Consistent Taxation for a Polluting Monopoly*

Guiomar Martín-Herrán† and Santiago J. Rubio‡

May 11, 2017

*The authors would like to thank participants at the 2016 International Symposium on Dynamic Games and Applications (Urbino, Italy) for stimulating discussion and Franz Wirl for his interesting comments. Guiomar Martín-Herrán gratefully acknowledges financial support from the Spanish Ministry of Economics and Competitiveness under project ECO2014-52343-P and COST Action IS1104 "The EU in the new economic complex geography: models, tools and policy evaluation." Santiago J. Rubio also gratefully acknowledges financial support from the Spanish Ministry of Economics and Competitiveness under project ECO2013-45045-R, and Valencian Generality under project PROMETEO II/2014/054.

†Department of Applied Economics and IMUVa, University of Valladolid, Spain.

‡Department of Economic Analysis and ERI-CES, University of Valencia, Spain.

Abstract

This paper evaluates the effects of a limited regulatory commitment on the emission tax paid by a polluting monopoly comparing two alternative equilibria of a policy (differential) game: the stagewise feedback Stackelberg equilibrium (SFSE) and the Markov perfect equilibrium with limited commitment (LCMPE). For both equilibria no commitment for the entire time horizon is assumed. However, for the SFSE the regulator moves first in each period whereas for the LCMPE in a first stage the regulator and the monopolist *simultaneously* choose the emission tax and abatement effort respectively, and in a second stage the monopolist selects the output level. We find that the SFSE is not *intratime consistent*, i.e. it is not time consistent for the game played in each period. We also find that a limited commitment leads to lower taxation and abatement that yield larger production and emissions and, consequently, a larger steady-state pollution stock. Moreover, the increase of environmental damages because of the increase in the pollution stock more than compensates the increase in consumer surplus and the decrease in abatement costs resulting in a reduction of net social welfare when there is a limited commitment.

Keywords: monopoly, limited commitment, emission tax, abatement, stock pollutant

JEL Classification System: H23, L12, L51, Q52, Q55

1 Introduction

The characterization of the efficiency-inducing tax rules for polluting firms with market power was established by Benchekroun and Long (1998, 2002). In the first paper, the authors show for a linear-quadratic case that there exists a stationary linear tax rule that leads polluting oligopolists to implement the efficient outcome. The authors propose a three-step procedure. First, they assume that the government announces at the initial period the tax rule that is applicable to all firms, at all times. Second, the regulated market equilibrium is calculated as a Markov perfect Nash equilibrium. Finally, the parameters of the linear tax rule are computed imposing the output strategy selected by the firms to be equal to the efficient strategy. The authors show that the equations system defined by this condition has a unique solution and that the tax increases with the pollution stock and could be negative for low levels of the stock if there are just a few firms.

In Benchekroun and Long's papers the only way to control emissions is reducing production. In our paper, we extend the analysis of emission taxation for a polluting monopoly to consider the possibility that the firm devotes resources to abatement activities. With the aim of characterizing the optimal tax rule under these circumstances, we calculate a stagewise feedback Stackelberg equilibrium (SFSE) where the regulator is the leader and the monopolist is the follower of the game. Following a dynamic programming approach, the SFSE gives the leader only a stagewise first-mover advantage. In continuous time, this translates into an instantaneous advantage at each moment of time (period).[1] In our model, this equilibrium is time consistent in the sense defined in the seminal paper by Kydland and Prescott (1977) and also satisfies subgame perfection. However, it is not clear whether these properties of the SFSE eliminate or not the credible commitment problem of the Stackelberg game played in each period. To complete the analysis of emission taxation we investigate this issue proposing a stronger definition of time consistency that we call *intratime consistency.* We say that the optimal tax is intratime consistent if

[1]For a clear explanation of this type of feedback Stackelberg equilibrium, the interested reader can consult the excellent book by Haurie et al. (2012). Several examples can be found in Long (2010).

it is also time consistent for the Stackelberg game that the regulator and the monopolist play in each period. To the best of our knowledge, this requirement has not been taken into account in the previous literature. To check the intratime consistency of the SFSE, we calculate a Markov perfect equilibrium with limited commitment (LCMPE) where the regulator and the monopolist first simultaneously choose the emission tax and abatement respectively, before the monopolist selects the production level.[2]

The results establish that the tax rule of the SFSE is not intratime consistent. Thus, if the regulator is unable to commit in each period there exists an incentive to deviate from the optimal tax rule corresponding to the SFSE. This means, at least in our model, that the time consistency does not guarantee the intratime consistency although the opposite is true. Moreover, as expected the tax rule does not implement the efficient outcome. With two control variables, production and abatement, two policy instruments are necessary to induce the firm to act efficiently. In other words, restricting to the analysis of emission taxation we are placing our study in a second-best policy setting.

To advance in the analysis, we solve in the second part of the paper a linear-quadratic (LQ) differential game for an end-of-the-pipe abatement technology. The comparison of equilibria establishes that the steady state for the pollution stock with commitment is lower than with limited commitment and consequently the steady-state emissions are also lower.[3] Therefore, a reduced commitment increases the accumulation of emissions in the environment yielding larger damages at the steady state. Moreover, we find that the tax rule with commitment gives larger taxes for all values of pollution stock lower than the steady-state value of the pollution stock for the limited commitment equilibrium. In order words, a reduced commitment moves down the tax rule used by the regulator. In fact, it can lead to a change in the sign of the optimal policy converting a tax in a

[2]We would like to point out that when the regulator uses an emission tax to control emissions, a minimum of commitment is required to influence the economic behavior of the firm. This minimum requirement of commitment is given in our model by the fact that the production level is selected by the firm after the tax has been chosen by the regulator. For this reason we call to this equilibrium a limited commitment equilibrium.

[3]By commitment hereinafter we refer to the solution of the game given by the SFSE.

subsidy.[4]

The welfare consequences of a limited commitment cannot be determined analytically except when the initial pollution stock is zero. In this case, the comparison of the regulator's value functions shows that net social welfare is lower with limited commitment whereas net profits are larger. To complete the comparative analysis, we have developed a numerical example that shows that at the steady state the production is lower and the abatement larger for SFSE. This increases the consumer surplus and reduces the abatement costs for a limited commitment. However, the increase in production (gross emissions) and the decrease in abatement cause an increase in emissions that lead to larger pollution stocks and environmental damages resulting finally in a fall of the net social welfare. Nevertheless, it should be noticed that the numerical exercise also shows that this negative effect on welfare diminishes with abatement costs and that for large abatement costs both equilibria practically yield the same welfare level. Thus, our research indicates that a limited commitment has no cost in welfare terms if the abatement costs are large or reduces welfare if this is not the case. Then, we can conclude that the commitment value defined as the difference between the welfare of the commitment solution and the welfare of the solution with limited commitment is non negative.

1.1 Literature Review

The literature addressing the taxation of polluting firms with market power in a dynamic framework includes only a few papers: Xepapadeas (1992), Kort (1996), Stimming (1999) Feenstra et al. (2001) and Yanese (2009). All these papers except Yanase (2009) assume

[4] As in Benchekroun and Long (1998) we also find that the optimal tax with commitment increases with the stock of pollution but that it is negative when the pollution stock is low. Nevertheless, we show that if environmental damages are not very low the steady-state tax is positive. The subsidy operates to correct the market power of the firm when this distortion is more important than the distortion caused by the negative externality (pollution), i.e. when the pollution stock is low. However, for the limited commitment equilibrium it cannot be discarded that a subsidy applies at the steady state. In a dynamic model a subsidy is compatible with a positive abatement because this depends not only on the tax but also on the shadow price of the pollution stock that defines along with the tax the marginal benefit of abatement.

that damages are caused by a flow pollutant and focus on the investment in abatement technology. The environmental policy is exogenously determined and the research assesses the effects of a stricter environmental policy and the comparison of taxes vs emission standards. However, in Yanese (2009) the environmental policy is endogenously determined. The author examines a non-cooperative policy game between national governments in a model of international pollution control of a stock pollutant in which duopolists compete myopically in a third country and expense resources in abatement activities. The results establish that an emission tax produces a more distortionary outcome than that produced by a standard, i.e. it generates more pollution and lower welfare.

The literature on the time consistency of the environmental policy is much more abundant. This issue has been studied in a competitive framework by Biglaiser et al. (1995), Marsiliani and Renström (2000) and Abrego and Perroni (2002). Biglaiser et al. (1995) following a differential game approach where competitive firms invest in an abatement technology to reduce the emissions originated by a polluting input show that the first-best tax, the Pigovian tax, is time consistent although this is not the case for tradable permits. The authors focus on a flow pollutant and constant marginal damages. With constant marginal damages, the first-best tax is constant and the strategic effect of the investment vanishes what explains the time consistency of the Pigovian tax. The other authors, however, find in different contexts that the time-inconsistency is a problem of the environmental policy. In a framework of imperfect competition this issue has been addressed by Petrakis and Xepapadeas (1999, 2003), Poyago-Theotoky and Teerasuwannajak (2002), Puller (2006) and more recently by Moner-Colonques and Rubio (2016). All these papers study the effects of the lack of commitment for a flow pollutant in a static model obtaining different results. According to these results, the government commitment is not always welfare improving. They show that the effects of the time inconsistency of the environmental policy on investment and welfare depend on the features of the emission function, the degree of product differentiation and the policy instrument used to control emissions. This paper extends the analysis of the time consistency of an emission tax to cover the case of a stock pollutant.

Another strand of the literature has focused on the importance of commitment in a setting where a monopolistic upstream firm engages in R&D activities and sells abatement technology to polluting downstream firms (see, among others Laffont and Tirole (1996), Denicolò (1999), Requate (2005), Montero (2011) and Wirl (2014)).[5] Wirl (2014) develops the analysis in a dynamic game solving a differential policy game between a monopoly that provides a clean technology for a polluting competitive industry and a regulator that uses an emission tax or emission permits to control a flow pollutant. The author finds that although the monopoly can be enforced to price taking behavior, the first-best policy is time inconsistent. The inability of the regulator to commit leads to too slow and to too little expansion of the clean technology regardless of the instrument applied to control pollution. In contrast with our model, in Wirl's (2014) policy game the tax does not depend on the investment and vice versa and consequently the stagewise feedback Stackelberg equilibrium coincides with the Markov perfect Nash equilibrium. Thus, we could say that the SFSE is intratime consistent but because moving first does not give any strategic advantage to the regulator in his model.

