirect expected utility) between the current distribution (equation 1) and the new one (equation 25). For simplicity, the $P$ argument has been suppressed in equation 26. For the non-marginal increase in $E$, the *ex-ante* willingness to accept compensation ($WTA_E$) can also be defined as follows:

$$\mathcal{E}_1[U] = rV(Y_C + WTA_E, E_0 - L) + (1 - r)V(Y_C + WTA_E, E_0), \tag{27}$$

where, $\mathcal{E}_1[U] = rV(Y_C, E_1 - L) + (1 - r)V(Y_C, E_1)$. $WTA_E$ is the minimum certain compensation that the individual is willing to accept in either state such that the individual is indifferent (in terms of indirect expected utility) between the current distribution (equation 1) and the new one (equation 25).

**Proposition 1.** It can be shown that willingness to pay (per unit of change) approaches the marginal willingness to pay when the discrete changes become very small:

$$\lim_{\Delta E \to 0, \Delta Y_C \to 0} \quad \frac{WTP_E}{\Delta E} \to MWTP_E.$$

*Proof*. See Appendix A.3 for proof. □

# 4 Results

The results will be presented in two parts. Section 4.1 presents the model under environmental certainty. Section 4.2 introduces environmental uncertainty where no insurance is available against such uncertainty.

## 4.1 Environmental certainty

In this section, the maximization problem is similar to what is shown in equation 8, but, with environmental certainty (i.e., $r = 0$, which means $E$ is supplied in either state). Therefore, the consumer maximizes $U(X, E)$ subject to the constraint $PX = Y_C$. Since the consumer spends all income on buying $X$ (recall that $E$ is provided free of cost), we have the following demand function for the market good:

$$X^*(P, Y_C) = \frac{Y_C}{P}. \tag{28}$$

The above demand function is used to derive the indirect utility function. Now, using the definition in equation 15, for the nested CES utility function, we derive:

$$MWTP_E(P, Y_C, E) = \frac{(1-\alpha)P}{\alpha}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}} > 0. \tag{29}$$

*Derivation.* See appendix A.4. □

**Proposition 2.** The comparative static results with respect to a change in the reference level of environmental good ($E$) and maximum certain income ($Y_C$) are:

$$\frac{\partial MWTP_E}{\partial E} = -\frac{(1-\alpha)P}{\alpha\sigma E}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}} < 0, \tag{30}$$

$$\frac{\partial MWTP_E}{\partial Y_C} = \frac{(1-\alpha)P}{\alpha\sigma Y_C}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}} > 0, \tag{31}$$

because, $1 > \alpha > 0$ and $Y_C, P, E > 0$. The signs of the above derivatives are intuitively reasonable. $MWTP_E$ decreases (increases) as the fixed supply of environmental good increases (decreases) and $MWTP_E$ increases (decreases) as consumer's maximum certain income increases (decreases). Although, the rate at which $MWTP_E$ increases with income depends, critically, on the degree of substitution possible between the two goods. To see this, calculate the partial derivative:

$$\frac{\partial^2 MWTP_E}{\partial Y_C^2} = \frac{(1-\sigma)(1-\alpha)P}{\alpha\sigma^2 Y_C^2}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}}. \tag{32}$$

Note the following proposition.

**Proposition 3.** Higher (lower) the elasticity of substitution between the two goods, lower (higher) the rate at which marginal willingness to pay increases with rising reference income.

$$\frac{\partial^2 MWTP_E}{\partial Y_C^2} \begin{cases} < 0 & \text{if } \sigma > 1 \\ = 0 & \text{if } \sigma = 1 \cdot \\ > 0 & \text{if } \sigma < 1 \end{cases} \tag{33}$$

This implies that, if the market and non-market goods are close substitutes, $MWTP_E$ will increase at a slower rate as income increases and *vice verse*. In the intermediate case, for a Cobb-Douglas utility function, $MWTP_E$ increases linearly (constantly) with income. Baumgärtner et al. (2013) also shows similar relationship between WTP and mean household income.

**Proposition 4.** The comparative static derivatives with respect to income distribution and insurance perfection parameters are:

$$\frac{\partial MWTP_E}{\partial \text{var}[Y]} = \frac{(1-\alpha)P}{\alpha\sigma Y_C}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}} \cdot \frac{\partial Y_C}{\partial Y_{\text{var}[Y]}} < 0, \tag{34}$$

$$\frac{\partial MWTP_E}{\partial \mathcal{E}[Y]} = \frac{(1-\alpha)P}{\alpha\sigma Y_C}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}} \cdot \frac{\partial Y_C}{\partial \mathcal{E}[Y]} > 0, \tag{35}$$

$$\frac{\partial MWTP_E}{\partial \phi} = \frac{(1-\alpha)(1-s)PL_Y}{\alpha\sigma Y_C}\left(\frac{Y_C}{EP}\right)^{\frac{1}{\sigma}} > 0, \tag{36}$$

where, $\frac{\partial Y_C}{\partial \mathcal{E}[Y]} > 0$ and $\frac{\partial Y_C}{\partial \text{var}[Y]} < 0$ from model assumptions and properties of $Y_C$ (see equation 4). Therefore, $MWTP_E$ is decreasing (increasing) in variance (mean level) of income. $MWTP_E$ is also increasing in the degree of credit market perfection ($\phi$); because, $\phi$ increases the maximum certain income (see equation 3) which increases the $MWTP_E$ (see equation 31).

