Pareto inefficiency and dynamic bargaining in common property resource games with asymmetric players*

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Abstract

The problem of aggregation of preferences for asymmetric players having different utilities and discount rates is addressed. It is illustrated how time-consistent cooperative strategies can be Pareto dominated by noncooperative feedback Nash equilibria. A dynamic bargaining procedure with nonconstant weights is presented. Nonconstant weights are obtained as the maximizers of a Nash welfare function. Three common property resource games are analyzed. Linear strategies are derived for appropriated weight functions including, in particular, those obtained from a dynamic Nash bargaining procedure.

Keywords: Asymmetric players, heterogeneous discounting, nonconstant discounting, dynamic bargaining, differential games, common property resource games

JEL codes: C73, C71, C78, Q30, Q20

1 Introduction

In the study of intertemporal choices it is customary in economics to consider the so-called Discounted Utility (DU) model, introduced in Samuelson (1937). In the DU model, agent's time preferences are characterized by a single parameter, the constant discount rate of time preference. However, empirical observations seem to show that predictions of the DU model sometimes disagree with the actual behavior of decision makers (see e.g. Frederick *et al.* (2002) and references therein). In addition, if there are several players, although it is typically assumed that all economic agents have the same rate of time preference, there is no reason to believe that consumers, firms or countries have identical time preferences for utility streams. In that case a natural question arises: how to aggregate preferences? A social planner could simply add the intertemporal utility functions of all agents. In the absence of a social planner, if cooperation is permitted, economic agents could decide to coordinate their strategies in order to optimize their collective payoff. However, in this

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framework, as in hyperbolic discounting, a problem of time-consistency arises: what is optimal for the coalition or the society at time t will be no longer optimal at time s, for $s > t$.

There are many papers that have addressed the issue of aggregating preferences of heterogeneous agents in a dynamic setting. Li and Lofgren (2000) and Gollier and Zeckhauser (2005) have restricted their attention to efficient but time-inconsistent policies (although Hara (2013) proves that, in a continuous time equilibrium model, the more heterogeneous the consumers are in their impatience, the more dynamically consistent the representative consumer is). Recent papers (Ekeland and Lazrak (2010), Ekeland *et al.* (2015), Karp (2016)) have justified the introduction of weighted sums of exponentials in nonconstant discounting models with one decision maker as a way to represent the time preferences in overlapping generation models[1]. Breton and Keoula (2014), Millner and Heal (2014) and Karp (2016) search for time-consistent policies in case players are symmetric in their preferences on consumption (i.e. utility functions) but heterogeneous in their time preferences. Sorger (2006), Marín-Solano and Shevkoplyas (2011), de-Paz *et al.* (2013), Ekeland *et al.* (2013) and Marín-Solano (2015) propose different approaches in the study of time-consistent equilibria in case players are asymmetric, not just in their discount rates, but also in their utility functions.

Our starting point in this paper is to consider the most natural way to address the problem of finding time-consistent policies if agents which heterogeneous discount functions can cooperate - or if there is a social planner aggregating their preferences -. We simply add the individuals intertemporal utility functions and look for - as in hyperbolic discounting - time-consistent policies. In this approach there are implicit two fundamental assumptions: all players can cooperate at every instant of time t, and the different t-coalitions (coalitions at time t) lack precommitment power for future decision rules. Somehow, in the setting of intergenerational models, if we interpret in an instant of time as a generation, previous papers assume intragenerational but not intergenerational cooperation. As in nonconstant discounting - that can can be seen as a problem of aggregating time preferences if agents share the same instantaneous utility function but discount the future at different rates -, time-consistent equilibria are computed by finding subgame perfect equilibria in a noncooperative sequential game where agents are the different t-coalitions (representing, for instance, different generations). However, the time-consistent "cooperative" solutions obtained in this way can have some important drawbacks that we list below.

First of all, such time-consistent solutions are not Pareto efficient, in general. In the study of collective decisions of individuals choosing a common consumption stream, Jackson and Yariv (2015) show that every Pareto efficient and non-dictatorial method of aggregating utility functions is time-inconsistent if time preferences of the individuals are heterogeneous[2]. Hence, unless discount rates are equal, time-consistent policies cannot be obtained, in general, by maximizing a weighted sum (with constant weights) of the initial intertemporal utility functions of all players (the "0-coalition"). In any case, this result indicates that there is a problem of rationality of Pareto efficient solutions in this setting: unless there is precomitment power, they will not be followed in the future. The (rational) time-consistent cooperative equilibria is not Pareto efficient but is

[1]For example, Karp (2016) studies a climate linear-in state differential game model with nonconstant discounting in the analysis of the relative importance of international cooperation and intergenerational altruism. Discount functions are constructed from different representations for the aggregate preferences of OLGs with paternalistic or pure altruism, and exponentially distributed or deterministic lifetimes. In his paper, all agents share the same discount function.

[2]Previously, Zuber (2011) had proven that Paretian social preferences are consistent and stationary if, and only if, all agents have the same discount rate of time preference.

constrained Pareto efficient, in the sense that this solution at time t is Pareto efficient for the t-coalition, constrained to the future behavior of the agents[3].

A natural consequence of the constrained Pareto efficiency of time-consistent cooperative solutions is that it is advantageous for the whole group to collaborate at each time t. Hence, if cooperation is not forbidden and the joint preferences are represented by the sum of players intertemporal utilities, the rational choice for them is to follow the decision rule described by the time-consistent cooperative solution. However, Marín-Solano (2015) showed with a simple example that time-consistent cooperative solutions can be inefficient for the group: joint payments can be higher if players act in a fully noncooperative way. In those cases, players are better off if cooperation is forbidden than if it is allowed. In addition, time-consistent cooperative equilibria seem to be extremely cumbersome to compute, as was illustrated in de-Paz *et al.* (2013) in the simplest problem of dynamic management of an exhaustible resource with different isoelastic utility functions.

In this paper we first present an extension of the result in Marín-Solano (2015), by illustrating that noncooperative Nash equilibria can Pareto dominate time-consistent cooperative equilibria (which are just constrained Pareto efficient). Then, we propose an alternative to the time-inconsistent Pareto efficient solutions and to the time-consistent cooperative equilibria. The idea consists in introducing nonconstant weights for the players. Since the games played at different moments are different games (the initial conditions change along time), it seems natural to assume that players at different times can have different weights (related to different bargaining powers) in the joint coalition, depending on the initial conditions at each game. The idea of introducing time-varying or time-dependent weights in a problem with heterogeneous agents is not new. Sorger (2006) proposed a new solution concept, the recursive Nash bargaining solution, in dynamic games with heterogeneous players in a discrete time setting with changing weights. According to this solution, knowing the decision rule at future periods $s = t+1, t+2, \dots$, agents look for a weighted Nash bargaining solution in which the status quo or threat point is given by the payoffs of the players if they do not cooperate just at period t. In a continuous time setting, Marín-Solano (2014) suggested to introduce nonconstant weights in problems with rather general discount functions: knowing the decision rule of players in all future s-coalitions, members of the t-coalition (for $t < s$) decide which is the "optimal" policy at time t for general weight functions. More recently, Yeung and Petrosyan (2015) consider also nonconstant weights in a discrete time setting. Note that the way nonconstant weights are introduced in the above mentioned papers is different to that in the previous literature with time-consistent preferences, in which weights are applied to the instantaneous utilities, not to the intertemporal utilities.

From the viewpoint of a social planner, what looks more natural is to simply add the intertemporal utilities of all agents, and to assign them - at least in principle, in the interest of fairness - equal weights. A social planner should require sound reasons to depart from this choice. But this solution becomes unsatisfactory in case it is clearly inefficient, or if it is simply very difficult to compute. In those situations, the introduction of a solution with unequal and nonconstant weights solving these problems can be justified. In any case, our bargaining solution fits better in a context without a social planner, in which agents bargain to form (or not) a coalition with no precommitment for the future. In problems with time-inconsistent preferences (e.g. hyperbolic discounting)

[3]Although in Marín-Solano and Shevkoplyas (2011) these solutions were described as Pareto efficient solutions in differential games with heterogeneous agents, it seems more appropriate to characterize them, following Karp (2007) for a related problem (with one agent with nonconstant discounting), as a Constrained Pareto Efficient Equilibria.

it is customary to assume that agents at time t cannot commit themselves for their future actions at time $s > t$. In a similar way, if some weights are bargained by the agents in order to cooperate among them at time t, it seems natural to assume that they cannot commit for future weights in the game played at future moments $s > t$. Instead, they would depend on the state of the system at time s.

Our contributions in this paper are the following.