Other papers with uncertainty or asymmetric information where environmental policy is compared under commitment and no commitment are Tarui and Polasky (2005), Ulph and Ulph (2013) and D'Amato and Dijkstra (2015). Tarui and Polasky (2005) and D'Amato and Dijkstra (2015) abstract for the market conditions where the polluting firms operate assuming that the target of the firm is to minimize the abatement costs plus the investment costs and regulation costs. Ulph and Ulph (2013) assume that a representative firm maximizes consumer surplus.

Finally, we would like to point out that the problem of credible commitment in climate policy has also been addressed by several scholars. A nice survey is Brunner at al. (2012). More recently, Bertinelli et al. (2017), Gerlagh and Liski (2017) and Rezai and van der Ploeg (2017) have studied the impact of commitment on climate policies in different settings. Bertinelli et al. (2017) solves a differential game between two countries with heterogeneous strategies to address the consequences of unilateral commitment.

[5] Golombek at al. (2010) consider that the supply of abatement equipment services are monopolistic competitive. They show that if the government can optimally subsidize R&D today, there is no time inconsistency problem.

Gerlagh and Liski (2017) and Rezai and van der Ploeg (2017) analyze the consequences of commitment in the setting of integrated assessment models. Gerlagh and Liski (2017) find that commitment is not welfare improving.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the stagewise feedback Stackelberg equilibrium and Section 4 the limited commitment Markov perfect equilibrium. In Section 5 we compare the two equilibria for a linear-quadratic differential game to evaluate the effects of a limited commitment. Section 6 offers some concluding remarks and points out lines for future research.

2 The Model

We consider a monopoly that faces a market demand represented by the decreasing inverse demand function $P(q(t))$ where $q(t)$ is the output at time t. The production process generates pollution emissions, however the firm can devote resources to abatement activities represented by $w(t)$. In this case, both the emission and cost functions depend on the output and abatement effort. The emission function is represented by $s(q(t), w(t))$ with $\partial s/\partial q$ positive and $\partial s/\partial w$ negative, and the cost function by $C(q(t), w(t))$ with $\partial C/\partial q$ and $\partial C/\partial w$ positive. The second-order partial derivatives are assumed to be positive or zero. The focus of the paper is on a stock pollutant that evolves according to the following differential equation

$$\dot{x}(t) = s(t) - \delta x(t) = s(q(t), w(t)) - \delta x(t), \qquad x(0) = x_0 \geq 0, \tag{1}$$

where $x(t)$ stands for the pollution stock and $\delta > 0$ for the decay rate of pollution. The environmental damages are given by the function $D(x(t))$ that is assumed strictly convex. Thus, the policy game we analyze in this paper is a differential game between a welfare maximizing regulator and a profit maximizing monopolist. The regulator chooses the level of the tax rate to apply on emissions and the monopolist chooses the abatement effort and production. Finally, we would like to point out that the focus in this paper is on a *second-best emission tax.* As is well known, since there are two control variables to adjust because of the distortions that characterize a polluting monopoly, the regulator would need two instruments to implement the first-best or efficient solution: a subsidy per unit of production could be used to correct for market power and a tax on emissions to correct for the pollution externality. However, we assume that the menu of policy instruments the regulator can use is restricted to only one policy instrument, a tax on emissions.

In the next section, we calculate the stagewise feedback Stackelberg equilibrium (SFSE) and characterize the emission tax applied by the regulator when it moves first in each period of the game. In Section 4 we compute the equilibrium of the game when the regulator does not benefit from this strategic advantage.

3 The Stagewise Feedback Stackelberg Equilibrium

This equilibrium is based on the assumption that the regulator moves first in each period. To find the regulator's optimal policy, we apply backward induction, substituting the monopolist's reaction function in the regulator's Hamilton-Jacobi-Bellman (HJB) equation, and computing the optimal strategy by maximizing the right-hand side of this equation. The resulting outcome is a stagewise feedback Stackelberg solution, which is a Markov-perfect equilibrium. For this kind of equilibria no commitment is required for the entire temporal horizon. However, we would like to point out that in this case an intratime commitment is assumed because the regulator is the leader of the policy game played in each period.

The output selection occurs in the last stage. The monopolist chooses its output to maximize the discounted present value of net profits

$$\max_{\{q(t)\}} \int_0^{\infty} e^{-rt} \left\{ P(q(t))q(t) - C(q(t), w(t)) - \tau(t)s(q(t), w(t)) \right\} dt, \tag{2}$$

subject to differential equation (1) where r is the time discount rate and $\tau(t)$ the emission tax. We assume that the firm acts strategically at this stage because it is aware that the dynamics of the stock will be taken into account by the regulator to set up the tax.

The solution to this dynamic optimization problem must satisfy the following HJB equation

$$rV(x(t)) = \max_{\{q(t)\}} \{P(q(t))q(t) - C(q(t), w(t)) - \tau(t)s(q(t), w(t)) + V'(x(t))(s(q(t), w(t)) - \delta x(t))\}, \tag{3}$$

where $V(x(t))$ stands for the maximum discounted present value of profits for the current value, $x(t)$, of the pollution stock.

From the first-order condition for the maximization of the right-hand side of the HJB equation, we get[6]

$$P'q + P = \frac{\partial C}{\partial q} + (\tau - V'(x))\frac{\partial s}{\partial q}, \tag{4}$$

where the left-hand side of the condition stands for the marginal revenue and the right-hand side represents the marginal costs. These costs include the marginal cost of production, the tax and the shadow price of the pollution stock. The latter is given by the reduction in the present value of the firm's profits because of an increase in the pollution stock caused by an increase in production. Observe that the last two terms are multiplied by the effect of an increase in production on emissions. Condition (4) implicitly defines the dependence of the output with respect to the tax, abatement effort and pollution stock: $q(\tau, w, x)$.

In a second stage, the firm selects the level of abatement that maximizes the present value of net profits that can be written as follows

$$\max_{\{w\}} \int_0^\infty e^{-rt} \{P(q(\tau, w, x))q(\tau, w, x) - C(q(\tau, w, x), w) - \tau s(q(\tau, w, x), w)\} \, dt \tag{5}$$

where $q(\tau, w, x)$ is implicitly defined by (4).[7]

From the first-order condition for the maximization of the right-hand side of the HJB equation with respect to w taking into account condition (4), we get

$$\frac{\partial C}{\partial w} = -(\tau - V'(x))\frac{\partial s}{\partial w}. \tag{6}$$

[6] Time argument will be eliminated when no confusion arises.

[7] We could consider that both decisions output and abatement selections are taken simultaneously but we have distinguished them to facilitate the comparison with the timing of the Markov perfect equilibrium with limited committed.

The left-hand side represents the marginal cost of abatement and on the right-hand side appear the marginal benefits that includes the marginal reduction in taxes because of the reduction in emissions caused by an increase in abatement that is given by the tax rate, and the increase in the present value of the firm's profits because of the reduction in the stock caused by an increase in abatement. Notice that $V'(x)$ is a marginal cost when we are considering an increase in production, and it stands for a marginal benefit when we are evaluating an increase in abatement. The same occurs for the tax rate. Conditions (4) and (6) implicitly define the firm's strategies: $q(\tau, x)$ and $w(\tau, x)$.

In the first stage, the regulator selects the emission tax rate that maximizes net social welfare defined as the sum of consumer surplus and monopoly profits plus tax revenues minus environmental damages

$$\max_{\{\tau\}} \int_0^\infty e^{-rt} \left\{ \int_0^q P(y)dy - Pq(\tau, x) + \pi(q(\tau, x), w(\tau, x), \tau) \right.$$
$$\left. + \tau s(q(\tau, x), w(\tau, x)) - D(x) \right\} dt,$$

where π stands for the firm profits. Notice that consumer expense and firm revenue on one hand and firm tax expense and regulator tax revenue on the other hand, cancel out. Therefore, this optimization problem can be rewritten as

$$\max_{\{\tau\}} \int_0^\infty e^{-rt} \left\{ \int_0^q P(y)dy - C(q(\tau, x), w(\tau, x)) - D(x) \right\} dt. \tag{7}$$

The solution to this dynamic optimization problem must satisfy the following HJB equation

$$rW(x) = \max_{\{\tau\}} \left\{ \int_0^q P(y)dy - C(q(\tau, x), w(\tau, x)) - D(x) \right.$$
$$\left. + W'(x)(s(q(\tau, x), w(\tau, x)) - \delta x) \right\} \tag{8}$$

where $W(x)$ stands for the maximum discounted present value of the net social welfare for the current value, $x(t)$, of the pollution stock.

From the first-order condition for the maximization of the right-hand side of the HJB equation, we get

$$\left(P - \frac{\partial C}{\partial q} + W'(x)\frac{\partial s}{\partial q} \right) \frac{\partial q}{\partial \tau} - \left(\frac{\partial C}{\partial w} - W'(x)\frac{\partial s}{\partial w} \right) \frac{\partial w}{\partial \tau} = 0. \tag{9}$$

Doing the expressions between parenthesis equal to zero the efficient conditions are obtained. However, these conditions are incompatible with conditions (4) and (6) that characterize the maximization of the discounted present value of profits by the firm and the SFSE is inefficient. As expected, the second-best policy does not implement the efficient outcome.