**Proposition 5.** To understand the effect of elasticity of substitution calculate:

$$\frac{\partial MWTP_E}{\partial \sigma} = -\frac{MWTP_E}{\sigma^2} \cdot \ln\left[\frac{Y_C}{EP}\right] = -\frac{MWTP_E}{\sigma^2} \cdot \ln\left[\frac{\left(\frac{Y_C}{P}\right)}{E}\right], \tag{37}$$

where, the sign of the derivative will depend on the sign of the ratio of real income $\left(\frac{Y_C}{P}\right)$

to quantity of environmental good ($E$). We can summarize:

$$\frac{\partial MWTP_E}{\partial \sigma} \begin{cases} < 0 & \text{if and only if} \quad \frac{Y_C}{P} > E \\ = 0 & \text{if and only if} \quad \frac{Y_C}{P} = E. \\ > 0 & \text{if and only if} \quad \frac{Y_C}{P} < E \end{cases} \tag{38}$$

Therefore, $MWTP_E$ is increasing (decreasing) in elasticity of substitution if and only if income (per unit of market good) is larger (smaller) than the reference level of environmental good. We can also state:

$$\sigma \rightarrow \begin{cases} 0 \text{ (Leontief)} & MWTP_E \rightarrow \infty \\ 1 \text{ (Cobb-Douglas)} & MWTP_E \rightarrow \frac{(1-\alpha)Y_C}{\alpha E} \\ \infty \text{ (Linear)} & MWTP_E \rightarrow \frac{(1-\alpha)P}{\alpha} \end{cases} \tag{39}$$

## 4.2 Environmental uncertainty

In this section, there is uncertainty in environmental good supply with no insurance, $E$ follows the distribution in equation 1. The consumer faces the full choice problem as shown in equation 8. Using equation 22 and the nested CES utility function, the indirect utility function under uncertainty is:

$$\begin{aligned} \mathcal{E}[U^*] = & \frac{r}{1-\rho}\left[\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(E_0 - L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}} \\ & + \frac{1-r}{1-\rho}\left[\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E_0^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}}. \end{aligned} \tag{40}$$

Using the definitions in equations 22, 23 and 24 three marginal willingness to pay amounts can be calculated (see appendix A.5).

**Proposition 6.** The marginal willingness to pay for a change in $r$ is:

$$MWTP_r = \frac{P}{(1-\rho)\alpha}\left(\frac{Y_C}{P}\right)^{\frac{1}{\sigma}} \cdot \frac{\left(\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}} - \left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0-L}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}}\right)}{r\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0-L}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}} + (1-r)\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}}} > 0. \quad (41)$$

Note that $MWTP_r = 0$ if there is no uncertainty in $E$, that is $L = 0$.

**Proposition 7.** Using equation 23, marginal willingness to pay for small change in amount of loss:

$$MWTP_L = \frac{\frac{(1-\alpha)rP}{\alpha}\left[\frac{Y_C}{P(E_0-L)}\right]^{\frac{1}{\sigma}}\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0-L}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}}}{r\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0-L}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}} + (1-r)\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}}} > 0. \quad (42)$$

Note that $MWTP_L = 0$ if there is no uncertainty in $E$, that is $L = 0$.

**Proposition 8.** Using equation 24, marginal willingness to pay for change in realized quantity of environmental good:

$$MWTP_E = \frac{(1-\alpha)P}{\alpha}\left(\frac{Y_C}{P}\right)^{\frac{1}{\sigma}} \cdot \frac{r(E_0-L)^{-\frac{1}{\sigma}}\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0-L}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}} + (1-r)E_0^{-\frac{1}{\sigma}}\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}}}{r\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0-L}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}} + (1-r)\left[1+\frac{1-\alpha}{\alpha}\left(\frac{E_0}{Y_C/P}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}}} > 0. \quad (43)$$

If there is no uncertainty in environmental supply, that is $L = 0$ and $E = E_0$ in either state, then, the above can be simplified to:

$$\lim_{L\to 0} \quad MWTP_E \to \frac{(1-\alpha)P}{\alpha}\left(\frac{Y}{E_0 P}\right)^{\frac{1}{\sigma}}. \quad (44)$$

which is the $MWTP_E$ under environmental certainty (with $E = E_0$, see equation 29).

**Proposition 9.** Based on numerical simulation using standard parameter values, the following can be stated:

Table 1: Comparative static properties of $MWTP_E$

| $z$ | $Y_C$ | $E_0$ | $r$ | $L$ | $\sigma$ | $\alpha$ | $\rho$ | var$[Y]$ | $\mathcal{E}[Y]$ | $\phi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{\partial MWTP_E}{\partial z}$ | $+$ | $-$ | $+$ | $+$ | $-$ if $\frac{Y_C}{P} > E_0$<br>$0$ if $\frac{Y_C}{P} = E_0$<br>$+$ if $\frac{Y_C}{P} < E_0$ | $-$ | $+$ | $-$ | $+$ | $+$ |

The positive effect of maximum certain income ($Y_C$) on $MWTP_E$ qualitatively similar to the environmental certainty case (see proposition 2). In addition, the rate of adjustment of $MWTP_E$ with changing income depends on (i) degree of substitutability between the two goods and (ii) value of the relative risk aversion parameter $\rho$ (see figure 1 in appendix A.5.1). The role of substitutability is observed also under environmental certainty (see proposition 3), but the effect of risk aversion is, of course, valid only under uncertainty. Figure 1 shows that, *ceteris paribus*, as risk aversion increases (from 1 to 4), all the three curves shift up ($MWTP_E$ is increasing in risk aversion).