- First, we provide an example in which noncooperative payoffs Pareto dominate payoffs obtained from time-consistent (rational) cooperative strategies.
- Second, we describe bargaining solutions with nonconstant weights. Nonconstant weights are derived by taking as threat points (in case of disagreement) the noncooperative feedback (Markov Perfect) Nash equilibria. Hence, disagreement leads to the noncooperative outcome at perpetuity. If solutions exist to this problem, they are, by construction, individually rational. Although in the search of fully time-consistent solutions it seems more "realistic" to adopt the Sorger's approach (Sorger (2006)), we do not follow it since we are interested in solving the problem of Pareto dominance by noncooperative outcomes.
- Finally, we compute the bargaining solutions in some common property resource games with a productive asset. For different and general time-distance discount functions, we study these problems for families of weight functions guaranteeing the existence of linear strategies in the stock of the resource. Some of these nonconstant weight functions can be obtained as the maximizers of a (possibly asymmetric) Nash welfare function.

The paper is structured as follows. Section 2 revises the cooperative solution concept for problems with asymmetric players studied in the previous literature. Since this solution is not strictly cooperative, nor Pareto efficient, following Marín-Solano (2015) it is defined as a "t-cooperative equilibrium rule" for the t-coalition[4]. A brief review of the main results in Marín-Solano (2014) on nonconstant weights is also included. Section 3 discusses the Pareto inefficiency of the t-cooperative equilibrium. Section 4 introduces and characterizes the so-called dynamic Nash t-bargaining solutions. Section 5 consider three applications related to the joint management of a (nonrenewable and renewable) common property natural resource. Section 6 concludes the paper.

2 Preliminaries

2.1 The t-cooperative equilibrium

Let us consider an intertemporal decision problem with several agents in which players can coordinate their strategies in order to optimize their collective payoff. Alternatively, a social planner can look for the solution maximizing the aggregate preferences of all economic agents. If there is a unique and constant discount rate of time preference for all agents, Pareto efficient solutions can be obtained by solving a standard optimal control problem. However, in the case of different discount rates, joint preferences become time inconsistent. As in hyperbolic discounting, when agents lack commitment power but they can cooperate at every instant of time, they act at different times t as

[4] In de-Paz *et al.* (2013) this solution was defined as a time-consistent equilibrium under partial cooperation, since cooperation among players is considered just at each instant (or period, in a discrete time setting) of time. However, this definition seems to be a bit confusing, since partial cooperation is usually understood in game theory as a situation in which not all the players decide to cooperate to form the whole coalition, but just some of them.

sequences of independent coalitions (the t-coalitions). Following Marín-Solano (2015), we call the time-consistent solution for this problem a t-cooperative equilibrium. If we identify a t-coalition as the "t-agent" in a problem with hyperbolic (or general time-distance) discounting, the t-cooperative equilibrium rule is the decision followed by that "sophisticated agent". Hence, this solution assumes or allows for cooperation among players at every time t, but is a non-cooperative (Markov Perfect) equilibrium for the non-cooperative sequential game defined by these infinitely many t-coalitions.

Let N be the number of players, and $x = (x^1, \ldots, x^n) \in X \subset \mathbf{R}^n$ the vector of state variables. For each player $i \in \{1, \ldots, N\}$, let $c_i = (c_i^1, \ldots, c_i^m) \in U_i \subset \mathbf{R}^{m_i}$ be her vector of control (decision) variables, $u_i(x, c_1, \ldots, c_N)$ her instantaneous utility function, and $\theta_i(s-t) \geq 0$ ($\theta_i(t,t) = 1$) her discount function representing how the player discounts, at time t, utilities enjoyed at time s. Although our results can be extended to more general discount functions, for many of our discussions we will take constant discount rates, $\theta_i(s-t) = e^{-\rho_i(s-t)}$. The time-horizon τ can be finite or infinite ($\tau \in \{T, \infty\}$). The intertemporal utility function for player i at time t is given by

$$J_i(x; c_1, \ldots, c_N; t) = \int_t^\tau \theta_i(s-t) u_i(x(s), c_1(s), \ldots, c_N(s))\, ds\ , \quad \text{with} \tag{1}$$

$$\dot{x}(s) = g(x(s), c_1(s), \ldots, c_N(s))\ ,\ x(t) = x_t\ . \tag{2}$$

We assume that functions u_i, g and θ_i are of class C^1, and $\int_0^\tau \theta_i(s)\, ds < \infty$.

In a cooperative setting, we aggregate preferences as

$$J^c(x, c, t) = \sum_{i=1}^N J_i(x, c, t) = \sum_{i=1}^N \int_t^\tau \theta_i(s-t) u_i(x(s), c_1(s), \ldots, c_N(s))\, ds\ . \tag{3}$$

If different weights $\lambda_i \geq 0$ are assigned to the players, the joint intertemporal utility function is given by $J^c(x, c, t) = \sum_{i=1}^N \lambda_i J_i(x, c, t)$. In this section we will assume that all weights are equal, so we take $\lambda_1 = \cdots = \lambda_N = 1$. The generalization of our results to different constant weights is straightforward.

Since the discount functions are different, independently on the constancy or not of each player's discount rate $\rho_i(s) = \theta_i'(s)/\theta_i(s)$, the joint time preferences are time inconsistent and, in order to find time-consistent solutions, the problem is solved as a noncooperative sequential game with a continuum of "players" (each "player" is each coalition at time t). Hence, we can follow the ideas in Karp (2007), Ekeland and Lazrak (2010) or Yong (2011), who suggested three different procedures in order to find subgame perfect equilibria for this sequential game. As in Ekeland and Lazrak (2010), in the infinite horizon case ($\tau = \infty$), we restrict our attention to stationary convergent Markovian strategies, i.e. strategies $c_i = \phi_i(x)$ such that there exists $x_\infty < \infty$ and a neighbourhood V of x_∞ such that, for every $x_0 \in V$, the solution to (2) along $c = \phi(x) = (\phi_1(x), \ldots, \phi_N(x))$ converges to x_∞. For convergent strategies, the integral in (1) converges.

If $c^*(s) = \phi(x(s))$ is a continuously differentiable equilibrium rule for the problem (3) subject to (2), by denoting $x_t = x$, the corresponding value function is

$$V(x, t) = \sum_{i=1}^N \int_t^\tau \theta_i(s-t) u_i(x(s), \phi(x(s), s))\, ds\ .$$

Next, for $\epsilon > 0$ and $\bar{c} = (\bar{c}_1, \ldots, \bar{c}_N)$, $\bar{c}_i \in U_i$, let

$$c_\epsilon(s) = \begin{cases} \bar{c} & \text{if} \quad s \in [t, t+\epsilon]\ , \\ \phi(x(s), s) & \text{if} \quad s > t+\epsilon\ . \end{cases} \tag{4}$$

Although the differentiability condition on the equilibrium rule is no strictly necessary (see e.g. Marín-Solano and Shevkoplyas (2011) and Ekeland *et al.* (2015)), we will impose this regularity condition in the paper, that is verified in the examples analyzed in Section 5.

If the t-coalition has the ability to precommit its behavior during the period $[t, t+\epsilon]$, the valuation along the perturbed control path c_ϵ is given by

$$V_{c_\epsilon}(x,t) = \sum_{i=1}^{N} \left\{ \int_t^{t+\epsilon} \theta_i(s-t) u_i(x(s), \bar{c})\, ds \right.$$

$$\left. + \int_{t+\epsilon}^{\tau} \theta_i(s-t) u_i(x(s), \phi(x(s), s))\, ds \right\} .$$

Definition 1 *A decision rule $c^*(s) = \phi(x(s), s)$ is a t-cooperative equilibrium (t-CE) if, for any admissible c_ϵ given by (4),*

$$\lim_{\epsilon \to 0^+} \frac{V(x,t) - V_{c_\epsilon}(x,t)}{\epsilon} = P(x, \phi, \bar{c}, t) \geq 0 .$$

In the definition above, note that the maximum in $\bar{c}$ of $P(x, \phi, \bar{c}, t)$ is achieved for the equilibrium rule $\bar{c} = \phi(x,t)$ (Ekeland and Lazrak (2010)).

In de-Paz *et al.* (2013) it is proved that, if a decision rule $(c_1, \ldots, c_N) = (\phi_1(x,s), \ldots, \phi_N(x,s))$ is such that the functions

$$V_i(x,t) = \int_t^{\tau} \theta_i(s-t) u_i(x(s), \phi_1(x(s), s), \ldots, \phi_N(x,s))\, ds \tag{5}$$

are of class C^1 and

$$(\phi_1(x,t), \ldots, \phi_N(x,t)) \tag{6}$$

$$= \mathrm{argmax}_{\{c_1,\ldots,c_N\}} \left\{ \sum_{i=1}^{N} u_i(x, c_1, \ldots, c_N) + \sum_{i=1}^{N} \nabla_x V_i(x,t) \cdot g(x, c_1, \ldots, c_N) \right\} ,$$

then $c = \phi(x,t)$ is a t-CE rule.