Conditions (4) and (6) can be rewritten as

$$P - \frac{\partial C}{\partial q} = (\tau - V'(x))\frac{\partial s}{\partial q} - P'q,$$

and

$$\frac{\partial C}{\partial w} = -(\tau - V'(x))\frac{\partial s}{\partial w}.$$

Eliminating $P - \partial C/\partial q$ and $\partial C/\partial w$ in (9) using these expressions yields

$$\left(-P'q + (\tau - V'(x) + W'(x))\frac{\partial s}{\partial q}\right)\frac{\partial q}{\partial \tau} + (\tau - V'(x) + W'(x))\frac{\partial s}{\partial w}\frac{\partial w}{\partial \tau} = 0. \tag{10}$$

Taking common factor gives

$$\tau^c = \frac{-\frac{P}{|\eta|}\frac{\partial q}{\partial \tau}}{\frac{\partial s}{\partial q}\frac{\partial q}{\partial \tau} - \frac{\partial s}{\partial w}\frac{\partial w}{\partial \tau}} - (W'(x) - V'(x)), \tag{11}$$

where η is the price elasticity of demand and the superscript c stands for committed equilibrium. Expression (11) is the dynamic version of the condition that characterizes the second-best emission tax derived by Barnett (1980) for a flow pollutant. When the firm has not market power, the price elasticity is infinite and the first-best emission tax, the Pigouvian tax, is equal to the difference between the social shadow price of the pollution stock, $-W'(x)$, and the private shadow price, $-V'(x)$. However, for a monopoly the absolute value of the elasticity is positive and consequently the first term on the right-hand side of (11) is negative provided that $\partial q/\partial \tau$ is negative and $\partial w/\partial \tau$ is positive and then we can conclude that[8]

Proposition 1 *The second-best emission tax with commitment is lower than the difference between the social and the private valuations of the pollution stock whenever the tax reduces the gross and net emissions.*

[8] Although these are the expected signs for these partial derivatives, they depend on the sign of cross effects (cross second-order partial derivatives) of the cost and emission functions.

Moreover, expressions (10) and (11) allow us to interpret condition (9). Notice that according to (11), $\tau - V'(x) + W'(x)$ must be negative. Then, if the tax has a negative impact on emissions and production, the first term in brackets in (10) must be positive and consequently the first term between parenthesis in (9) must be positive and the second term negative. Thus, the optimal tax balances the reduction in consumer surplus net of the social shadow price of the pollution stock caused by the decrease in output induced by the increase in the tax rate with the increase in welfare caused by the augmentation in abatement provoked by the increase in the tax rate. This increase in welfare is explained by the fact that at the equilibrium the marginal cost of abatement is lower that the social shadow price of the pollution stock that represents the increase in the discounted present value of welfare because of a decrease in the stock. Thus, when the abatement increases the reduction in the social shadow price of the pollution stock more than compensated the augmentation in abatement costs.

4 The Markov Perfect Equilibrium with Limited Commitment

With a reduced commitment, the regulator will choose the intratime consistent tax rate after the firm has selected the level of abatement. This means that the monopolist moves first in each period and that it could use this strategic advantage to influence the environmental policy in its own interest. In this section, we compute the equilibrium of this game to evaluate the consequences of the lack of commitment.

We assume as in the previous section that output selection occurs after the choice of the tax and abatement. Thus, no changes occur at this stage with respect to the previous game and the first-order condition (4) also characterizes the LCMPE. However, now in the second stage is the regulator who selects the level of the tax rate that maximizes net social welfare

$$\max_{\{\tau\}} \int_0^\infty e^{-rt} \left\{ \int_0^q P(y)dy - C(q(\tau, w, x), w) - D(x) \right\} dt.$$

where $q(\tau, w, x)$ is implicitly defined by the condition (4). Notice that with a limited commitment the regulator cannot influence the abatement. The solution to this dynamic optimization problem must satisfy the following HJB equation

$$rW(x) = \max_{\{\tau\}} \left\{ \int_0^q P(y)dy - C(q(\tau, w, x), w) - D(x) \right.$$
$$\left. + W'(x)(s(q(\tau, w, x), w) - \delta x) \right\}. \tag{12}$$

From the first-order condition for the maximization of the right-hand side of the HJB equation, we obtain the following expression

$$\left(P - \frac{\partial C}{\partial q} + W'(x) \frac{\partial s}{\partial q} \right) \frac{\partial q}{\partial \tau} = 0. \tag{13}$$

The same condition can be directly derived from (9) adding the requirement that $\partial w / \partial \tau = 0$. Using (4), we can characterize the intratime consistent emission tax[9]

$$\tau^{nc} = \frac{-\frac{P}{|\eta|}}{\frac{\partial s}{\partial q}} - (W'(x) - V'(x)). \tag{14}$$

This condition can be directly derived from (11) for $\partial w / \partial \tau = 0$. The intratime consistent emission tax is also lower than the difference between the social shadow price of the pollution stock and the private shadow price because $\partial s / \partial q$ is negative. In this case this result applies regardless the sign of the effect of the tax on the production and abatement. Thus, we can conclude that

Proposition 2 *The second-best emission tax with limited commitment is lower than the difference between the social and the private valuations of the pollution stock.*

When the regulator has a limited commitment, in the first stage, the firm selects the abatement level that maximizes the discount present value of profits taking into account the time evolution of the pollution stock.

$$\max_{\{w\}} \int_0^\infty e^{-rt} \left\{ P(q(w, x))q(w, x) - C(q(w, x), w) - \tau(w, x)s(q(w, x), w) \right\} dt$$

subject to differential equation (1) where $q(w, x)$ and $\tau(w, x)$ are defined by (4) and (13).

[9] Where the superscript nc is used to represent the solution of the game with limited commitment.

From the first-order condition for the maximization of the right-hand side of the HJB equation and taking into account condition (4), we obtain

$$\frac{\partial C}{\partial w}=-(\tau-V'(x))\frac{\partial s}{\partial w}-\frac{\partial \tau}{\partial w}s, \tag{15}$$

where $\partial\tau/\partial w$ is the strategic effect that measures the influence that the firm can exert on the environmental policy through the abatement.

Thus, comparing conditions (13) and (15) that characterizes the LCMPE with conditions (6) and (9) that the SFSE must satisfied we can conclude that the two equilibria will not coincide and that consequenlty

Proposition 3 *The SFSE is not intratime consistent.*

In other words, if the regulator is unable to commit in each period there is an incentive to deviate from the optimal tax rule corresponding to the SFSE.

Condition (15) can be rewritten as follows

$$\frac{\partial C}{\partial w}=V'(x)\frac{\partial s}{\partial w}-\left(\tau\frac{\partial s}{\partial w}+\frac{\partial \tau}{\partial w}s\right)=V'(x)\frac{\partial s}{\partial w}-\frac{\partial T}{\partial w}, \tag{16}$$

where $T=\tau s$ stands for the firm's taxes.

When the firm chooses the abatement before the regulator sets up the tax, the firm will use the abatement strategically to influence on its fiscal spending. Again the sign of this strategic effect depends on the cross-effects of the cost and emission functions. Nevertheless, we would like to highlight that the no coincidence of the two equilibria and consequently our Proposition 3 does not depend critically on this strategic effect. Suppose that $\partial\tau/\partial w$ is zero, as it occurs in the LQ policy game we analyze below, then condition (15) of the LCMPE coincides with condition (6) of the SFSE but the conditions that characterize the regulator's behavior are different yielding two different equilibria. The intratime inconsistency of the SFSE occurs even if the firm cannot use its strategic advantage to influence the environmental policy.

5 The LQ Policy Game

The LQ differential game we analyze in this section is an extension of the LQ differential game studied by Benchekroun and Long (1998) to include abatement activities. It considers a monopolist that faces a linear (inverse) demand function given by $P(t) = a - q(t)$, where $P(t)$ is the price and $q(t)$ is the output at time t. The production process generates pollution emissions. After an appropriate choice of measurement units we can say that each unit of output generates one unit of pollution. The emissions can be reduced without declining output if the monopoly employs an abatement technology. The abatement technology is assumed to be the end-of-the-pipe type.[10] For this type of abatement technology the emission function is $s(q(t), w(t)) = q(t) - w(t)$ where $w(t)$ stands for the emission reduction achieved operating the abatement technology. On the other hand, we assume an additive and separable cost function $C(q(t), w(t)) = cq(t) + \gamma w(t)^2/2$. The abatement technology has decreasing returns to scale, with the parameter γ measuring the extent of such decreasing returns, and the production technology presents constant returns to scale, with the parameter c standing for the marginal cost of production. In this case, the stock of pollution follows the dynamic equation

$$\dot{x}(t) = s(q(t), w(t)) - \delta x(t) = q(t) - w(t) - \delta x(t), \quad x(0) = x_0 \geq 0. \tag{17}$$

The disutility from environmental deterioration is given by the damage function $D(x(t)) = dx(t)^2/2,\ d > 0$.

Next, we calculate the SFSE of this differential game and characterize the optimal tax with commitment.

[10] This assumption has been extensively used in the literature among others by Petrakis and Xepapadeas (2003), Poyago-Theotoky and Teerasuwannajak (2002) and more recently by Moner-Colonques and Rubio (2016) in a static context. In a dynamic context has been used for instance by Yanese (2009).