The negative effect of reference level of environmental good ($E_0$) on $MWTP_E$ is also similar to the certainty case (see proposition 2). In addition, the curvature of this negative relationship between $MWTP_E$ and $E_0$ changes depending on the degree of substitutability between the two goods (see figure 2 in appendix A.5.1). Figure 2 also shows the full range of effect that elasticity of substitution has on $MWTP_E$. Consider quantity $E_0^1$, for example, and see that $MWTP_E$ is decreasing in elasticity of substitution. On the other hand, at $E_0^2$ (for example), $MWTP_E$ is increasing in elasticity of substitution; everything, of course, depends on the relative sizes of $\frac{Y_C}{P}$ and $E_0$ (similar to proposition 5 under certainty).

Both the environmental loss parameters (size and probability of loss) positively affect $MWTP_E$; intuitively, an increase (decrease) in probability and size of loss increases (decreases) the likelihood and severity of environmental loss and, therefore, the consumer is willing to pay a larger (smaller) amount to secure the marginal increase in $E_0$. The effect of the income distribution parameters (var$[Y]$, $\mathcal{E}[Y]$ and $\phi$) are qualitatively similar to the certainty case (see proposition 4). The effect of relative risk aversion parameter ($\rho$) is also intuitively reasonable; higher the risk aversion, the higher the $MWTP_E$.

We are also interested to see how different elasticities of $MWTP_E$ change with changes in elasticity of substitution and risk aversion. There is a growing literature on whether income elasticity of marginal willingness to pay is constant or changing; see, e.g., Barbier et al. (2015) for a useful review. Following this discussion, we want to see how elasticities change with respect to $\sigma$ and $\rho$ in this model. Based on numerical simulation using standard parameter values, the following can be stated:

**Proposition 10.** Income elasticity of $MWTP_E$ is non-constant over income and risk aversion for different values of elasticity of substitution (see figures 3 and 4 in appendix A.5.1).

**Proposition 11.** Environmental elasticity of $MWTP_E$ is non-constant over quantity of environmental good and risk aversion (see figure 5 in appendix A.5.1).

# 5 Discussion and conclusion

To sum up, we have derived the following results. In the model with environmental certainty (Section 4.1), we have derived a positive marginal WTP for the environmental good. The marginal effect of change in environmental supply follows from the assumption of diminishing marginal utility of the environmental good. The marginal effect of a change in income depends on the elasticity of substitution between the two goods.

In case of environmental uncertainty (Section 4.2), the marginal WTP for the quantity of environmental good is increasing in income and decreasing in the quantity of the environmental good. It is also increasing in the size and probability of environmental loss as long as the elasticity of substitution between the two goods is less than the inverse of the risk aversion parameter.

It is widely recognized that climate change will fundamentally affect many of the underlying functions and processes that support and maintain earth's ecosystems. This will result in loss of ecosystem services that are vital to human livelihood. In the context of policy-making, it is imperative to understand the effect of such losses on social welfare, taking into account that the quantity available of such goods is often exogenous and uncertain to the individual user. This paper attempts to provide a theoretical framework for such a valuation exercise. Our model is generic and can be adapted to different specific valuation and decision contexts. Given data on the probability distribution of (uncertain) ecosystem services, our model can be used to calculate the economic value of these ecosystem services, taking risk and risk-aversion explicitly into account.

# Acknowledgments

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# Appendix

## A.1 Model of income uncertainty and insurance

As an example of income uncertainty, consider the following income distribution:

$$Y = \begin{cases} Y_0 - L_Y & \text{with probability } s \\ Y_0 & \text{with probability } 1-s \end{cases} . \tag{A.1}$$

Thus, the consumer enjoys a reference income level of $Y_0 > 0$ and faces a potential loss of $L_Y$ with probability $s$, and no loss with probability $1 - s$ (where $0 \leq L_Y \leq Y_0$ and $0 \leq s \leq 1$). Here, the expected income and income variance are:

$$\mathcal{E}[Y] = Y_0 - sL_Y , \tag{A.2}$$

$$\text{var}[Y] = s(1-s)Y_L^2 , \tag{A.3}$$

and $Y_{\min} = Y_0 - L_Y$ is the minimum income in any state. Let $\phi$ with $0 \leq \phi \leq 1$ denote the degree of perfection of insurance and credit-and-savings markets, where $\phi = 0$ means that such markets do not exist or income insurance or credit-and-saving is available only at prohibitively high costs, and $\phi = 1$ means that markets are perfect so that income insurance or credit-and-saving is available at zero real costs. Then the consumer's maximum certain income is given by:

$$Y_C := \phi \mathcal{E}[Y] + (1-\phi)(Y_0 - L_Y) = Y_0 - (1-(1-s)\phi)L_Y \ . \tag{A.4}$$