If $\tau = \infty$, equilibrium strategies are stationary and the value functions V_i are time-independent. If, in addition, $\theta_i(s-t) = e^{-\rho_i(s-t)}$ (constant discount rates), we can write the dynamic programming equation (DPE)[5]

$$\sum_{i=1}^{N} \rho_i V_i(x) = \max_{\{c_1,\ldots,c_N\}} \left\{ \sum_{i=1}^{N} u_i(x, c_1, \ldots, c_N) + \sum_{i=1}^{N} \nabla_x V_i(x) \cdot g(x, c_1, \ldots, c_N) \right\} . \tag{7}$$

At first look, Equation (7) is very similar to the standard DPE. Obviously, this is the standard HJB when $\rho_1 = \cdots = \rho_N = \rho$. However, the standard DPE in the present value formulation, providing the optimal solution from the viewpoint of the coalition at the initial time, is given by

$$-\frac{\partial V^P(x,t)}{\partial t} \tag{8}$$

$$= \max_{\{c_1,\ldots,c_N\}} \left\{ \sum_{i=1}^{N} e^{-\rho_i t} u_i(x, c_1, \ldots, c_N) + \sum_{i=1}^{N} \nabla_x V_i^P(x,t) \cdot g(x, c_1, \ldots, c_N) \right\} .$$

[5]This dynamic programming equation can also be obtained by extending in a natural way the derivation in Karp (2007) (see de-Paz *et al.* (2013)).

The key point is that, if discount rates are not equal, we cannot isolate the exponentials in the right hand term of equation (8). As a result, we cannot write a current value formulation of (8). On the contrary, if the discount rates are not all equal, a present value formulation of the DPE (7) giving rise to time-consistent solutions does not exist. The time-consistent solutions to (7) are stationary (time independent) Markov strategies, whereas there are not stationary (time-independent) Markov strategies solving equation (8).

2.2 The case of general nonconstant weights

The idea of introducing nonconstant weights is to look for t-cooperative equilibria with just one but very important difference: weights of players in each t-coalition are not necessarily equal nor constant. In general, they will be the result of a bargaining process at every time t. Since the initial conditions for each t-coalition depend on the state of the system at time t, if $x(t) = x_t$, following Marín-Solano (2014), the problem of the t-coalition is to "maximize"

$$J(x_t, t) = \sum_{i=1}^{N} \lambda_i(x_t, t) \int_t^{\tau} \theta_i(s-t) u_i\left(x(s), c_1(s), \ldots, c_N(s)\right) ds \tag{9}$$

subject to (2), where $\lambda_i(x_t, t) \geq 0$, for every $i = 1, \ldots, N$. Coefficients $\lambda_i(x_t, t)$ represent the weight of agent i at state x_t and time t in the t-coalition[6]. Hence, at time t, given the initial state x_t, and knowing which will be the equilibrium decision rule of future s-agents, the members of the t-coalition decide their decision rule and bargain their current weight. Note that we are not weighting the instantaneous utilities ("inside" the integral, see e.g. Munro (1979) or Hara (2013)), but we are introducing nonconstant weights in the expression of the intertemporal utility function (9) (i.e., "outside" the integral).

Remark that there are two sources of time-inconsistency in Problem (9). First, there is the time-consistency problem related to the presence of different discount functions (and hence changing time preferences of the different t-coalitions). In addition, the introduction of changing weights generates also time-inconsistent preferences, since the joint intertemporal utility to be optimized changes along time. This applies also to the problem with constant and equal discount rates of time preference, in which the only source of time-inconsistency are the changing weights. Problems with one decision maker where instantaneous utilities of each t-agent depend also on the current state x_t were studied in Yong (2011).

If $c_i(s) = \phi_i(x(s), s)$ is a continuously differentiable equilibrium rule, for $i = 1, \ldots, N$, the joint value function is

$$V(x, t) = \sum_{i=1}^{N} \lambda_i(x, t) V_i(x, t) \ , \tag{10}$$

where

$$V_i(x, t) = \int_t^{\tau} \theta_i(s-t) u_i(x(s), \phi_1(x(s), s), \ldots, \phi_N(x(s), s))\, ds \ . \tag{11}$$

We assume that, along the equilibrium rule, the value function (10) is finite (i.e. the integral converges). In addition, functions $\lambda_i(x, t)$, $i = 1, \ldots, N$, are continuously differentiable in all their arguments, and equilibrium rules are assumed to be (at least) continuous.

[6] It is customary to demand $\sum_{i=1}^{N} \lambda_i = 1$. We do not impose this condition since the solution depends just on the relative weights.

Next, for $\epsilon > 0$, if the t-coalition precommits its behavior during the period $[t, t+\epsilon]$ by following the decision rule $c_\epsilon(s)$, for $s \geq t$ (see (4)), the valuation along the perturbed control path c_ϵ is

$$V_{c_\epsilon}(x,t) = \sum_{i=1}^{N} \lambda_i(x_t,t) \left\{ \int_t^{t+\epsilon} \theta_i(s-t) u_i(x(s), \bar{c})\, ds \right.$$
$$\left. + \int_{t+\epsilon}^{\infty} \theta_i(s-t) u_i(x(s), \phi(x(s),s))\, ds \right\} . \qquad (12)$$

Definition 2 *A decision rule $c^*(s) = \phi(x(s))$ is called a dynamic t-bargaining solution (t-BS) if*

$$\lim_{\epsilon \to 0^+} \frac{V(x,t) - V_{c_\epsilon}(x,t)}{\epsilon} \geq 0 \ ,$$

with $V(x,t)$ and $V_{c_\epsilon}(x,t)$ given by (10-11) and (12), respectively, for any admissible $c_\epsilon(s)$.

We use the expression *t-bargaining* since weights $\lambda_i(x_t, t)$ are chosen by each t-coalition. The following proposition characterizes the dynamic t-bargaining solutions. We refer to Marín-Solano (2014) for a proof.

Proposition 1 *If there exist N value functions of class C^1 given by (11) where*

$$(c_1, \ldots, c_N) = (\phi_1(x,t), \ldots, \phi_N(x,t))$$
$$= argmax_{\{c_1,\ldots,c_N\}} \left\{ \sum_{i=1}^{N} \lambda_i(x,t) \left(u_i(x, c_1, \ldots, c_N) + \nabla_x V_i(x) \cdot g(x, c_1, \ldots, c_N, t) \right) \right\}$$

are continuous functions, and there exists a unique absolutely continuous curve $x\colon [0,\infty] \to X$ solution to $\dot{x}(t) = g(x(t), \phi_1(x(t),t), \ldots, \phi_N(x(t),t))$ with $x(0) = x_0$, then $(c_1, \ldots, c_N) = (\phi_1(x,t), \ldots, \phi_N(x,t))$ is a dynamic t-bargaining solution.

Marín-Solano (2014) assumes in the previous proposition that weights $\lambda_i(x,t)$ in (9) are exogenously given. If weights are derived through a bargaining process, the dependence of $\lambda_i(x,t)$ in (x,t) can be seen through a dependence on the value functions $V_i(x,t)$ of all the agents. Then, in the derivation of dynamic t-bargaining solutions, we can take weight functions to be of the form $\lambda_i(x, z_1, \ldots, z_N, t)$, with $(z_1, \ldots, z_N) = (V_1(x,t), \ldots, V_N(x,t))$. It is easy to check that the problem of "maximizing" (constrained to the future behavior)

$$\sum_{i=1}^{N} \lambda_i(x, V_{1,c_\epsilon}(x,t), \ldots, V_{N,c_\epsilon}(x,t)) V_{i,c_\epsilon}(x,t)$$

is equivalent to Problem (9) with weights $\Lambda_i(x,t) = \sum_{j=1}^{N} \frac{\partial \lambda_j}{\partial z_i} V_j(x,t) + \lambda_i|_{(z_1,\ldots,z_N)=(V_1(x,t),\ldots,V_N(x,t))}$. Then we can solve the problem by applying Proposition 1 to the new weights $\Lambda_i(x,t)$.

3 Pareto inefficiency of the t-cooperative equilibria

The solutions to (8) are Pareto efficient in the "classical" sense, according to the preferences of the joint coalition at the beginning of the game. Therefore, if we compare the joint payoffs with those provided by Markovian (feedback) Nash equilibria if players act in a noncooperative way, the solution to (8) is "group efficient": at initial time, it gives higher payoffs to the coalition in case of cooperation. This result is clear since the corresponding maximization problem includes all strategies (and the noncooperative strategies, in particular). However, if commitment is not allowed, this solution becomes time-inconsistent. In general, if cooperation is permitted, (8) and (5-6) provide the right solutions to the joint "maximization" problem in case of precommitment (at $t = 0$) and no commitment, respectively. However, the time-consistent (in case of no commitment) t-cooperative equilibrium can be group inefficient: joint payoffs can be lower in the t-cooperative equilibrium than in the feedback noncooperative Nash equilibrium (Marín-Solano (2015)). In order to achieve this surprising result, it seems that players must be very asymmetric, both in the discount rates and in the instantaneous utility functions. In Karp (2016), where players are symmetric in their utility functions, cooperation cannot be disadvantageous.