5.1 The Stagewise Feedback Stackelberg Equilibrium

For this LQ policy game, the first-order conditions (4) and (6) define the dependence of the production and abatement with respect to the tax

$$
\begin{aligned}
q(\tau, x) &= \frac{1}{2}\left(a - c - (\tau - V'(x))\right), &(18)\\
w(\tau, x) &= \frac{1}{\gamma}(\tau - V'(x)). &(19)
\end{aligned}
$$

Therefore, $\partial q/\partial \tau = -1/2$ and $\partial w/\partial \tau = 1/\gamma$, and consequently Prop. 1 holds, i.e. the second-best emission tax with commitment is lower than the difference between the social and the private valuations of the pollution stock. Taking into account these partial derivatives, condition (9) defines the optimal tax rule

$$
\tau(x) = V'(x) - \frac{\gamma(a-c) + 2(\gamma+2)W'(x)}{\gamma+4}. \tag{20}
$$

Next, substituting $\tau - V'$ in (18) and (19) the optimal strategies for the output and abatement read

$$
\begin{aligned}
q(x) &= \frac{\gamma+2}{\gamma+4}(a - c + W'(x)), &(21)\\
w(x) &= -\frac{\gamma(a-c) + 2(\gamma+2)W'(x)}{\gamma(\gamma+4)}. &(22)
\end{aligned}
$$

Notice that as both output and abatement depend on $\tau - V'$, finally the optimal strategies of these two control variables are independent of the first derivative of the monopolist's value function. The emissions can be obtained as the difference between output (gross emissions) and abatement

$$
s(x) = q(x) - w(x) = \frac{\gamma(\gamma+3)(a-c) + (\gamma+2)^2 W'(x)}{\gamma(\gamma+4)}. \tag{23}
$$

Now, substituting the optimal strategies (21) and (22) in the regulator's HJB equation (8), the following nonlinear differential equation is obtained

$$
rW(x) = \frac{\gamma+3}{2(\gamma+4)}(a-c)^2 + \frac{\gamma+3}{\gamma+4}(a-c)W'(x) + \frac{(\gamma+2)^2}{2\gamma(\gamma+4)}W'(x)^2 - \frac{d}{2}x^2 - \delta x W'(x). \tag{24}
$$

In order to solve this equation, we guess a quadratic representation for the value function W:

$$W^c(x) = \frac{A_1^c}{2}x^2 + B_1^c x + C_1^c, \tag{25}$$

which implies that $dW^c(x)/dx = A_1^c x + B_1^c$ and where A_1^c, B_1^c and C_1^c are unknowns to be determined.[11]

The substitution of $W^c(x)$ and $dW^c(x)/dx$ into (24) yields a system of Riccati equations that must hold for every x. Then if this system of equations for the coefficients of the value function has a solution, the optimal strategies for output and abatement would be

$$\begin{aligned} q^c(x) &= \frac{\gamma+2}{\gamma+4}\left(a - c + B_1^c + A_1^c x\right), & (26)\\ w^c(x) &= -\frac{1}{\gamma(\gamma+4)}\left(\gamma(a-c) + 2(\gamma+2)B_1^c + 2(\gamma+2)A_1^c x\right), & (27) \end{aligned}$$

which are obtained from (21) and (22). Finally, we obtain the dynamics of the state variable in terms of the coefficients of the value function substituting (26) and (27) in (17)

$$\dot{x} = \frac{\gamma+3}{\gamma+4}(a-c) + \frac{(\gamma+2)^2}{\gamma(\gamma+4)}B_1^c + \left(\frac{(\gamma+2)^2}{\gamma(\gamma+4)}A_1^c - \delta\right)x. \tag{28}$$

Thus, if we look for a stable solution, the following condition should be satisfied

$$\frac{d\dot{x}}{dx} < 0 \rightarrow \frac{d\dot{x}}{dx} = \frac{(\gamma+2)^2}{\gamma(\gamma+4)}A_1^c - \delta < 0.$$

Applying this stability condition, we find that the system of Riccati equations has only one stable solution given by the following values for the coefficients of the regulator's value function

$$\begin{aligned} A_1^c &= \frac{\gamma(\gamma+4)(r+2\delta) - \sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2} < 0, & (29)\\ B_1^c &= \frac{\gamma(\gamma+3)(a-c)A_1^c}{\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c} < 0, & (30)\\ C_1^c &= \frac{(\gamma+3)(a-c)^2[\gamma\,(\gamma+4)\,(r+\delta)\,(\gamma(r+\delta) - 2A_1^c) + (\gamma+2)^2(A_1^c)^2]}{2r(\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c)^2} > 0. & (31) \end{aligned}$$

[11] Recall that the superscript c stands for commitment and is associated with the solution of the game given by the SFSE.

Using these coefficients and taking into account the Riccati equation for A_1^c, the steady-state pollution stock can be calculated resulting in

$$x_{SS}^c = \frac{(a-c)\gamma(\gamma+3)(r+\delta)}{(\gamma+2)^2 d + \gamma\delta(\gamma+4)(r+\delta)}. \tag{32}$$

This expression clearly establishes an inverse relationship between the pollution stock at the steady state and d, the slope of the marginal damages curve. Thus, we can conclude the larger the marginal damages the lower the accumulation of emissions at the steady state.

Moreover, the optimal strategies for output, abatement and emissions are

$$q^c(x) = \frac{(a-c)(\gamma+2)(A_1^c - \gamma(r+\delta))}{(\gamma+2)^2 A_1^c - \gamma(\gamma+4)(r+\delta)} + \frac{(\gamma+2)A_1^c}{\gamma+4}x, \tag{33}$$

$$w^c(x) = \frac{(a-c)(\gamma(r+\delta) + (\gamma+2)A_1^c)}{(\gamma+2)^2 A_1^c - \gamma(\gamma+4)(r+\delta)} - \frac{2(\gamma+2)A_1^c}{\gamma(\gamma+4)}x, \tag{34}$$

$$s^c(x) = \frac{(a-c)\gamma(\gamma+3)(r+\delta)}{\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c} + \frac{(\gamma+2)^2 A_1^c}{\gamma(\gamma+4)}x, \tag{35}$$

where $A_1^c < 0$ is given by (29). It is easy to show that if d is larger than the critical value

$$d_w^c = \frac{\gamma(r+\delta)[(r+\delta)(\gamma+2) + (\gamma+4)(r+2\delta)]}{(\gamma+4)(\gamma+2)}, \tag{36}$$

the abatement is positive for all x. Then, we can conclude that

Proposition 4 *If d is larger than d_w^c and the pollution stock is lower than*

$$x_s^c = \frac{\gamma(a-c)(\gamma+3)(r+\delta)}{(\gamma+2)^2(\delta A_1^c + d)} > x_{SS}^c,$$

then the production and emissions decrease and the abatement increases with the pollution stock.

Proof. See Appendix. ∎

⇒ FIGURE 1 ⇐

In Fig. 1 we have represented the optimal strategies and the steady state. The figure also illustrates the dynamics of the model. If the initial pollution stock is lower than the steady-state pollution stock, x^c_{SS}, the production is decreasing and the abatement is increasing resulting in decreasing emissions. The emissions are larger than the natural decay of pollution stock and the stock of pollution increases until the steady state is reached. If the initial pollution stock is larger than x^c_{SS} but lower than x^c_s, the production is increasing and the abatement decreasing yielding increasing emissions. In this case, the emissions are lower than the natural decay and the pollution stock decreases until the steady state is reached.[12]

5.1.1 The Second-Best Emission Tax with Commitment

The features of the model allow to calculate the optimal strategies for production, abatement and net emissions without solving the monopolist's HJB equation. However, the next step, the calculation of the regulator optimal policy, cannot be given without solving this equation. With this aim, we substitute the tax given by (20), the output defined by (21) and the abatement specified by (22) in the monopolist's HJB equation given by (3) obtaining the following differential equation

$$\begin{aligned} rV(x) &= \frac{2\gamma^2+9\gamma+8}{2(\gamma+4)^2}(a-c)^2 + \frac{2\,(\gamma+3)\,(\gamma+2)}{(\gamma+4)^2}(a-c)W'(x) \\ &+ \frac{(\gamma+2)^3}{\gamma(\gamma+4)^2}W'(x)^2 - V'(x)\delta x. \end{aligned} \tag{37}$$

In order to solve this equation, we also guess a quadratic representation

$$V^c(x) = \frac{A^c_2}{2}x^2 + B^c_2 x + C^c_2, \tag{38}$$

that yields $dV^c(x)/dx = A^c_2 x + B^c_2$. The substitution of $V^c(x)$ and $dV^c(x)/dx$ into (37) yields a system of Riccati equations whose solution is

$$A^c_2 = \frac{2(\gamma+2)^3}{\gamma(r+2\delta)(\gamma+4)^2}(A^c_1)^2 > 0, \tag{39}$$

$$B^c_2 = \frac{2(a-c)\gamma\,(\gamma+3)\,(\gamma+2)A^c_1}{(\gamma+4)(\gamma(\gamma+4)(r+\delta)-(\gamma+2)^2 A^c_1)} < 0, \tag{40}$$

[12] We bound the feasible values for d and x to guarantee that the control variables take non-negative values. To keep the model as simple as possible we do not address the characterization of corner solutions in this paper.

$$C_2^c = \frac{(a-c)^2[(2\gamma^2+9\gamma+8)\gamma^2(r+\delta)^2 - 2\gamma(\gamma+4)(\gamma+2)(r+\delta)A_1^c + (\gamma+2)^3(A_1^c)^2]}{2r(\gamma(\gamma+4)(r+\delta)-(\gamma+2)^2A_1^c)^2} > 0. \tag{41}$$

Then eliminating $V'(x)$ and $W'(x)$ in (20) using the coefficients of the value functions, the optimal policy is obtained.[13]

Proposition 5 *The optimal policy is given by the following rule*

$$\tau^c(x) = -\frac{\gamma(a-c)}{\gamma+4} + \frac{2(\gamma+2)d}{(r+2\delta)(\gamma+4)}x. \tag{42}$$

If environmental damages are large enough, in particular if d is larger than

$$d_\tau^c = \frac{\gamma\delta(\gamma+4)(r+2\delta)(r+\delta)}{(r(\gamma+4)+2\delta)(\gamma+2)},$$

there exists a threshold value for the stock of pollution, x_τ^c, given by the following expression

$$x_\tau^c = \frac{\gamma(a-c)(r+2\delta)}{2(\gamma+2)d} < x_{SS}^c$$

such that the optimal policy consists of applying a decreasing subsidy for $x < x_\tau^c$ and an increasing tax for $x > x_\tau^c$.