## A.2 Pseudo choice model under certainty

In case of a nested-CES utility function, the Lagrange for the pseudo choice problem is:

$$\mathcal{L}(X,E,\lambda) = \frac{1}{1-\rho}\left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}} + \lambda[\hat{Y} - PX - EP_E], \tag{A.5}$$

with the first order conditions:

$$\mathcal{L}_X : \alpha X^{-\frac{1}{\sigma}}\left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}} - \lambda P = 0, \tag{A.6}$$

$$\mathcal{L}_E : (1-\alpha)E^{-\frac{1}{\sigma}}\left[\alpha X^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1-\sigma\rho}{\sigma-1}} - \lambda P_E = 0, \tag{A.7}$$

$$\mathcal{L}_\lambda : \hat{Y} - PX - EP_E = 0. \tag{A.8}$$

Divide equation A.6 by equation A.7 and re-arrange to obtain

$$E = X\left[\frac{(1-\alpha)P}{\alpha P_E}\right]^{\sigma}. \tag{A.9}$$

Substitute in to equation A.8 and re-arrange to obtain the Marshallian demand system for the pseudo choice problem:

$$\hat{X}(P,P_E,\hat{Y}) = \frac{\hat{Y}}{P + \left[\frac{(1-\alpha)P}{\alpha P_E}\right]^{\sigma}} \tag{A.10}$$

$$\hat{E}(P,P_E,\hat{Y}) = \frac{\hat{Y}\left[\frac{(1-\alpha)P}{\alpha P_E}\right]^{\sigma}}{P + \left[\frac{(1-\alpha)P}{\alpha P_E}\right]^{\sigma}} \tag{A.11}$$

## A.3 $WTP$ for non-marginal changes under uncertainty

Re-write the definition of *ex-ante* willingness to pay (or option price):

$$r[V(Y_1, E_1 - L) - V(Y, E_0 - L)] = (1 - r)[V(Y, E_0) - V(Y_1, E_1)], \tag{A.12}$$

where, $Y_1 \equiv Y + \Delta Y$ which means $\Delta Y = -WTP_E$; because, willingness to pay is the amount of income that is sacrificed by the consumer to secure the increase in the reference level of environmental supply (from $E_0$ to $E_1$). Using Taylor's series approximation, for large changes in $Y$ and $E$:

$$\begin{aligned} V(Y + \Delta Y, E_0 + \Delta E - L) \approx\ & V(Y, E_0 - L) + \frac{\partial V(Y, E_0 - L)}{\partial Y}(\Delta Y) \\ & + \frac{\partial V(Y, E_0 - L)}{\partial E}(\Delta E) + ... + \text{h.o.t...}, \end{aligned} \tag{A.13}$$

$$\begin{aligned} V(Y + \Delta Y, E_0 + \Delta E) \approx\ & V(Y, E_0) + \frac{\partial V(Y, E_0)}{\partial Y}(\Delta Y) \\ & + \frac{\partial V(Y, E_0)}{\partial E}(\Delta E) + ... + \text{h.o.t....} \end{aligned} \tag{A.14}$$

Substitute the above in equation A.12 and re-arrange to obtain:

$$\begin{aligned} & r\left[\frac{\partial V(Y, E_0 - L)}{\partial Y}\right] WTP_E + (1 - r)\left[\frac{\partial V(Y, E_0)}{\partial Y}\right] WTP_E \\ & \approx r\left[\frac{\partial V(Y, E_0 - L)}{\partial E}\right] \Delta E + (1 - r)\left[\frac{\partial V(Y, E_0)}{\partial E}\right] \Delta E. \end{aligned} \tag{A.15}$$

The above can be re-arranged to yield:

$$\frac{WTP_E}{\Delta E} \approx \frac{rV_E^L + (1 - r)V_E^0}{rV_Y^L + (1 - r)V_Y^0}, \tag{A.16}$$

which means:

$$\lim_{\substack{\Delta E \to 0 \\ \Delta Y \to 0}} \quad \frac{WTP_E}{\Delta E} = \frac{rV_E^L + (1 - r)V_E^0}{rV_Y^L + (1 - r)V_Y^0} = MWTP_E. \tag{A.17}$$

The above is the marginal willingness to pay found in equation 24.

## A.4 Willingness to pay under certainty

Under certainty, the consumer's problem is:

$$\begin{aligned} \max_X \quad & U(X, E), \\ \text{s.t.} \quad & PX \leq Y_C, \end{aligned} \tag{A.18}$$

where the consumer spends all income in buying $X$ as $E$ is provided free of cost. Thus, the consumer's optimal bundle is $(X^*, E) = \left(\frac{Y_C}{P}, E\right)$. The indirect utility function is

$$V(P, Y_C, E) = \frac{1}{1-\rho}\left[\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}}, \tag{A.19}$$

Now, calculate the partial derivatives:

$$\frac{\partial V}{\partial E} = (1-\alpha)E^{-\frac{1}{\sigma}}\left[\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}-1},$$

$$\frac{\partial V}{\partial Y_C} = \frac{\alpha}{P}\left(\frac{Y_C}{P}\right)^{-\frac{1}{\sigma}}\left[\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)E^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(1-\rho)}{\sigma-1}-1}.$$

Substituting the above partial derivatives in equation 15 and re-arranging yields the $MWTP_E$ expression found in equation 29.