From a different viewpoint, the t-cooperative equilibria solving (7) can be seen as constrained (to the future behavior of the agents) Pareto efficient. In general, in problems with time-inconsistent preferences, agents at different moments are treated as different agents. If there are, e.g., 2 players and 3 periods, this means that we have a game with $2 \times 3 = 6$ players. Equations (8) and (7) are not addressed to compute Pareto efficient solutions for this 6-player game. It is not clear for us how to compute the Pareto efficient solutions for this new game in a general setting (in dynamic games with an arbitrary number of periods, or in a differential game) and, in any case, they will be time-inconsistent, in general. In this section we show with a simple example that the payoffs given by the (constrained Pareto efficient) t-cooperative equilibrium rule can be Pareto dominated by those obtained by playing in a completely noncooperative way. The key point is that, if commitment is not allowed but cooperation is permitted, both the cooperative solutions to the DPE (8) and the noncooperative Nash equilibria become, in general, time-inconsistent. In the latter case, given the decision rule of future s-agents (for $s > t$), it is optimal for agents at time t to cooperate if their objective is to maximize their joint payoff.

As a combination of the previous properties, and although it seems paradoxical, if cooperation is permitted, time consistent players will follow the t-cooperative decision rule also in case non-cooperation is advantageous. In order to avoid this result, an everlasting unique regulator could commit the behavior of future agents for the whole time horizon, or to forbid coordination among players when necessary. In Section 4 we present an alternative, a time-consistent dynamic bargaining procedure, that can be useful, at least in some cases, in finding time-consistent and individually rational cooperative equilibria.

Next we show how the t-cooperative equilibrium rule can be, not just Pareto inefficient, but Pareto dominated by completely noncooperative outcomes. For computational reasons and to better illustrate the result, we take an example with just 2 players and in a discrete time setting with finite number of periods. At initial time, there is a resource x_0 to be consumed along time $t = 0, \ldots, T-1$. If c_{it} denotes the quantity of the resource that agent i (for $i = 1, 2$) consumes in period t, so x_{t+1} is the remaining quantity of the resource, then $x_{t+1} = x_t - c_{1t} - c_{2t}$. Let $u_i(c_{it})$ be the (strictly increasing and concave) utility function of player i from consumption at period t, and $\delta_i \in (0, 1]$ the discount factor of player i. First we restrict our attention to the case with $T = 2$.

***t*-cooperative equilibria**. For the calculation of the t-cooperative equilibrium rules, we first solve the problem at final time,

$$\max_{(c_{11}, c_{21})} u_1(c_{11}) + u_2(c_{21})$$

$$\text{subject to} \quad x_2 = x_1 - c_{11} - c_{21} \ , \quad \text{with } x_1 \text{ given} \ .$$

Since the utility functions are strictly incresings, we can assume that at final time the resource is exhausted, so that $x_2 = 0$. Let $(c^c_{11}, c^c_{21}) = (\phi^c_{11}(x_1), \phi^c_{21}(x_1))$ the solution to the above optimization problem. Then, at time $t = 0$ we solve

$$\max_{(c_{10},c_{20})} u_1(c_{10}) + u_2(c_{20}) + \delta_1 u_1(\phi^c_{11}(x_1)) + \delta_2 u_2(\phi^c_{21}(x_1))$$

$$\text{subject to} \quad x_1 = x_0 - c_{10} - c_{20} \ , \quad \text{with } x_0 \text{ given} \ .$$

We denote the corresponding solution as $(c^c_{10}, c^c_{20}) = (\phi^c_{10}(x_0), \phi^c_{20}(x_0))$.

Nash equilibria. In the final period we do not impose any fairness condition, so that players can share the resource x_1 in every possible way. If we denote by $(c^n_{11}, c^n_{21}) = (\phi^n_{11}(x_1), \phi^n_{21}(x_1))$, $\phi^n_{11}(x_1) + \phi^n_{21}(x_1) = x_1$ a division of the resource, we then compute the Nash equilibria at time $t = 0$ as usual: given $\phi^n_{j0}(x_0)$, agent $i \neq j$ mazimizes $u_i(c_{i0}) + \delta_1 u_i(\phi^n_{i1}(x_1)$ subject to $x_1 = x_0 - c_{i0} - \phi^n_{j1}(x_1)$. The corresponding decision rule is $(c^n_{10}, c^n_{20}) = (\phi^n_{10}(x_0), \phi^n_{20}(x_0))$.

For this problem we have computed some of the t-cooperative equilibria and noncooperative equilibria in case $u_1(c_1) = \ln c_1$ and $u_2(c_2) = \ln c_2^\lambda$. Let $U^c_i(x_0) = u_i(\phi^c_{i0}) + \delta_i u_i(\phi^c_{i1}(x_1))$ and $U^n_i(x_0) = u_i(\phi^n_{i0}) + \delta_i u_i(\phi^n_{i1}(x_1))$, for $i = 1, 2$, be the payoffs of the two players under the t-cooperative and the noncooperative decision rules. In the case of the noncooperative equilibria, we consider linear strategies $\phi^n_{it}(x_t) = a_{it} x_t$. In our analysis we take $x_0 = 10$, $\delta_1 = 0.95$, $1 \leq \lambda \leq 10$ and $0.05 \leq \delta_2 \leq 0.95$. For these values of the parameters, in most of cases, t-cooperative payoffs are not Pareto dominated by the noncooperative payoffs. However, if players are sufficiently asymmetric, both in their preferences (utility functions) and degree of impatience (discount rates), we have found many situations in which $U^c_1(x_0) < U^n_1(x_0)$ and $U^c_2(x_0) < U^n_2(x_0)$, i.e., noncooperative payoffs are Pareto superior to time-consistent cooperative payoffs. Table 1 presents some of these situations.

Table 1: Two players and 2 periods with $x_0 = 10$ and $\delta_1 = 0.95$

λ	δ_2	a^1_1	a^2_1	a^1_0	a^2_0	$U^c_1(10)$	$U^n_1(10)$	$U^c_2(10)$	$U^n_2(10)$
4	0.15	0.6	0.4	0.1207	0.7646	-0.2875	-0.1667	7.6206	7.6691
4	0.2	0.4	0.6	0.1492	0.7089	-0.2309	-0.1383	7.7010	7.7052
5	0.1	0.7	0.3	0.0873	0.8297	-0.7752	-0.6516	9.7608	9.8841
5	0.15	0.4	0.6	0.1207	0.7646	-0.6884	-0.5519	9.8113	9.8905
6	0.1	0.5	0.5	0.0873	0.8297	-1.1268	-0.9712	11.9549	12.1674
6	0.1	0.6	0.4	0.0873	0.8297	-1.1268	-0.7980	11.9549	12.0335
7	0.1	0.4	0.6	0.0873	0.8297	-1.4303	-1.1832	14.1528	14.3229
7	0.1	0.5	0.5	0.0873	0.8297	-1.4303	-0.9712	14.1528	14.1953
8	0.1	0.3	0.7	0.0873	0.8297	-1.6967	-1.4565	16.3526	16.4924
8	0.1	0.4	0.6	0.0873	0.8297	-1.6967	-1.1832	16.3526	16.3691
8	0.05	0.7	0.3	0.0477	0.9069	-1.8693	-1.8297	16.4195	16.8412
8	0.05	0.8	0.2	0.0477	0.9069	-1.8693	-1.7028	16.4195	16.6790
9	0.1	0.2	0.8	0.0873	0.8297	-1.9340	-1.8417	18.5534	18.6741
9	0.1	0.3	0.7	0.0873	0.8297	-1.9340	-1.4565	18.5534	18.5539
9	0.05	0.6	0.4	0.0477	0.9069	-2.1233	-1.9761	18.6408	19.0758
9	0.05	0.7	0.3	0.0477	0.9069	-2.1233	-1.8297	18.6408	18.9463
9	0.05	0.8	0.2	0.0477	0.9069	-2.1233	-1.7028	18.6408	18.7638
10	0.05	0.5	0.5	0.0477	0.9069	-2.3523	-2.1493	20.8631	21.3069
10	0.05	0.6	0.4	0.0477	0.9069	-2.3523	-1.9761	20.8631	21.1953
10	0.05	0.7	0.3	0.0477	0.9069	-2.3523	-1.8297	20.8631	21.0514

We have performed the same analysis for the case of 3 periods and the number of situations in which both players are better off not collaborating at all seems to be similar. Table 2 presents some of these results.