Proof. The value of the pollution stock x_τ^c is calculated doing $\tau^c(x) = 0$ and the difference of this value with the steady-state pollution stock is

$$x_{SS}^c - x_\tau^c = \frac{(a-c)\gamma[(r(\gamma+4)+2\delta)(\gamma+2)d - \gamma\delta(\gamma+4)(r+2\delta)(r+\delta)]}{2(\gamma+2)[d(\gamma+2)^2d + \gamma(\gamma+4)(r+\delta)\delta]d}$$

that is positive for $d > d_\tau^c$. ■

The intuition of this result is straightforward. When the pollution stock is zero, the marginal damages are also zero and the inefficiency of the monopoly is caused only by its market power. It is well known that in this case the monopoly reduces its output to take advantage of a larger price selecting a level of production lower than the efficient

[13] Notice that the monopolist's value function has a minimum for a positive value of the pollution stock. It is easy to check that this minimum is larger than x_s^c ensuring that dV^c/dx is negative in the interval $[0, x_s^c]$.

level. Then, the optimal policy consists of setting up a subsidy to stimulate production.[14] Thus, when the pollution stock is zero or when is low the environmental problem is not relevant and the regulator applies exclusively an industrial policy. However, once the emissions accumulate causing environmental damages, the inefficiency of the polluting monopoly is also caused by a negative externality. In other words, there are two market failures operating at the same time. The point is that a negative externality induces the firm to produce more than the efficient level and in this case, as is also well known, the optimal policy, when the firm is competitive, consists of applying an emission tax to reduce the firm's output and emissions. Thus, the sign of the optimal policy applied by the regulator when the two market failures are acting at the same time can be negative (a subsidy) or positive (a tax) depending on the stock pollution level and also on the importance of the marginal damages. Prop. 5 defines a threshold value for d that implies that a tax is applied at the steady state. In other words, it implies that environmental damages are serious enough to justify that the environmental policy (taxation) dominates the industrial policy (subsidization) at the steady state.

The comparison of d_w^c and d_τ^c yields an ambiguous sign. For this reason, we assume that $d > \max\{d_w^c, d_\tau^c\}$. The consequences of this assumption are that the non-negative constraint is satisfied by the control variables of the model in the interval $[0, x_s^c]$ because $d > d_w^c$, and that the optimal policy at the steady state consists of setting a tax on emissions given that $d > d_\tau^c$.

5.2 The Markov Perfect Equilibrium with Limited Commitment

For this equilibrium, condition (13) directly defines the optimal strategy from production

$$q(x) = a - c + W'(x). \tag{43}$$

[14] Observe that a subsidy is compatible with a positive abatement effort because it depends not only on the tax but also on the shadow price of the pollution stock. According to (6), w depends on the difference $\tau - V'$ where V' is negative so that this difference can be positive even when τ is negative.

Substituting in condition (4) gives the tax rule

$$\tau(x) = V'(x) - (a - c) - 2W'(x). \tag{44}$$

Notice that the tax does not depend on abatement which implies that production does not depend on abatement either. This independence of the tax with respect to the abatement eliminates the possibility of using the abatement in the first stage for influencing the tax. Thus, for our model we may define the LCMPE as the solution of a two-stage game where in the first stage the regulator and the firm decide simultaneously the tax rate and abatement respectively and in the second stage the firm chooses the level of production.[15] In this case, condition (15) that characterizes the optimal abatement for the LCMPE coincides with the condition (6) that characterizes the optimal abatement for the SFSE. Therefore, $w(\tau, x)$ is again given by (19).

Finally, eliminating $\tau - V'$ in (19) using (44), we obtain the optimal strategy for abatement

$$w(x) = -\frac{1}{\gamma}(a - c + 2W'(x)). \tag{45}$$

Notice that again the optimal strategies of both variables are independent of the first derivatives of the monopolist's value function. The emission can be calculated as the difference between output (gross emissions) and abatement yielding

$$s(x) = q(x) - w(x) = \frac{1}{\gamma}\left((\gamma + 1)(a - c) + (\gamma + 2)W'(x)\right). \tag{46}$$

Now, substituting the optimal strategies (43) and (45) in the HJB equation (8), the following nonlinear differential equation for the regulator's value function is obtained

$$rW(x) = \frac{\gamma - 1}{2\gamma}(a - c)^2 + \frac{\gamma - 1}{\gamma}(a - c)W'(x) + \frac{1}{2}W'(x)^2 - \frac{d}{2}x^2 - \delta x W'(x). \tag{47}$$

We also guess in this section a quadratic representation for the value function W[16]:

$$W^{nc}(x) = \frac{A_1^{nc}}{2}x^2 + B_1^{nc}x + C_1^{nc}, \tag{48}$$

[15] As we have already pointed out, the possibility of using the abatement to influence the emission tax depends on the sign of the cross effects between production and abatement in the emission and cost functions. With an additive and separable cost function and a linear emission function, these cross effects are zero and the equilibrium in the first stage is an equilibrium in dominant strategies.

[16] Recall that the superscript nc stands for limited commitment and is associated with the solution of the game given by the LCMPE.

whose first derivative is $dW^{nc}(x)/dx = A_1^{nc}x + B_1^{nc}$.

The substitution of the first derivative and the proposed value function in (47) gives a system of Riccati equations. If this system of equations for the coefficients of the value function has a solution, the optimal strategies for the output and the abatement are

$$q^{nc}(x) = a - c + B_1^{nc} + A_1^{nc}x, \tag{49}$$
$$w^{nc}(x) = -\frac{1}{\gamma}\left(a - c + 2B_1^{nc} + 2A_1^{nc}\right), \tag{50}$$

that are derived from (43) and (45) by substitution of the first derivative of the value function. Using these two optimal strategies to eliminate the output and the abatement in (17), the dynamics of the pollution stock is

$$\dot{x}^{nc} = \frac{1+\gamma}{\gamma}(a-c) + \frac{\gamma+2}{\gamma}B_1^{nc} + \left(\frac{\gamma+2}{\gamma}A_1^{nc} - \delta\right)x, \tag{51}$$

so that the stability condition implies the following constraint on A_1^{nc}

$$\frac{d\dot{x}}{dx} < 0 \rightarrow \frac{d\dot{x}}{dx} = \frac{\gamma+2}{\gamma}A_1^{nc} - \delta < 0.$$

Only one of the roots of the first equation of the system of Riccati equations satisfies this constraint

$$A_1^{nc} = \frac{1}{2}\left(r + 2\delta - \sqrt{(r+2\delta)^2 + 4d}\right) < 0. \tag{52}$$

The other two coefficients of the value functions can be written as a function of A_1^{nc}

$$B_1^{nc} = \frac{(\gamma-1)(a-c)A_1^{nc}}{\gamma(r+\delta-A_1^{nc})} < 0 \text{ for } \gamma > 1, \tag{53}$$
$$C_1^{nc} = \frac{(\gamma-1)(a-c)^2(\gamma(r+\delta)^2 - 2(r+\delta)A_1^{nc} + (A_1^{nc})^2)}{2r\gamma^2(r+\delta-A_1^{nc})^2} > 0. \tag{54}$$

For this solution of the Riccati equations, the steady-state pollution stock is

$$x_{SS}^{nc} = \frac{(a-c)(\gamma(\gamma+1)(r+\delta) - 2A_1^{nc})}{\gamma(r+\delta-A_1^{nc})(\gamma\delta - (\gamma+2)A_1^{nc})} > 0. \tag{55}$$

The optimal strategies for production, abatement and emissions are

$$q^{nc}(x) = \frac{(a-c)(\gamma(r+\delta) - A_1^{nc})}{\gamma(r+\delta-A_1^{nc})} + A_1^{nc}x, \tag{56}$$
$$w^{nc}(x) = -\frac{(a-c)(\gamma(r+\delta) + (\gamma-2)A_1^{nc})}{\gamma^2(r+\delta-A_1^{nc})} - \frac{2A_1^{nc}}{\gamma}x, \tag{57}$$
$$s^{nc}(x) = \frac{(a-c)(\gamma(\gamma+1)(r+\delta) - 2A_1^{nc})}{\gamma^2(r+\delta-A_1^{nc})} + \frac{(\gamma+2)A_1^{nc}}{\gamma}x, \tag{58}$$

where $A_1^{nc} < 0$ is given by (52). Using the optimal strategy for the abatement, it is easy to check that if γ is larger than 2 and environmental damages are large enough, in particular if d is larger than

$$d_w^{nc} = \frac{\gamma^2(r+\delta)^2 + \gamma(r+\delta)(2\delta+r)(\gamma-2)}{(\gamma-2)^2}, \tag{59}$$

the abatement is positive for all x. Then, we can conclude that

Proposition 6 *If γ is larger than 2, d larger than d_w^{nc} and the pollution stock is lower than*

$$x_s^{nc} = \frac{(a-c)(\gamma(\gamma+1)(r+\delta) - 2A_1^{nc})}{\gamma(\gamma+2)(d+\delta A_1^{nc})} > x_{SS}^{nc},$$

the production and emissions decrease and the abatement increases with the pollution stock.

The proof of this proposition follows step by step the proof of Prop. 4. For this reason we omit it. Notice that the optimal strategies for the LCMPE present the same qualitative features that those derived in the previous section for the SFSE.

5.2.1 The Second-Best Emission Tax with Limited Commitment

As occurs for the solution of the game with commitment, the optimal strategies for production, abatement and net emissions can be computed without solving the monopolist's HJB equation. However, to derive the optimal policy it is necessary to solve this equation. With this aim, we substitute the tax given by (44), the output defined by (43) and the abatement specified by (45) in the monopolist's HJB equation obtaining the following differential equation for the monopolist's value function

$$rV(x) = \frac{1+2\gamma}{2\gamma}(a-c)^2 + \frac{2(\gamma+1)}{\gamma}(a-c)W'(x) + \frac{\gamma+2}{\gamma}W'(x)^2 - V'(x)\delta x. \tag{60}$$

We also guess a quadratic representation for this case

$$V^{nc}(x) = \frac{A_2^{nc}}{2}x^2 + B_2^{nc}x + C_2^{nc},$$

whose first derivative is $dV^{nc}(x)/dx = A_2^{nc}x+B_2^{nc}$. The substitution of $V^{nc}(x),\ dV^{nc}(x)/dx$ and also $dW^{nc}(x)/dx$ into (60) results in a system of Riccati equations whose solution is

$$A_2^{nc} = \frac{2(\gamma+2)}{\gamma(r+2\delta)}(A_1^{nc})^2 > 0, \tag{61}$$

$$B_2^{nc} = \frac{2(a-c)((\gamma+1)(r+\delta)\gamma A_1^{nc} - 2(A_1^{nc})^2)}{\gamma^2(r+\delta)(r+\delta-A_1^{nc})} < 0, \tag{62}$$

$$C_2^{nc} = \frac{(a-c)^2[\gamma^2(1+2\gamma)(r+\delta)^2 - 2\gamma(\gamma+2)(r+\delta)A_1^{nc} + (\gamma^2-2\gamma+4)(A_1^{nc})^2]}{2r\gamma^3(r+\delta-A_1^{nc})^2} > 0. \tag{63}$$

Next, we derive the optimal policy eliminating $V'(x)$ and $W'(x)$ using the coefficients of the value functions.[17]

Proposition 7 *The optimal policy is given by the following rule*

$$\begin{aligned}\tau^{nc}(x) &= \frac{(a-c)(\gamma^2(r+\delta)^2 - \gamma(\gamma+4)(r+\delta)A_1^{nc} + 4(A_1^{nc})^2)}{\gamma^2(r+\delta)(A_1^{nc}-r-\delta)} \\ &+ \frac{2(\gamma+2)(A_1^{nc})^2 - 2\gamma(r+2\delta)A_1^{nc}}{\gamma(r+2\delta)}x. \end{aligned}\tag{64}$$

where A_1^{nc} is given by (52). $\tau^{nc}(x)$ increases with the pollution stock but is negative for $x=0$, i.e. the optimal policy consists of setting up a subsidy for $x=0$.