## A.5 Willingness to pay under uncertainty

The conditional indirect utility function under uncertainty is as shown in equation 40. Marginal willingness to pay for $r$, $L$ and $E$ are calculated using the equations 22, 23 and 24 from section 3.2. Calculate the following partial derivatives:

$$\frac{\partial \mathcal{E}[U^*]}{\partial E_0} = r(1-\alpha)(E_0-L)^{-\frac{1}{\sigma}}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)\left(E_0-L\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma\rho}{\sigma-1}}$$
$$+(1-r)(1-\alpha)E_0^{-\frac{1}{\sigma}}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)E_0^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma\rho}{\sigma-1}} > 0 \tag{A.20}$$

$$\frac{\partial \mathcal{E}[U^*]}{\partial r} = \frac{1}{1-\rho}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)\left(E_0-L\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma-\sigma\rho}{\sigma-1}}$$
$$-\frac{1}{1-\rho}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)E_0^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma-\sigma\rho}{\sigma-1}} < 0 \tag{A.21}$$

$$\frac{\partial \mathcal{E}[U^*]}{\partial L} = -r(1-\alpha)(E_0-L)^{-\frac{1}{\sigma}}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)\left(E_0-L\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma\rho}{\sigma-1}} < 0 \tag{A.22}$$

$$\frac{\partial \mathcal{E}[U^*]}{\partial Y_C} = \frac{r\alpha}{P}\left(\frac{Y_C}{P}\right)^{-\frac{1}{\sigma}}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)\left(E_0-L\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma\rho}{\sigma-1}}$$
$$+\frac{(1-r)\alpha}{P}\left(\frac{Y_C}{P}\right)^{-\frac{1}{\sigma}}\left(\alpha\left(\frac{Y_C}{P}\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)E_0^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma\rho}{\sigma-1}} > 0 \tag{A.23}$$

Substitute the above partial derivatives in equations 22, 23 and 24 and simplify to obtain the expressions found in equations 41, 42 and 43.

#### A.5.1 Simulation results

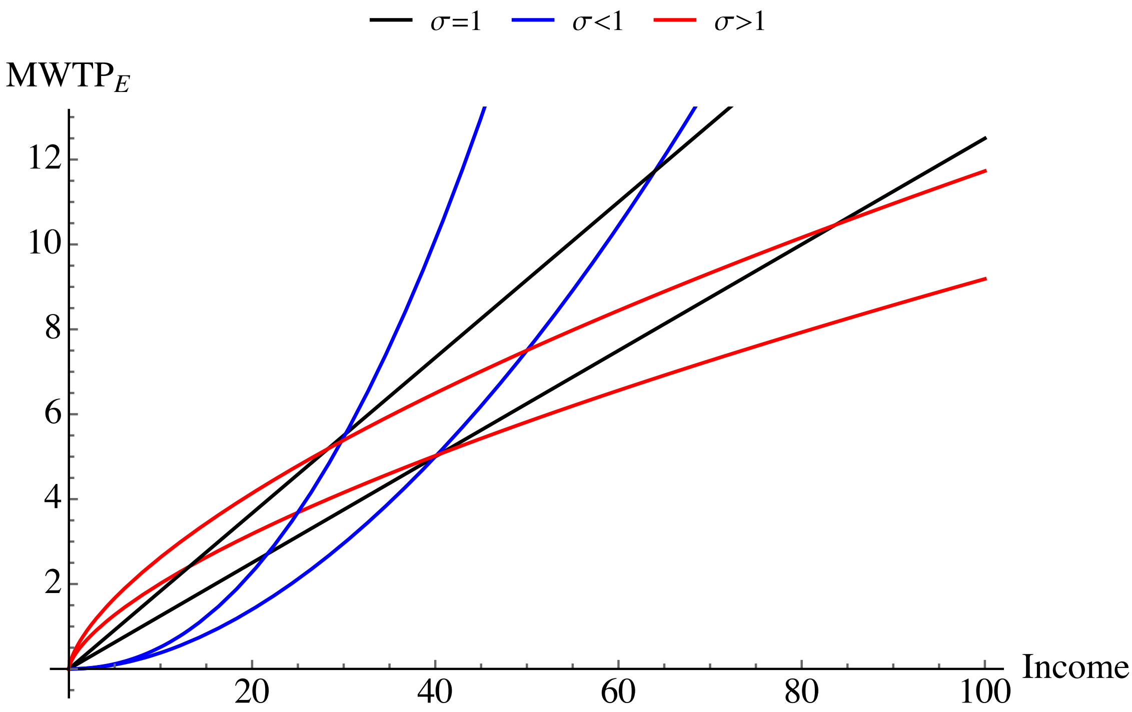



Figure 1: Marginal willingness to pay plotted over maximum certain income

$(\alpha = r = 0.5, \sigma = 1, L = 15, P = 5, E_0 = 20$; $\rho$ increases from 1 to 4)