Table 2: Two players and 3 periods with $x_0 = 10$ and $\delta_1 = 0.95$

λ	δ_2	a_2^1	a_2^2	a_1^1	a_1^2	a_0^1	a_0^2	$U_1^c(10)$	$U_1^n(10)$	$U_2^c(10)$	$U_2^n(10)$
5	0.25	0.6	0.4	0.1739	0.6609	0.1139	0.6751	-2.3562	-2.2346	9.2753	9.3485
6	0.2	0.7	0.3	0.1492	0.7089	0.0946	0.7302	-3.0410	-2.9082	11.3611	11.5659
6	0.2	0.8	0.2	0.1492	0.7089	0.0946	0.7302	-3.0410	-2.7877	11.3611	11.4686
6	0.25	0.3	0.7	0.1739	0.6609	0.1139	0.6751	-2.8711	-2.8602	11.4191	11.4280
7	0.2	0.4	0.6	0.1492	0.7089	0.0946	0.7302	-3.5037	-3.4132	13.5039	13.6876
7	0.2	0.5	0.5	0.1492	0.7089	0.0946	0.7302	-3.5037	-3.2118	13.5039	13.6366
7	0.2	0.6	0.4	0.1492	0.7089	0.0946	0.7302	-3.5037	-3.0473	13.5039	13.5741
8	0.15	0.6	0.4	0.1207	0.7646	0.0736	0.7901	-4.1607	-4.1571	15.6486	16.0867
8	0.15	0.7	0.3	0.1207	0.7646	0.0736	0.7901	-4.1607	-4.0180	15.6486	16.0349
8	0.15	0.8	0.2	0.1207	0.7646	0.0736	0.7901	-4.1607	-3.8975	15.6486	15.9619
8	0.15	0.9	0.1	0.1207	0.7646	0.0736	0.7901	-4.1607	-3.7912	15.6486	15.8372
8	0.2	0.3	0.7	0.1492	0.7089	0.0946	0.7302	-3.9090	-3.6729	15.6486	15.6923
9	0.15	0.4	0.6	0.1207	0.7646	0.0736	0.7901	-4.5415	-4.5231	17.8023	18.1797
9	0.15	0.5	0.5	0.1207	0.7646	0.0736	0.7901	-4.5415	-4.3217	17.8023	18.1427
9	0.15	0.6	0.4	0.1207	0.7646	0.0736	0.7901	-4.5415	-4.1571	17.8023	18.0975
9	0.15	0.7	0.3	0.1207	0.7646	0.0736	0.7901	-4.5415	-4.0180	17.8023	18.0393
9	0.15	0.8	0.2	0.1207	0.7646	0.0736	0.7901	-4.5415	-3.8975	17.8023	17.9572
9	0.15	0.9	0.1	0.1207	0.7646	0.0736	0.7901	-4.5415	-3.7912	17.8023	17.8168
10	0.15	0.3	0.7	0.1207	0.7646	0.0736	0.7901	-4.8838	-4.7827	19.9564	20.2343
10	0.15	0.4	0.6	0.1207	0.7646	0.0736	0.7901	-4.8838	-4.5231	19.9564	20.1996
10	0.15	0.5	0.5	0.1207	0.7646	0.0736	0.7901	-4.8838	-4.3217	19.9564	20.1586
10	0.15	0.6	0.4	0.1207	0.7646	0.0736	0.7901	-4.8838	-4.1572	19.9564	20.1084
10	0.15	0.7	0.3	0.1207	0.7646	0.0736	0.7901	-4.8838	-4.0180	19.9564	20.0437

Although these results are quite surprising at first sight, they are not new in the economics literature. Related inefficiency results have been found in previous studies on disadvantageous cooperation (Rogoff (1985)), disadvantageous monopsony (Maskin and Newbery (1990)) or disadvantageous monopoly (Karp (1996)). In a different context with hyperbolic preferences, Krusell *et al.* (2002) and Hiraguchi (2014) found that aggregate welfare in the market outcome weakly dominates the outcome under a social planner.

4 Dynamic Nash t-Bargaining Solution

In the dynamic t-bargaining solution, weights are, in principle, arbitrary functions, that can be chosen according to different criteria. In this section we propose to use Nash bargaining theory as a possible way to fix them. In Nash bargaining theory, payments are obtained as the maximizers of a Nash bargaining function (Nash (1953)). These payments implicitly characterize the weights of players in the whole coalition. Munro (1979) proposed to use Nash bargaining theory with constant weights in a transboundary resource model with asymmetric players. In his model, the threat (status quo or disagreement) point is a noncooperative Nash equilibrium, and weights are bargained at the beginning of the game. In order to avoid the problems of dynamic inconsistency (Haurie (1976)) and time inconsistency of the solution, he assumed full commitment of economic agents. We propose to introduce nonconstant weights - as the result of repeated negotiations that take place at each moment t - as a natural way to eliminate this (unrealistic in most of cases) full commitment.

Let us assume that, at time t, players know which will be their decision rule $c = \phi(x(s), s)$ for future $s > t$ as a reaction to their current decision rule. Then, as in the classical Nash bargaining solution, they compare what they get cooperating at time t (or during the time interval $[t, t+\epsilon]$, with ϵ arbitrarily small) with what they receive otherwise (the status quo). Let us denote the threat point by $(W_1(x,t), \ldots, W_N(x,t))$. When players at time t try to reach an agreement and to derive their corresponding actions, they choose their policy in the time interval $[t, t+\epsilon]$ as the maximizer of some "distance" between what they obtain in case of agreement and in case of disagreement. We assume that this distance is measured according to the generalized Nash welfare function with strictly positive bargaining powers $\eta_1, \ldots, \eta_N$

$$\prod_{i=1}^{N} [V_i(x, c_1, \ldots, c_N, t) - W_i(x,t)]^{\eta_i} \ .$$

For $\eta_1 = \cdots = \eta_N = 1$ we recover the unique bargaining solution satisfying the four classical axioms (in a "static game"): invariance to equivalent utility representations, symmetry, independence of irrelevant alternatives and Pareto efficiency. However, as Sorger (2006) arguments, in a 2-player game, the Nash bargaining solution corresponding to the relative bargaining power $\eta = \eta_2/\eta_1 = \ln \beta_1 / \ln \beta_2$ (where β_i^t is the discount function of player i) is the limit (as time delay between offers approaches to 0) of the unique perfect equilibrium outcome of a Rubinstein-type alternating-offers model (see, e.g., Binmore (1987)). Then it seems natural to introduce different bargaining powers related to the discount rates.

Definition 3 *Let $W(x,t) = (W_1(,tx), \ldots, W_N(x,t))$ be a set of threat value functions. A decision rule $(c_1, \ldots, c_N) = (\phi_1(x,t), \ldots, \phi_N(x,t))$ is called a dynamic Nash t-bargaining solution corresponding to the threat point $W(x,t)$ if $V_i(x, \phi_1(x,t), \ldots, \phi_N(x,t)) - W_i(x,t) > 0$, for all $i = 1, \ldots, N$, and*

$$\lim_{\epsilon \to 0^+} \frac{\Pi(x,t) - \Pi_{c_\epsilon}(x,t)}{\epsilon} \geq 0 \ ,$$

where

$$\Pi(x,t) = \prod_{i=1}^{N} [V_i(x, \phi_1(x,t), \ldots, \phi_N(x,t)) - W_i(x,t)]^{\eta_i}$$

and, for all admissible c_ϵ given as in (4),

$$\Pi_{c_\epsilon}(x,t) = \prod_{i=1}^{N} [V_{i,c_\epsilon}(x, c_\epsilon, t) - W_i(x,t)]^{\eta_i} \ .$$

It can be easily checked that, provided that $V_i(x, \phi_1(x,t), \ldots, \phi_N(x,t)) - W_i(x,t) > 0$, the dynamic Nash t-bargaining solutions are the dynamic t-bargaining solution defined in Definition 2 with weights given by

$$\lambda_i(x,t) = \eta_i (V_i(x, \phi_1(x,t), \ldots, \phi_N(x,t)) - W_i(x,t))^{\eta_i - 1}$$

$$\cdot \prod_{j \neq i} (V_j(x, \phi_1(x,t), \ldots, \phi_N(x,t)) - W_j(x,t))^{\eta_j} ,$$

or, since what are relevant are the relative weights λ_i/λ_j,

$$\lambda_i(x) = \frac{\eta_i}{V_i(x, \phi_1(x), \phi_N(x)) - W_i(x)} . \tag{13}$$

Hence, we can use Proposition 1 for the computation of dynamic Nash t-bargaining solutions.

It remains to define the threat value functions. In our examples in Section 5 we follow the "classical" approach in which threat points are those obtained in case agents do not cooperate in the whole time interval $[0, \infty)$ and follow Markov Perfect Nash equilibria (MPNE). This means that, if players do not cooperate at time t, they will not cooperate forever. This approach seems to be rather unrealistic. What seems more realistic is to assume that, if players do not cooperate at time t, they will not cooperate along a (finite) time interval. However, since our objective in this paper is to look for dynamic bargaining solutions that are not Pareto dominated by MPNE, we will take as threat points the valuations according to the MPNE, so $W_i(x,t) = W_i^n(x,t)$, for $i = 1, \ldots, N$. By construction, in this case, if there are bargaining powers $\eta_1, \ldots, \eta_N$ such that the corresponding Nash t-bargaining solutions exist, then the related payoffs are individually rational in the classical sense.