Although in this case it is not possible to derive an *explicit* value for the parameter d above which the optimal policy at the steady state is a tax, the fact that τ^{nc} increases with the pollution stock suggests that this is a feasible outcome of the game with limited commitment. The intuition behind this result is the same that in the case of with commitment. Notice that there are no qualitative differences between the optimal strategies obtained for both policy games: output and emissions decrease with the pollution stock, whereas abatement and the tax augment. In this section is assumed that $\gamma > 2$ and that $d > d_w^{nc}$. These two assumptions guarantee that abatement is positive for $x = 0$ so that we can conclude that the control variables of the model satisfy the non-negative constraint in the interval $[0, x_s^{nc}]$.[18]

[17] Again the monopolist's value function has a minimum for a positive value of the pollution stock that is larger than x_s^{nc}, ensuring that dV^{nc}/dx is negative in the interval $[0, x_s^{nc}]$.

[18] It is easy to show that $d_w^{nc} > d_w^c$. Then if d is larger than d_w^{nc} is also larger than d_w^c and, according to Prop. 4, the optimal abatement is also positive for the SFSE.

5.3 Comparing the Optimal Emission Taxes

In this section, we investigate the effects of a limited commitment on the optimal policy and social welfare. To guarantee an interior solution we assume that $\gamma > 2$ and that $d > \max\{d_\tau^c, d_w^{nc}\}$.[19] We begin the comparison calculating the difference between the steady-state values of the pollution stock. The result is

Lemma 1 *The steady state for the pollution stock with commitment is lower than with limited commitment, i.e.* $x_{SS}^c < x_{SS}^{nc}$ *and consequently the steady-state emissions are also lower.*

Proof. See Appendix. ■

Next, we study the effects of a limited commitment on the optimal policy. First, we compare the steady-state values of the emission tax. The comparison gives the following result

Proposition 8 *If environmental damages are large enough, in particular if* d *is larger than*

$$d_{SS}^{\tau} = \frac{\gamma(\gamma+4)(r+\delta)(r+2\delta)(2\gamma^2(r+\delta)+2\gamma(3r-\delta)-16\delta)}{4(\gamma+2)(\gamma^2(r+3\delta)+\gamma(5r+13\delta)+8(r+2\delta))}, \tag{65}$$

then the steady-state tax with commitment is larger than the steady-state tax with limited commitment, i.e. $\tau^{nc}(x_{SS}^{nc}) < \tau^c(x_{SS}^c)$.

To show this result we calculate first the steady-state value of the emission tax with commitment substituting the steady-state value of pollution stock in the tax rule (42). Then using the tax rule of the LCMPE given by (64), we calculate the value of the pollution stock for which this tax rule yields the steady-state value of the emission tax with commitment, and show that this value of the pollution stock is larger than the steady-state pollution stock corresponding to the LCMPE. Thus, taking into account that the tax rules are increasing with respect to the pollution stock, it can be concluded

[19] Remember that $d > d_\tau^c$ guarantees that the optimal policy at the steady state is to tax emissions for the SFSE. Moreover, $d > d_w^{nc}$ along with $\gamma > 2$ yield a positive abatement for both the SFSE and LCMPE.

that the steady-state value of the emission tax with limited commitment must be lower than the value that the tax with commitment takes at the steady state. This is the sketch of the proof. Nevertheless, for the interested reader the details can be checked at the Appendix.

This proposition defines a sufficient condition so that this result could hold for values of d below the lower bound (65). The numerical exercise that closes this section shows that this condition is not very restrictive. Thus, we can expect that the steady-state tax rate with commitment is larger than the steady-state tax with limited commitment for a wide constellation of parameter values.

Now, we study the effects of a limited commitment on the optimal tax rule evaluating how the intersection point with the vertical axis, i.e. the subsidy for $x = 0$, and the slope of the optimal tax rule change. The results are

Lemma 2 *The slope of the optimal tax rule is lower with commitment than with limited commitment, while the optimal tax for a null pollution stock is greater with commitment than with limited commitment i.e.* $\tau^{nc}(0) < \tau^{c}(0) < 0,\ 0 < m^{c} < m^{nc}$*, where* m^{c} *and* m^{nc} *denote the slopes of the optimal tax strategy for the commitment and limited commitment equilibria, respectively.*

Proof. See Appendix. ■

If sufficient condition (65) is satisfied it is immediate to conclude from the previous results that

Corollary 1 *The optimal policy with commitment gives larger values for* τ *than the optimal policy with limited commitment in the interval* $[0, x_{SS}^{nc}]$*, i.e.* $\tau^{c}(x) > \tau^{nc}(x),\ \forall x \in [0, x_{SS}^{nc}]$.

Proof. According to Lemma 2, the optimal tax rules must intersect once. Then if we denote by x^{ip} the value of the pollution stock defined by the intersection point, $\tau^{c}(x)$ must be larger than $\tau^{nc}(x)$ for all x in the interval $[0, x^{ip})$ also by Lemma 2. Suppose that $x_{\tau SS}^{nc}$ is larger than or equal to x^{ip}. In this case according to Prop. 7, x_{SS}^{nc} should be lower than x^c_{SS} but this contradicts Lemma 1. Thus, x^{nc}_{SS} is lower than x^{ip} and $\tau^c(x)$ is larger than $\tau^{nc}(x)$ for all x in the interval $[0, x^{nc}_{SS}]$. ■

Next, we study the effects that a limited commitment has on the payoffs of both players. Unfortunately, the comparison of the payoffs is ambiguous because it involves the coefficients of the value functions corresponding to the two equilibria. Nevertheless, it is possible to get an unambiguous comparison when the initial pollution stock is zero.[20] In this case, we obtain the following result

Proposition 9 *The net social welfare is lower with limited commitment whereas net profits are larger.*

Proof. See Appendix. ■

Thus, a reduction in commitment has a cost in welfare terms. The limited commitment benefits the firm but does not increase social welfare. This result suggests that the increase in damages because of a larger pollution stock in the case of a limited commitment dominates any other variation in net social welfare caused by the other variables yielding in net terms a reduction in net social welfare.

To have an idea of whether this detrimental effect on welfare holds for initial values of the pollution stock different from zero, we carry out a numerical exercise. In this numerical example we keep constant r and δ and consider variations on γ and d, parameters that determine the slope of the marginal abatement costs and marginal environmental damages respectively. We assume two reasonable values for r and δ : $r = 0.05$ and $\delta = 0.10$.[21] First, we compute the lower bounds for parameter d defined in the paper to get an idea whether the conditions imposed on this parameter are very restrictive. It is very easy to check that for the parameter values of this example, the different lower bounds defined on parameter d are satisfied assuming $d > 1$. Next, as the steady state is globally stable

[20] Notice that when the pollution stock is zero, the comparison of the value functions reduces to the comparison of the coefficients C.

[21] Nevertheless, we have checked whether the sign of payoffs comparison could be affected by a change in the decay rate of pollution comparing also the payoffs for $\delta = 0.05$. For both values, 0.05 and 0.10, the sign remains the same. For this reason, the second exercise is omitted.

instead of comparing the players' payoffs for different initial values of the pollution stock we focus on the steady-state values because all the temporal paths from different initial values for the same equilibrium converge to the same steady state.

In Table 1a, the net social welfare for the steady-state pollution stocks are represented for $a = 1000$ and $c = 20$. In each box, the first figure stands for the steady-state value corresponding to the SFSE, whereas the second figure represents the corresponding steady-state value for the LCMPE. The comparison of the different figures is in the line of the result presented in Prop. 9: the net social welfare is lower with limited commitment. The figures also show that welfare decreases with the environmental damages and also with the abatement costs when the regulator moves first but not necessarily with a limited commitment. In this case, an increase in environmental damages for low values of the abatement costs has a positive effect on welfare. The explanation to this effect should be looked for in the impact that larger damages has on the steady-state pollution stock. We expect that for large damages the pollution stock is lower. Therefore, given the structure of the value function with the coefficients A_1 and B_1 negative, a reduction in the coefficient C_1 because of the increase in the parameter d could be more than compensated by the reduction in the steady-state pollution stock yielding a higher net social welfare.

$\gamma\backslash d$	2.50	5.00	10.00	20.00
2.50	2621776, 2237773	2615135, 2255993	2611813, 2269594	2610153, 2279578
5.00	1587199, 1524898	1577599, 1524884	1572799, 1526575	1570399, 1528638
10.00	890508, 877835	878768, 869256	872897, 865483	869962, 863976
20.00	482412, 479226	469400, 467204	462894, 461326	459640, 458490

Table 1a. Net social welfare for the steady-state pollution stocks.

In Table 1b we show the welfare losses in relative terms caused by a limited commitment. The figures show that the differences between the two equilibria are minimal when both damages and abatement costs are high. Moreover, it can be checked that welfare losses are more inelastic to environmental damages than to abatement costs. For instance, when $\gamma = 2.5$ an increase in d from 2.50 to 20.00 reduces the welfare losses in 1.98 percentage points. However, when $d = 2.5$ for an increase in γ from 2.50 to 20.00, the welfare losses go down by 13.99 percentage points.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	14.65	13.73	13.10	12.67
5.00	3.93	3.34	2.94	2.66
10.00	1.42	1.08	0.85	0.69
20.00	0.66	0.47	0.34	0.25

Table 1b. Welfare losses (%).