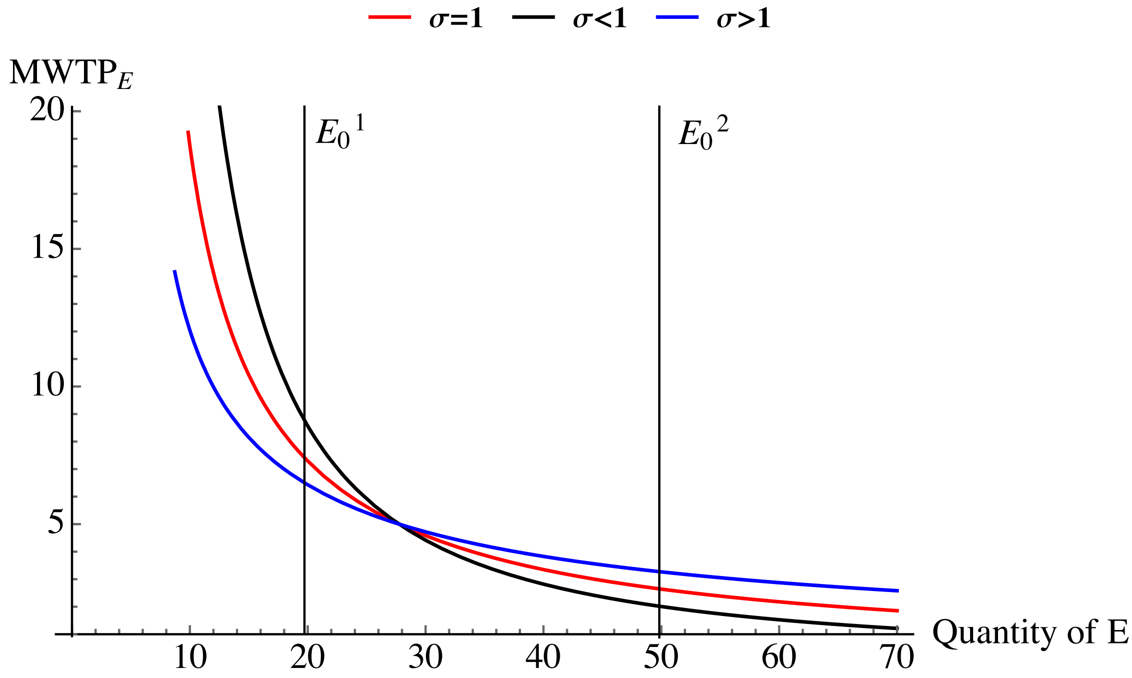
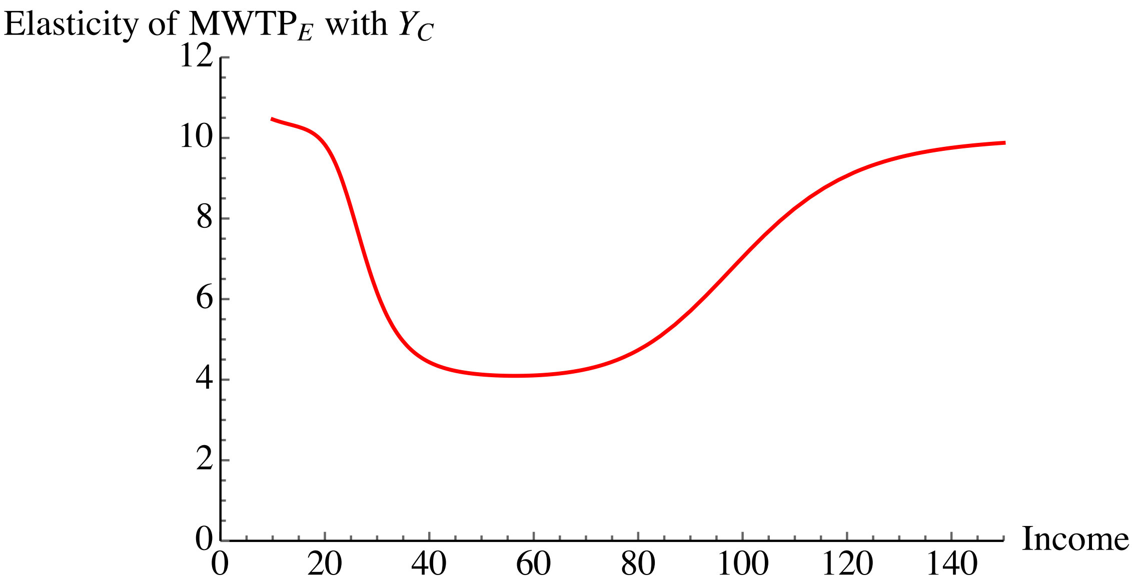
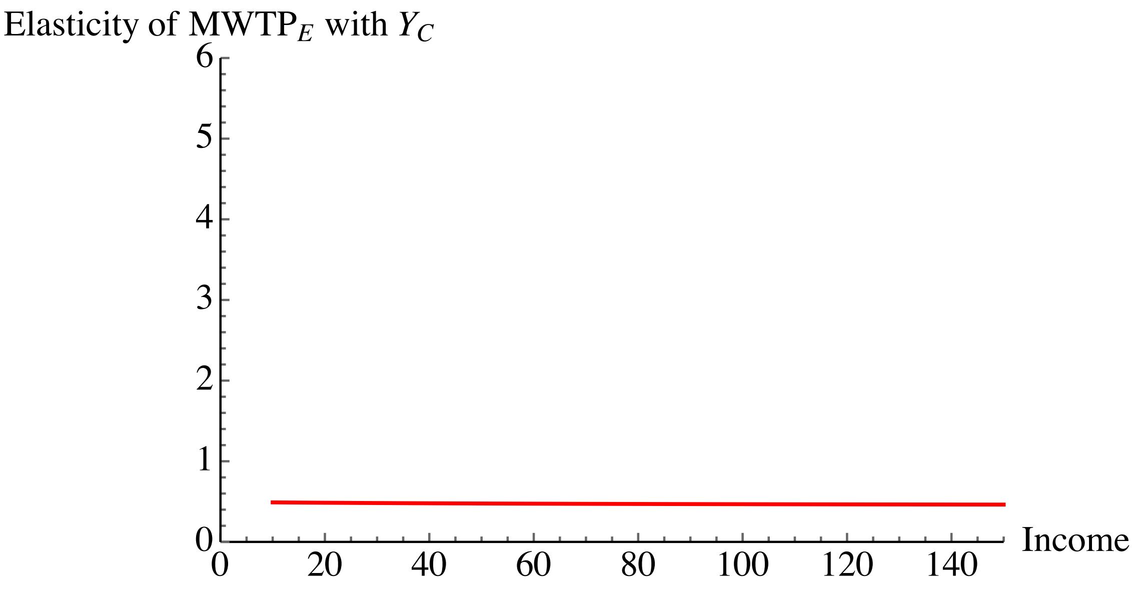