5 Common property resource games with asymmetric players

In this section we apply the results in the previous section to the search of time consistent equilibria in three common property resource games with heterogeneous agents coming from the literature of resource economics. The first model considers the joint management of an exhaustible natural resource in case players have isoelastic utilities with different marginal elasticities. The second model introduces a linear production function. This model can be useful in the analysis of a renewable resource near a stationary point. Alternatively, our analysis with a linear production function can be useful in the study of other models. For example, Benchekroun (2002) considered production functions constructed as the union of two segments, one increasing and the other decreasing. Also, several pollution models have a linear dynamics in the state variable. Finally, our third model studies the joint management of a renewable natural resource for log-utilities and production function $f(x) = x(a - b \ln x)$, with $a, b > 0$. For these models we provide conditions on the functional form of the (nonconstant) weights of players for the existence of linear strategies. These strategies have an important advantage: due to its simplicity, they are easier to implement in practical applications. In addition, as we show, linear strategies solve the dynamic Nash t-bargaining problem, with the threat point corresponding to the payoffs if players act according to noncooperative feedback (Markovian) strategies, for the above mentioned models.

5.1 An exhaustible resource model with general isoelastic utilities

First we study the management of an exhaustible natural resource in case players have general isoelastic utilities, with different marginal elasticities. The intertemporal utility function of Player i is given by

$$J_i = \int_t^\infty \theta_i(s-t) \frac{c_i^{1-\sigma_i}(s) - 1}{1 - \sigma_i} \, ds \tag{14}$$

with $\sigma_i > 0$, $\sigma_i \neq 1$, for $i = 1, \dots, N$. The stock of the resource evolves according to

$$\dot{x}(s) = -\sum_{i=1}^{N} c_i(s) , \quad x(t) = x_t . \tag{15}$$

In this model, the stock of the resource converges in all the solution concepts to the stationary state $x_\infty = 0$.

5.1.1 Noncooperative Markov Perfect Nash equilibrium

The problem in which players do not cooperate and have constant discount rates has been studied with full detail in the literature. We can easily derive the solution for the case of general time-distance discount functions. We restrict our attention to stationary linear strategies.

If $\phi_j^n(x)$ denotes an equilibrium strategy for Player $j = 1, \dots, N$, then Player i aims to solve

$$\max_{\{c_i\}} \left\{ \frac{c_i^{1-\sigma_i} - 1}{1 - \sigma_i} - W_i'(x) \left(c_i + \sum_{j \neq i} \phi_j^n(x) \right) \right\} ,$$

where $W_i(x)$ denotes the value function of Player i. From the maximization problem we obtain $c_i^n = (W_i'(x))^{-1/\sigma_i}$. Since we are looking for strategies of the form $c_i^n = \phi_i^n(x) = A_i^n x$, then $W_i(x) = x^{1-\sigma_i}/((A_i^n)^{\sigma_i}(1-\sigma_i)) + \beta_i^n$.

Next, note that the solution to $\dot{x}(s) = -\sum_{j=1}^N c_j^n(s) = -\sum_{j=1}^N A_j^n x(s)$ with the initial condition $x(t) = x_t$ is given by $x(s) = x_t \exp\left[-\sum_{j=1}^N A_j^n (s-t)\right]$. Then, $c_i^n(s) = A_i^n x_t \exp\left[-\sum_{j=1}^N A_j^n (s-t)\right]$. By substituting in the expression of the value function we obtain

$$\begin{aligned} W_i(x_t) &= \int_t^\infty \theta_i(s-t) \frac{(c_i^n(s))^{1-\sigma_i} - 1}{1-\sigma_i} \, ds \\ &= \frac{1}{1-\sigma_i} \int_t^\infty \theta_i(s-t) \left[\left(A_i^n x_t e^{-\sum_{j=1}^N A_j^n (s-t)} \right)^{1-\sigma_i} - 1 \right] ds \\ &= \frac{(A_i x_t)^{1-\sigma_i}}{1-\sigma_i} \int_o^\infty \theta_i(\tau) e^{-\sum_{j=1}^N A_j^n (1-\sigma_i)\tau} \, d\tau - \frac{1}{1-\sigma_i} \int_0^\infty \theta_i(\tau) \, d\tau . \end{aligned}$$

By identifying $x_t = x$, since $W_i(x) = x^{1-\sigma_i}/((A_i^n)^{\sigma_i}(1-\sigma_i)) + \beta_i^n$ we obtain

$$A_i^n = \frac{1}{\int_0^\infty \theta_i(\tau) e^{-\sum_{j=1}^N A_j^n (1-\sigma_i)\tau} \, d\tau} , \quad \beta_i^n = -\frac{1}{1-\sigma_i} \int_0^\infty \theta_i(\tau) \, d\tau . \tag{16}$$

In the standard case of constant discount rates, $\theta_i(\tau) = \exp(-\rho_i \tau)$, we have

$$c_i^n = A_i^n x = \left(\rho_i + \frac{(1-\sigma_i) \sum_{j=1}^N \rho_j}{1 - \sum_{j=1}^N (1-\sigma_j)} \right) x . \tag{17}$$

Remark 1 *In the derivation of (16) and (17) we have implicitly assumed that the integral converges. Obviously, A_i^n in (17) must be positive for the existence of MPNE.*

5.1.2 Dynamic t-bargaining solution

In general, t-cooperative equilibria seem to be extremely cumbersome to compute for this simple problem. In fact, no linear decision rules exist. In order to find analytical solutions, we introduce nonconstant weights. The following proposition characterizes the functional form of weight functions guaranteeing the existence of linear decision rules in the computation of the dynamic t-bargaining solutions described in Definition 2.

Proposition 2 *In Problem (14-15) linear dynamic t-bargaining solutions exist if, and only if, weight functions are of the form $\lambda_i(x) = \nu_i x^{\sigma_i} h(x)$, i.e. $\lambda_i(x)/\lambda_j(x) = (A_i^{\sigma_i}/A_j^{\sigma_j})x^{\sigma_i-\sigma_j}$, with $h(x)$ an arbitrary continuously differentiable function.*

Proof: See the Appendix. □

Next, for this family of weight functions, the corresponding decision rules are presented.

Corollary 1 *In Problem (14-15), for $\lambda_i(x) = \nu_i x^{\sigma_i} h(x)$, $i = 1, \ldots, N$, the corresponding dynamic t-bargaining solution is $c_i(x) = A_i x$ and $V_i(x) = \alpha_i x^{1-\sigma_i} + \beta_i$, where*

$$\beta_i = -\frac{1}{1-\sigma_i}\int_0^{\infty} \theta_i(\tau)\, d\tau$$

and coefficients α_i and A_i are the solution to the $2N$ equation system

$$\begin{aligned} A_i &= \left(\frac{\nu_i}{\sum_{j=1}^N \nu_j(1-\sigma_j)\alpha_j}\right)^{1/\sigma_i} \\ \alpha_i &= \frac{A_i^{1-\sigma_i}}{1-\sigma_i}\int_0^{\infty} \theta_i(\tau) e^{-\sum_{j=1}^N A_j(1-\sigma_i)\tau}\, d\tau\ . \end{aligned}$$

In particular, if $\theta_i(s) = \exp(-\rho_i s)$, then

$$\alpha_i = \frac{A_i^{1-\sigma_i}}{(1-\sigma_i)\left(\rho_i + (1-\sigma_i)\sum_{j=1}^N A_j\right)}\ .$$

Proof: It follows from (27) and (28).

□

Remark 2 *In the previous derivations we have assumed that $\sigma_i \neq 1$, for all $i = 1, \ldots, N$. If there are players with logarithmic utilities, previous calculations can be reproduced in a similar way.*

5.1.3 Dynamic Nash t-bargaining solution

Once we have computed both the MPNE and the dynamic t-bargaining solutions, we can use (13) for the calculation of the dynamic Nash t-bargaining solution. From the results in Section 5.1.1 and Section 5.1.2, for the case of constant but different discount rates, we have:

Proposition 3 *Let $\theta_i(s) = \exp(-\rho_i s)$, for $i = 1, \ldots, N$. The dynamic Nash t-bargaining is given by Corollary 1 with weight functions $\lambda_i(x) = \nu_i x^{\sigma_i - 1}$ (i.e. $h(x) = 1/x$) and coefficients ν_i, α_j and A_k, for $i, j, k = 1, \ldots, N$, solving*

$$\begin{aligned} \nu_i &= \frac{\eta_i}{\alpha_i - \alpha_i^n}\ , \\ A_i &= \left(\frac{\nu_i}{\sum_{j=1}^N \nu_j(1-\sigma_j)\alpha_j}\right)^{1/\sigma_i}\ , \\ \alpha_i &= \frac{A_i^{1-\sigma_i}}{(1-\sigma_i)\left(\rho_i + (1-\sigma_i)\sum_{j=1}^N A_j\right)}\ , \end{aligned}$$

where

$$\alpha_i > \alpha_i^n = \frac{1}{1-\sigma_i}\left(\rho_i + \frac{(1-\sigma_i)\sum_{j=1}^N \rho_j}{1-\sum_{j=1}^N (1-\sigma_j)}\right)^{-\sigma_i}\ .$$

Proof: See the Appendix. □

Proposition 3 provides a sound justification to the use of some of the (a priori) arbitrary weight functions introduced in Section 5.1.2. The use of these weight functions is not only necessary for the existence of linear decision rules, but weights of the form $\lambda_i(x) = \nu_i x^{\sigma_i - 1} h(x)$ are the natural outcome of the dynamic Nash t-bargaining procedure described in Definition 3, that extend the standard (asymmetric) Nash bargaining function to a setting in which agents bargain their weight at every instant of time.