To complete the comparison of payoffs we present in Table 2a the discounted present value of net profits. The figures support the result of Prop. 9: net profits are larger with limited commitment. Now, net profits decrease both with the environmental damages and abatement costs for both equilibria.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	$2163545, 3159753$	$2148909, 3018081$	$2141571, 2923792$	$2137898, 2860138$
5.00	$1419589, 1626478$	$1395844, 1548869$	$1383934, 1502295$	$1377970, 1473577$
10.00	$864460, 927885$	$832466, 873258$	$816417, 843692$	$808379, 827368$
20.00	$512546, 536979$	$474631, 489593$	$455609, 464248$	$446082, 451482$

Table 2a. Net profits for the steady-state pollution stocks.

Table 2b shows the increase in net profits in relative terms caused by a limited commitment. The percentages also decrease both with damages and abatement costs as it happened in the case of net social welfare. However, the increments of the net profits in percentage points are significantly larger than the reductions of social welfare also in percentage points for all the cases. For instance, for $\gamma = d = 2.5$ the reduction in welfare is 14.65% whereas the increase in net profits is 46.04%. This difference is explained by the variations in the optimal policy caused by a limited commitment. As shown in Fig. 2 just for these parameter values, when there is commitment the optimal policy consists of taxing emissions for $t > 1$ whereas with limited commitment the firm receives a subsidy all the time, accounting for the substantial increase in net profits the firm obtains when the regulator does not move first.[22] Nevertheless, the larger the abatement costs and

[22]Notice that if the firm receives a subsidy, net profits include subsidies.

the larger the damages, the lower is this effect because the difference between the two solutions substantially decreases.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	46.04	40.45	36.52	33.78
5.00	14.57	10.96	8.55	6.94
10.00	7.34	4.90	3.34	2.35
20.00	4.77	3.15	1.90	1.21

Table 2b. Increase in net profits (%).

Next, with the aim of having a better intuition about the effects of a limited commitment on welfare, we compute the steady-state values of production and abatement. In Table 3, the steady-state values for production are represented.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	219.98, 226.17	218.88, 223.24	218.33, 221.40	218.05, 220.22
5.00	143.41, 146.01	141.71, 143.53	140.86, 142.13	140.43, 141.33
10.00	86.06, 86.98	83.87, 84.50	82.77, 83.23	82.22, 82.53
20.00	49.60, 49.88	47.08, 47.27	45.81, 45.95	45.18, 45.27

Table 3. Steady-state values of production.

According to the figures that appear in this table, the steady-state value of production with a limited commitment is always larger than the corresponding value with commitment. Moreover, for both equilibria the output decreases both with respect to damages and abatement costs. However, an increase in damages has a lower effect on the steady-state value of production that the effect caused by an increase in abatement costs. In other words, a variation in abatement costs causes a stronger change in the level of production than the change caused by the same variation in environmental damages. Finally, it could be pointed out that the higher the damages and abatement costs, the lower the differences in production at the steady state.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	$216.01, 211.07$	$216.89, 213.41$	$217.34,\ 214.88$	$217.56,\ 215.82$
5.00	$138.63, 137.59$	$139.31, 138.59$	$139.66,\ 139.15$	$139.83,\ 139.47$
10.00	$80.79,\ 80.60$	$81.23, 81.10$	$81.45,\ 81.36$	$81.56,\ 81.49$
20.00	$44.04,\ 44.01$	$44.29,\ 44.27$	$44.42,\ 44.40$	$44.48,\ 44.47$

Table 4. Steady-state values of abatement.

The figures in Table 4 establish that the steady-state value of abatement when the regulator moves first is larger than the steady-state value corresponding to the LCMPE. They also establish that the abatement increases with damages and decreases with abatement costs. In other words, the larger the damages the larger the abatement, whereas the contrary occurs with the abatement costs. Again, the effect of a change in damages on the steady-state values is significantly weaker that the effect of a change in abatement costs, and the differences in the abatement at the steady state are minimal for big values of damages and abatement costs.

As the output is lower and the abatement is larger at the steady state for the SFSE, the emissions are higher with limited commitment as it has been established in Lemma 1. Moreover, as the output decreases with environmental damages and the abatement increases both things for both equilibria, the emissions are decreasing with respect to damages for both equilibria. However, the larger the abatement costs the larger the emissions for the SFSE, although the contrary occurs for the LCMPE. This difference is explained by the fact that the reduction in production with limited commitment is larger than the reduction in production with commitment when the abatement costs increase whereas the contrary occurs for the abatement. The result is that the net effect on emissions of an increase in abatement costs is different for each equilibrium.

Finally, we compare the optimal paths of the different variables of the model for $\gamma = d = 2.5$ and an initial pollution stock equal to zero. In Fig. 2 we plot the optimal paths of the tax. In green, the temporal trajectory of the tax with commitment and in black with limited commitment.

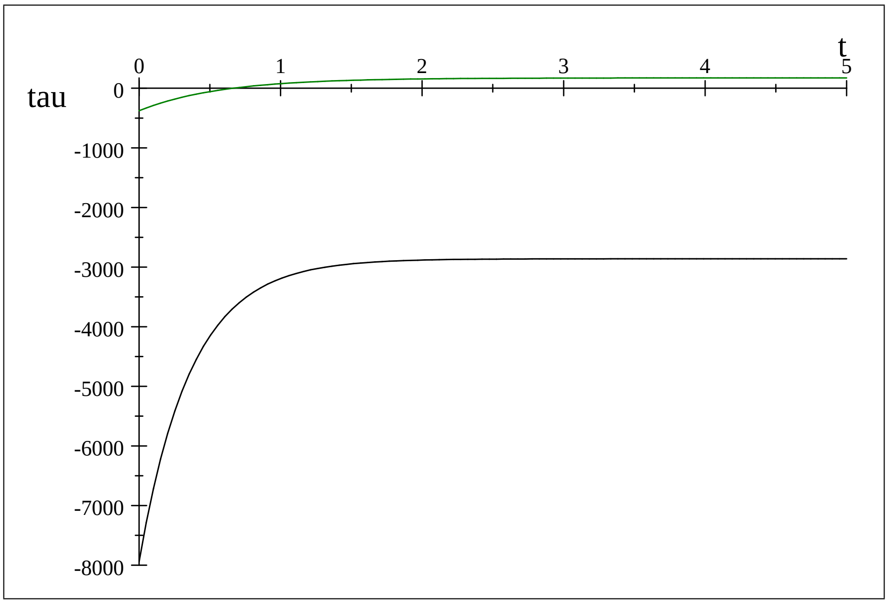

Figure 2. The optimal paths of the emission tax.

The figure shows that a reduced commitment drastically changes the type of the optimal policy. With commitment, the optimal policy for $x_0 = 0$ consists of subsidizing emissions to correct the effect of the monopolist's market power on production but in less that one period the subsidy becomes a tax and continues being a tax until reaching its steady state value of 173.25. However, with a limited commitment the optimal path yields a subsidy for all t. The path converges to a steady-state value for τ equal to -2860.42. As pointed out above this difference in the sign of the optimal policy explains the significant increase in net profits whit limited commitment. This numerical example shows a case where a subsidy applies at the steady state. In this case, the severity of environmental damages does not justify a tax be applied but the accumulation of emissions induces a reduction of the subsidy. Fig. 3 shows that the optimal path of production with limited commitment is above the optimal path with commitment. The subsidy incentives production whereas the tax has the opposite effect. The result is a level of production for the SFSE that is lower than the level of production for the LCMPE for all t.

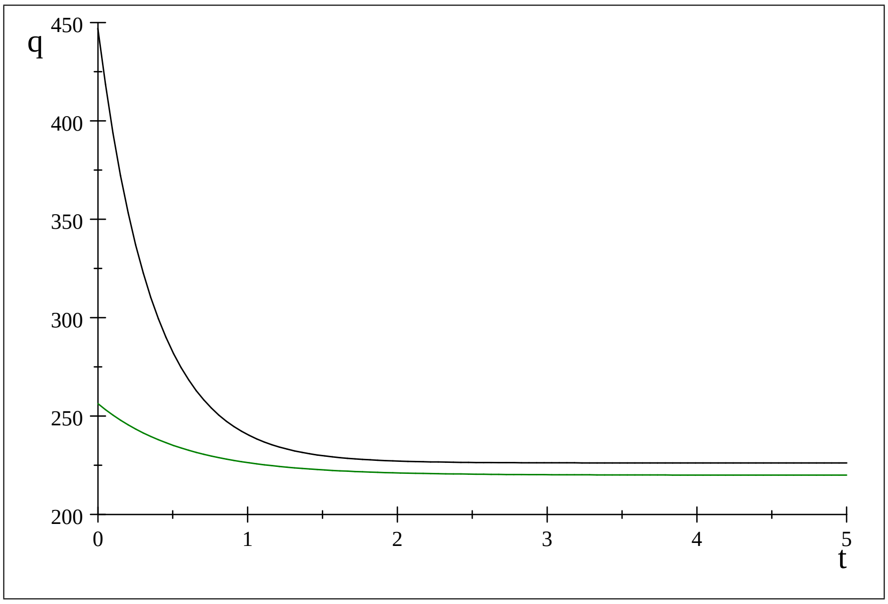

Figure 3. The optimal paths of production.

Fig. 4 shows that the abatement effort is lower for all t with limited commitment. The subsidy is an incentive to reduce abatement, and therefore, to emit more. Notice that if the production is larger and the abatement is lower with limited commitment, the emissions must be larger in this case.