Figure 2: Marginal willingness to pay plotted over quantity of environmental good ($\alpha = r = 0.5, \sigma = \rho = 1, L = P = 5, Y_C = 125$)

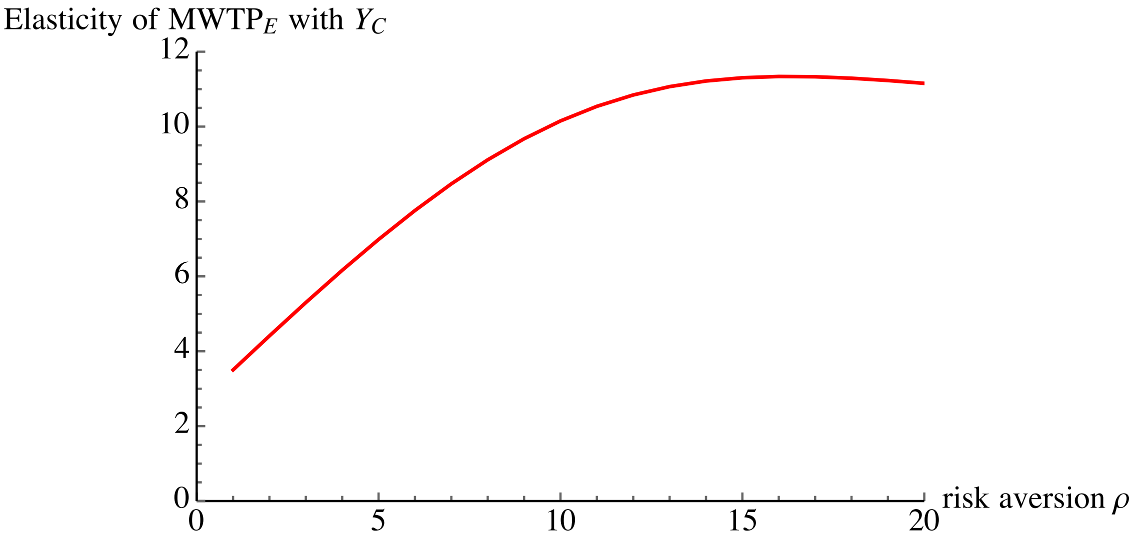



(a) $\alpha = r = 0.5, \sigma = \rho = 1, L = 15, P = 5, E_0 = 20$

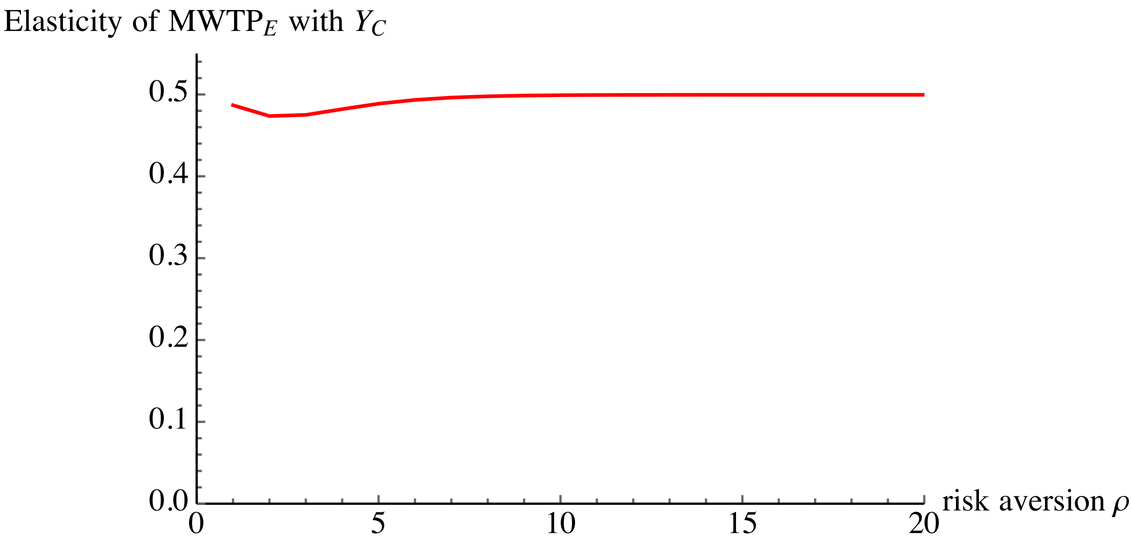



(b) $\alpha = r = 0.5, \sigma = 2, \rho = 1, L = 15, P = 5, E_0 = 20$

Figure 3: Plotting income elasticity of $MWTP_E$ over income

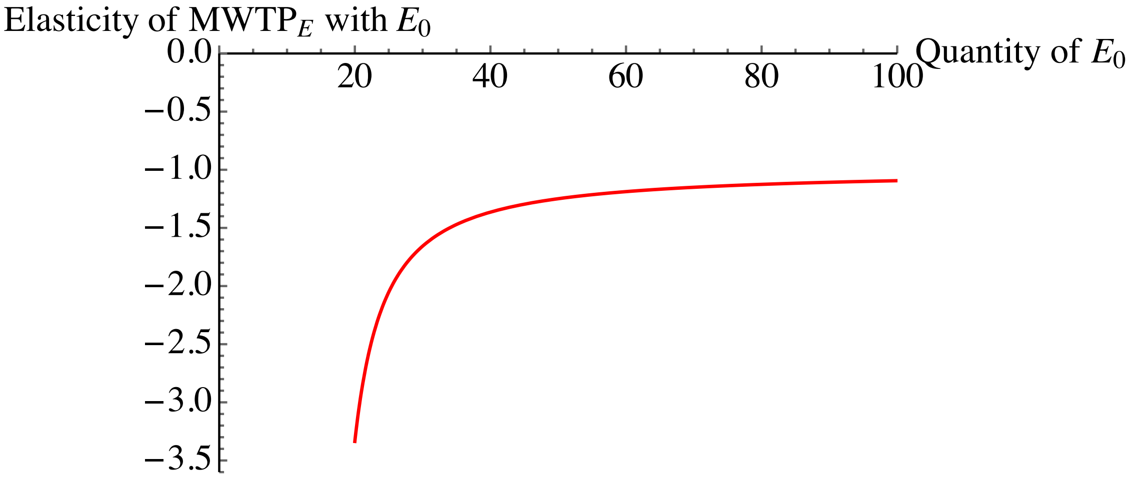