5.1.4 An extension: a linear production model

The results of Section 5.1 can be easily extended to the case of affine production functions of the form $f(x) = ax + d$, with a, d real numbers. The evolution of the stock of the resource is described by

$$\dot{x}(s) = ax + d - \sum_{i=1}^{N} c_i(s) , \quad x(t) = x_t , \tag{18}$$

where c_i denotes the harvest rate of agent i. Agents aim to maximize the sum of discounted utilities. Equation (18) can be interpreted as a model of a productive asset with constant gross return, or as a model analyzing the equilibrium harvest/consumption rates in the exploitation of a common access renewable resource near a stationary state x_∞. Note that, in the study of the dynamics near a x_∞, we can take the linear approximation $f(x) \approx f(x_\infty) + f'(x_\infty)(x - x_\infty)$. However, if $f(x)$ is an arbitrary production function, we can apply the results in Karp (2007) and Ekeland and Lazrak (2010) to deduce that, in general, multiple equilibria exist for this kind of problems. Then, in the analysis near a stationary state, first we have to choose a particular state in order to eliminate the indeterminacy in the values of the coefficients a and d in equation (18).

In the case of noncooperative MPNE, affine strategies $c_i^n = \phi_i^n(x) = A_i^n x + B_i$ exist. In the t-cooperative case, no affine decision rules exist unless marginal elasticities coincide. For the existence of affine (in the resource stock) dynamic t- bargaining decision rules, Proposition 2 can be easily generalized.

Proposition 4 *In Problem (14-18), affine dynamic t-bargaining solutions exist if, and only if, weight functions are of the form $\lambda_i(x) = (A_i x + B)^{\sigma_i} h(x)$, i.e. $\lambda_i(x)/\lambda_j(x) = (\nu_i/\nu_j)(x + B)^{\sigma_i - \sigma_j}$, with $h(x)$ an arbitrary continuously differentiable function.*

Proof: See the Appendix. □

5.2 A common property renewable natural resource model with log-utilities

As a final example, let us consider the problem of exploitation of a renewable natural resource with state dynamics

$$\dot{x}(s) = x(s)(a - b \ln x(s)) - \sum_{j=1}^{N} c_j(s) , \quad x(t) = x_t \tag{19}$$

and utility functions

$$J_i = \int_t^\infty \theta_i(s - t) \ln \left[(c_i(s))^{\mu_i} \right] ds , \tag{20}$$

with $\mu_i > 0$, for $i = 1, \ldots, N$.

For general time-distance discount functions with $\mu_i = 1$, noncooperative MPNE and t-cooperative equilibria were already derived in Marín-Solano (2014). The extension is straightforward and we summarize the results.

5.2.1 Markov Perfect Nash equilibrium

If players do not cooperate, stationary linear strategies exist and are given by

$$c_i^n = \phi_i^n(x) = \frac{x}{\int_0^\infty \theta_i(s)e^{-bs}\,ds}\ , \tag{21}$$

for $i = 1, \ldots, N$. The corresponding value functions are of logarithmic type,

$$V_i(x) = \alpha_i^n \ln x + \beta_i^n\ ,$$

where

$$\alpha_i^n = \mu_i \int_0^\infty \theta_i(s)e^{-bs}\,ds\ ,$$

$$\beta_i^n = \frac{\mu_i}{b}\left(a - \sum_{j=1}^{N} \frac{1}{\int_0^\infty \theta_j(s)e^{-bs}\,ds}\right)\int_0^\infty \theta_i(s)[1-e^{-bs}]\,ds$$
$$-\mu_i \ln\left(\int_0^\infty \theta_i(s)e^{-bs}\,ds\right)\int_0^\infty \theta_i(s)\,ds\ . \tag{22}$$

If $\theta_i(s) = \exp(-\rho_i s)$, we recover the classical results: $c_i^n(x) = (\rho_i + b)x$ and

$$V_i(x) = \left(\frac{\mu_i}{\rho_i + b}\right)\ln x + \frac{\mu_i}{\rho_i}\left[\frac{a - \sum_{j=1}^{N}(\rho_j + b)}{\rho_i + b} + \ln\,(\rho_i + b)\right]\ .$$

5.2.2 Dynamic bargaining solutions

Again, we restrict our attention to linear strategies. In such a case a unique stationary state exists, avoiding in this way the problems related to multiplicity of equilibria. The following proposition summarizes our results.

Proposition 5 *In Problem (19-20), linear decision rules exist if, and only if, weights are of the form $\lambda_i(x) = \nu_i h(x)$. Equilibrium rules are given by*

$$c_i = \frac{\lambda_i \mu_i x}{\sum_{j=1}^{N} \lambda_j \mu_j \int_0^\infty e^{-bs}\theta_j(s)\,ds} \tag{23}$$

and the corresponding value functions are of the form $V_i(x) = \alpha_i\, ln\, x + \beta_i$, where

$$\alpha_i = \mu_i \int_0^\infty \theta_i(s)e^{-bs}\,ds\ ,$$

$$\beta_i = \frac{\mu_i}{b}\left(a - \frac{\sum_{j=1}^{N}\lambda_j\mu_j}{\sum_{j=1}^{N}\lambda_j\mu_j\int_0^\infty \theta_j(s)e^{-bs}\,ds}\right)\int_0^\infty \theta_i(s)[1-e^{-bs}]\,ds \tag{24}$$
$$+\mu_i\left(ln\,(\lambda_i\mu_i) - \,ln\left(\sum_{j=1}^{N}\lambda_j\mu_j\int_0^\infty \theta_j(s)e^{-bs}\,ds\right)\right)\int_0^\infty \theta_i(s)\,ds\ .$$

Proof: See the Appendix. □

Next we briefly compare the dynamic bargaining solutions with the MPNE. First of all, note that $\alpha_i = \alpha_i^n$. Hence, in order to look for individually rational dynamic bargaining solutions admitting linear strategies, it suffices to look for weights verifying $\beta_i \geq \beta_i^n$, for every $i = 1, \ldots, N$. With respect to the harvesting rule, it is easy to check, using (21) and (23), that harvest rates verify $c_i^n(x) - c_i(x) > 0$ if, and only if,

$$\sum_{j=1}^{N} \lambda_j \mu_j \frac{\int_0^\infty e^{-bs}\theta_j(s)\,ds}{\int_0^\infty e^{-bs}\theta_i(s)\,ds} - \lambda_i \mu_i > 0 \ ,$$

and this is obviously satisfied. Hence, as in the standard case with equal and constant discount rates, harvest rates are higher in the pure noncooperative setting than in the problem with t-cooperation.

5.2.3 Dynamic Nash t-bargaining solutions

As a way to select particular weights, we can compute the dynamic Nash t-bargaining solution. Since $V_i(x) - W_i(x) = \beta_i - \beta_i^n$, the corresponding weights are given by

$$\lambda_i = \frac{\eta_i}{\beta_i - \beta_i^c} \ . \tag{25}$$

Then in order to find the solution, we have to solve the equation system relating β_i and λ_i, i.e. Equations (24) and (25), with β_i^n given by (22).

For instance, take the example in Marín-Solano (2015), in which there is group inefficiency. Value parameters are $a = b = 0$, $N = 2$, $\theta_1(s) = e^{-0.01s}$, $\theta_2 = e^{-0.2s}$, $\mu_1 = 0.01$ and $\mu_2 = 10$. Following Sorger (2006), we take the asymmetric bargaining powers $\eta_1 = 1$, $\eta_2 = \rho_1/\rho/2$ ($\rho_i = -\ln \beta_i$). From (22) we obtain $\beta_1^n = -25.6052$, $\beta_2 = -132.972$. Then by solving the equation system (24-25) we have: $\beta_1 = -23.1472$, $\beta_2 = -132.713$ with weights $\lambda_1 = 0.406842$, $\lambda_2 = 0.0397278$. Note that by assigning the more patient Player 1 a weight much higher than to the more impatient Player 2, we obtain a payoffs that are individually rational: $V_1(x) - W_1(x) = 2.45796$ and $V_2(x) - W_2(x) = 1.25856$. Hence, $V_1(x) + V_2(x) > W_1(x) + W_2(x)$ and the group inefficiency result of the t-cooperative equilibrium is avoided. It is important to realize that the existence of dynamic Nash bargaining solutions is not guaranteed (for these or other values of the bargaining powers η_i).