(a) $\alpha = r = 0.5, \sigma = 0.1, L = 15, P = 5, Y_C = 30, E_0 = 20$

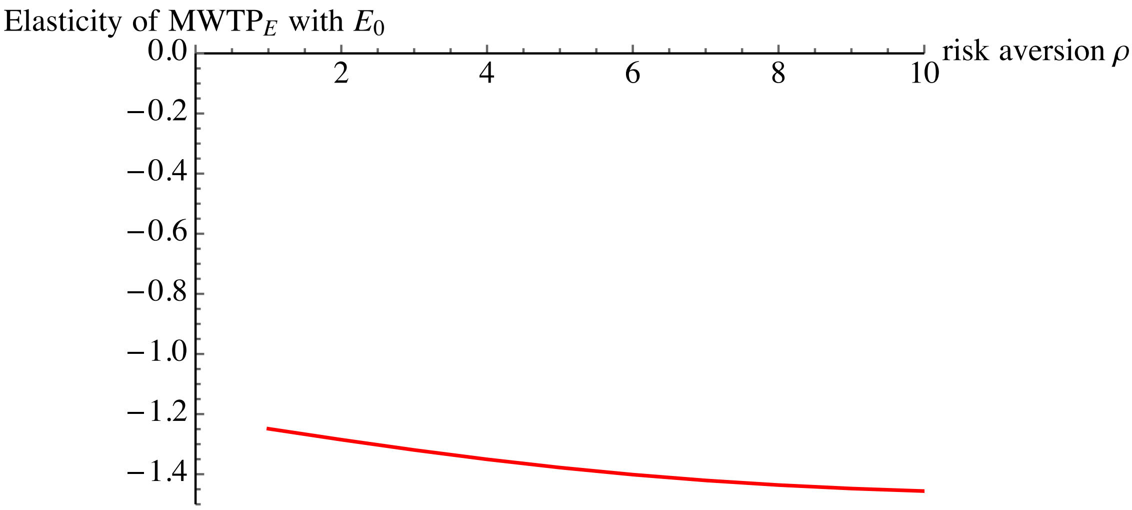



(b) $\alpha = r = 0.5, \sigma = 2, L = 15, P = 5, Y_C = 30, E_0 = 20$

Figure 4: Plotting income elasticity of $MWTP_E$ over risk aversion




(a) $\alpha = r = 0.5, \sigma = \rho = 1, L = 15, P = 5, Y_C = 50$




(b) $\alpha = r = 0.5, \sigma = 1, L = 15, P = 5, Y_C = 50, E_0 = 50$

Figure 5: Plotting environmental elasticity of $MWTP_E$

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