6 Conclusions

In this paper we have presented the limits of adding utilities if economic agents discount the future at different rates. Time-consistent equilibria can be strongly inefficient, since individual payments can be Pareto dominated by those obtained if players act in a completely noncooperative way. In those cases, players are better off if cooperation is forbidden than if it is allowed. Since time-consistent t-cooperative solutions are, by definition, constrained (to future decision rules) Pareto efficient at time t, players will collaborate at each time t and noncooperative solutions become, in general, time-inconsistent if collaboration is permitted. Therefore, although payments can be higher if players act in a completely noncooperative way, rational agents will follow the inefficient time-consistent cooperative decision rule. In order to obtain this strong inefficiency result discount rates must be necessarily different. Another drawback, thinking in practical applications, of time-consistent t-cooperative solutions is that, if players have different (instantaneous) utility functions, they are, in general, very difficult to compute.

In order to address (at least partially) these problems, non constant (state-dependent) weights are introduced. These non constant weights can be obtained as the maximizers of a Nash welfare function, by introducing in this way the concept of dynamic Nash t-bargaining solution. These ideas are applied to the study of three common property resource games: a model of an exhaustible natural resource with different isoelastic utilities, an extension of this model to the analysis of the joint management of a renewable resource near a stationary point, and the management of a renewable resource with log-utilities. For these models, for different general time-distance discount functions, linear (affine) strategies are derived for families of weight functions including, in particular, those obtained from the dynamic Nash t-bargaining procedure.

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APPENDIX

Proof of Proposition 2: From Proposition 1 we have to solve

$$\max_{\{c_1,\dots,c_N\}} \left\{ \sum_{i=1}^{N} \lambda_i(x) \frac{c_i^{1-\sigma_i}-1}{1-\sigma_i} - \left(\sum_{i=1}^{N} \lambda_i(x) V_i'(x)\right) \cdot \left(\sum_{j=1}^{N} c_j\right) \right\} .$$

By applying the first order condition we obtain

$$c_i(x) = \phi_i(x) = \left(\frac{\lambda_i(x)}{\sum_{j=1}^{N} \lambda_j(x) V_j'(x)} \right)^{1/\sigma_i} . \tag{26}$$

First, let us assume that linear strategies $\phi_i(x) = A_i x$ exist. Then, from (26) we obtain

$$\lambda_i(x) = \left(\sum_{j=1}^{N} \lambda_j(x) V_j'(x) \right) (A_i x)^{\sigma_i} ,$$

i.e. weights must be of the form $\lambda_i(x) = \nu_i x_i^{\sigma} h(x)$, with $h(x) = \sum_{j=1}^{N} \lambda_j(x) V_j'(x)$ so $\lambda_i(x)/\lambda_j(x) = (A_i^{\sigma_i}/A_j^{\sigma_j}) x^{\sigma_i - \sigma_j}$. Note that, in this case, by proceeding as in the derivation of the MPNE, the solution to (15) is given by $x(s) = x_t \exp\left[-\sum_{j=1}^{N} A_j (s-t)\right]$, and

$$V_i(x) = \frac{(A_i x_t)^{1-\sigma_i}}{1-\sigma_i} \int_0^{\infty} \theta_i(\tau) e^{-\sum_{j=1}^{N} A_j (1-\sigma_i)\tau}\, d\tau - \frac{1}{1-\sigma_i} \int_o^{\infty} \theta_i(\tau)\, d\tau . \tag{27}$$

Reciprocally, let $\lambda_i(x) = \nu_i x_i^{\sigma} h(x)$, for $i = 1, \dots, N$. Then, from (26) we have

$$c_i = \nu^{1/\sigma_i} \left(\sum_{j=1}^{N} \nu_j x^{\sigma_j} V_j'(x) \right)^{-1/\sigma_i} x .$$

We have to prove that there are value functions $V_1(x), \dots, V_N(x)$ such that the quantity $\sum_{j=1}^{N} \nu_j x^{\sigma_j} V_j'(x)$ is a constant number. Since we are interested in the existence of linear strategies, we take $V_i(x) = \alpha_i x^{1-\sigma_i} + \beta_i$. Solutions obtained from other choices of functions $V_i(x)$ will be nonlinear. Then $V_j'(x) = (1-\sigma_j)\alpha_j x^{-\sigma_j}$ and $\sum_{j=1}^{N} \nu_j x^{\sigma_j} V_j'(x) = \sum_{j=1}^{N} \nu_j (1-\sigma_j)\alpha_j$ so

$$c_i = \left(\frac{\nu_i}{\sum_{j=1}^{N} \nu_j (1-\sigma_j)\alpha_j} \right)^{1/\sigma_i} x . \tag{28}$$

Proof of Proposition 3: First of all, note that $V_i(x) - W_i(x) = (\alpha_i x^{1-\sigma_i} + \beta_i) - (\alpha_i^n x^{1-\sigma_i} + \beta_i^n) = (\alpha_i - \alpha_i^n) x^{1-\sigma_i}$, since $\beta_i = \beta_i^n = \int_o^{\infty} \theta_i(s)\, ds/(1-\sigma_i)$. Then, from (13),

$$\lambda_i(x) = \frac{\eta_i}{V_i(x) - W_i(x)} = \frac{\eta_i}{\alpha_i - \alpha_i^n} x^{\sigma_i - 1} .$$

The result follows from Corollary 1 and the expression of α_i^n in Section 5.1.1.

Proof of Proposition 4: By applying Proposition 1 we obtain

$$c_i(x) = \phi_i(x) = \left(\frac{\lambda_i(x)}{\sum_{j=1}^{N} \lambda_j(x) V_j'(x)} \right)^{1/\sigma_i} .$$

First, if there are affine strategies $\phi_i(x) = A_i x + B_i$, then

$$\lambda_i(x) = \left(\sum_{j=1}^{N} \lambda_j(x) V_j'(x) \right) A_i^{\sigma_i} \left(x + \frac{B_i}{A_i} \right)^{\sigma_i} .$$

Reciprocally, if $\lambda_i(x) = (A_i x + B_i)^{\sigma_i} h(x)$, then affine equilibrium rules exist with $h(x) = \sum_{j=1}^{N} \lambda_j(x) V_j'(x)$. It remains to prove $B_i = B_j = B$, for every $i, j = 1, \ldots, N$. By solving (15) we obtain

$$x(s) = x_t e^{\left(a - \sum_{j=1}^{N} A_j\right)(s-t)} + \frac{d - \sum_{j=1}^{N} B_j}{a - \sum_{j=1}^{N} A_j} \left(e^{\left(a - \sum_{j=1}^{N} A_j\right)(s-t)} - 1 \right) .$$

By substituting in the expression of the value function of player i.

$$V_i(x) = \frac{1}{1-\sigma_i} \int_0^{\infty} \theta_i(s) \left[A_i x(s-t) + B_i \right]^{1-\sigma_i} \, ds - \frac{1}{1-\sigma_i} \int_0^{\infty} \theta_i(s) \, ds .$$

By taking $B = B_i = d/(a - \sum_{j=1}^{N} A_j + N)$, for $i = 1, \ldots, N$, the expression above simplifies to $V_i(x) = \alpha_i (A_i x + B)^{1-\sigma_i} + \beta_i$, where

$$\begin{aligned} \alpha_i &= \frac{1}{1-\sigma_i} \int_0^{\infty} \theta_i(s) e^{\left(a - \sum_{j=1}^{N} A_j\right)(1-\sigma_i)s} \, ds , \\ \beta_i &= \frac{1}{1-\sigma_i} \int_0^{\infty} \theta_i(s) \, ds . \end{aligned}$$

Proof of Proposition 5: By applying Proposition 1 we obtain

$$c_i(x) = \frac{\mu_i \lambda_i(x)}{\sum_{j=1}^{N} \lambda_j(x) V_j'(x)} .$$

If $\lambda_i(x) = \nu_i h(x)$ are constant numbers, then linear strategies exist associated to the value functions $V_i(x) = \alpha_i \ln x + \beta_i$. Reciprocally, if linear strategies $c_i(x) = A_i x$ exist, then the value functions are give by $V_i(x) = \alpha_i \ln x + \beta_i$ and necessarily $\lambda_i(x) = \nu_i h(x)$. Then the values of A_i, α_i and β_i follow from Marín-Solano (2014).