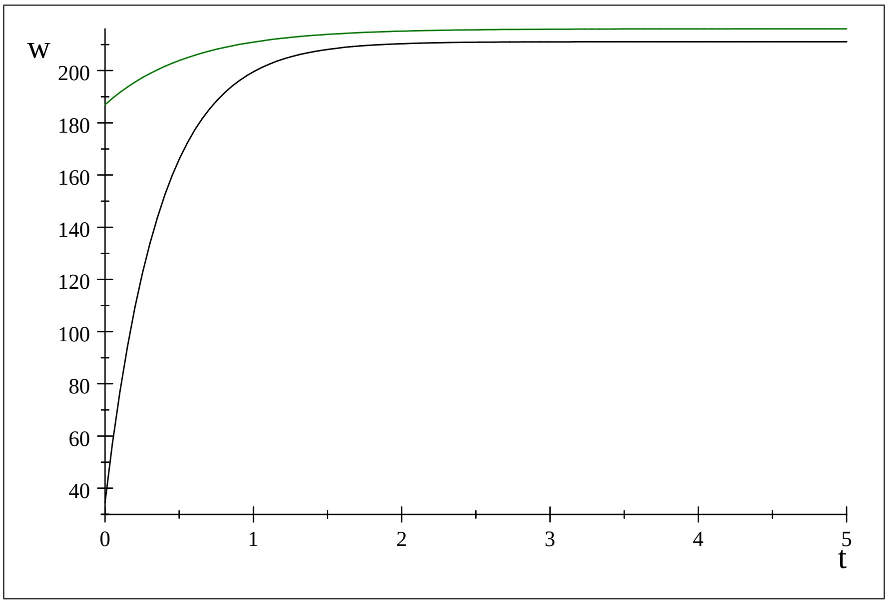

Figure 4. The optimal paths of abatement.

Finally, in Fig. 5 we represent the optimal paths for the pollution stocks. As emissions are larger with limited commitment, the pollution stock is larger for the LCMPE.

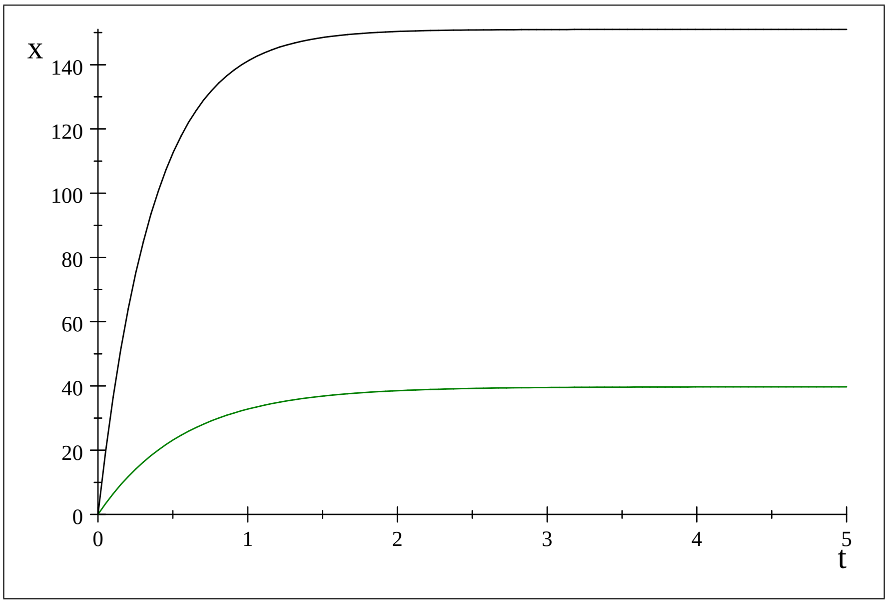

Figure 5. The optimal paths of pollution stock.

This dynamic analysis gives support to the intuition previously presented that the increase in damages caused by the increment in the pollution stock dominates the other changes in the different terms included in the net social welfare function yielding a reduction in welfare when the regulator does not move first. Summarizing, the reduction in commitment implies a change in the sign of the optimal policy turning a tax in a subsidy, increases output and reduces abatement causing an increase in emissions that leads to a larger pollution stock. The expansion of output increases consumer surplus and the decrease of abatement reduces abatement costs. Both changes are welfare improving but, as can be seen in Fig. 5, there is an important augmentation of the pollution stock that results in an increase of damages big enough as to yield finally a reduction in net social welfare.

6 Conclusions

This paper studies the effects that a limited commitment has on the optimal taxation of a polluting monopoly and its welfare implications. To evaluate these effects we compare two equilibria of a policy game between a regulator and a monopolist: the stagewise feedback Stackelberg equilibrium and the limited commitment Markov-perfect equilibrium. In the

SFSE, it is assumed that the regulator cannot commit for the entire time horizon of the game but he enjoys a strategic advantage in each period. However, in the LCMPE it is assumed that the regulator and the monopolist first simultaneously choose the emission tax and abatement respectively before the monopolist decides the production level. The two equilibria are different, establishing that although the SFSE is time consistent is not intratime consistent, i.e. it is not time consistent in each period. The comparison of these two equilibria for the LQ policy game analyzed in this paper establishes that the steady-state pollution stock with commitment is lower than with limited commitment. Thus, the lack of credibility in each period has a clear consequence on the accumulation of emissions leading to larger damages at the steady state. Moreover, we show that this lack of intratime commitment moves down the tax rule applied by the regulator. If environmental damages are not very low, the steady-state tax with commitment is larger than the steady-state tax with limited commitment that, on the other hand, could be negative, i.e. the optimal policy would be to apply a subsidy. The welfare implications of a limited commitment are unclear except when the initial pollution stock is zero. For this case, the net social welfare is lower with a limited commitment whereas net profits are larger.

To progress in the comparison we have computed a numerical example that shows that a limited commitment has a negative impact on social welfare also for the steady-state pollution stock. Thus, our analysis shows that a reduction in the regulator's commitment level has a *detrimental* effect on welfare. However, the numerical exercise also shows that this negative effect decreases with the abatement costs and that for large abatement costs the difference in welfare between the two equilibria is practically zero. Finally, we find that the steady-state value of production for the LCMPE is larger than the steady-state value of production for the SFSE whereas the contrary is true for abatement. Thus, a limited commitment leads to lower taxation and abatement and larger production. However, the increase in consumer surplus because of a larger production and the reduction in abatement costs is more than compensated by the increase in environmental damages originated by a larger pollution stock resulting in a reduction of net social welfare.

A limitation of our analysis is that we have assumed the simplest form of the emission function, i.e. one that is additively separable in production (gross emissions) and abatement. To overcome this limitation, an interesting extension to develop in the future would be to consider that abatement expenditures can reduce the emissions-to-output ratio or that emissions can be reduced by investing in abatement capital. This second approach would allow to study the dynamic interaction between the accumulation of emissions and the accumulation of abatement capital. A further step in this line of research would be to analyze the effects of a reduced commitment when the abatement technology is subject to stochastic innovation. Finally, it would be also interesting to know how the results would change if the market structure is an oligopoly.

Appendix

Proof of Proposition 3

To guarantee that abatement is positive for $x \geq 0$, $w^c(0)$ for the optimal strategy (34) must be positive since the slope of the strategy is positive. This requires that $\gamma(r+\delta)+(\gamma+2)A_1^c$ be positive. Substituting A_1^c by (29) this expression is negative if and only if

$$\gamma(r+\delta)+(\gamma+2)\frac{\gamma(\gamma+4)(r+2\delta)-\sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2+\gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2}<0,$$

that can be reordering yielding

$$2\gamma(r+\delta)(2+\gamma)+\gamma(4+\gamma)(r+2\delta)<\sqrt{\gamma(4+\gamma)[4d(2+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}.$$

Taking square in both side of the inequality gives

$$\gamma(r+\delta)[(r+\delta)(2+\gamma)+(4+\gamma)(r+2\delta)]-(4+\gamma)(2+\gamma)d<0,$$

that is negative when $d>d_w^c$. Thus, if $d>d_w^c$, then $\gamma(r+\delta)+(\gamma+2)A_1^c$ is negative and the abatement is positive and increasing for $x \geq 0$. On the other hand, according to (33) and (35), $q^c(0)$ and $s^c(0)$ are positive and the slope of their optimal strategies negative. Thus, the output and emissions are decreasing for $x \geq 0$. Moreover, if $q^c(0)$ and $s^c(0)$ are positive, $w^c(0)$ must be lower than $q^c(0)$ by definition. Also by definition the emissions are lower than the output so that they will be zero for a value of the pollution stock lower than the value for which the output becomes zero. To calculate the value of the pollution stock that makes zero the emissions we use the equation $s^c(x_s^c) = 0$ that yields the value that appears in the proposition. The difference of this value with the steady-state pollution stock is

$$x_{SS}^c - x_s^c = \frac{(a-c)\gamma(\gamma+3)(r+\delta)[(\gamma+2)^2\delta A_1^c - \gamma(\gamma+4)(r+\delta)\delta]}{[(\gamma+2)^2 d + \gamma(\gamma+4)(r+\delta)\delta](\gamma+2)^2(\delta A_1^c + d)},$$

that is negative for A_1^c negative provided that $\delta A_1^c + d$ is positive. According to (29) $\delta A_1^c + d$ is positive if and only if

$$\delta\frac{\gamma(\gamma+4)(r+2\delta) - \sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2} + d > 0,$$

that can be rewritten as

$$\gamma\delta(\gamma+4)(r+2\delta) + 2(\gamma+2)^2 d > \delta\sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]},$$

taking square in both sides of the inequality and simplifying yields

$$2\left(\gamma+2\right)^2 d(r\gamma\delta\left(\gamma+4\right) + 2(\gamma+2)^2 d) > 0,$$

that established that $\delta A_1^c + d$ is positive.

Proof of Lemma 1

The steady state for the pollution stock with commitment is given by (32) and the steady state for the pollution stock with limited commitment by (55). Taking into account that the first Riccati's equation for the LCMPE establishes that $(A_1^{nc})^2 = (2\delta + r)A_1^{nc} + d$ and using (52) for eliminating A_1^{nc}, (55) can be written as follows

$$x_{SS}^{nc} = \frac{(a-c)\left(\gamma(1+\gamma)(r+\delta) + \sqrt{4d + (r+2\delta)^2} - (r+2\delta)\right)}{\gamma\left(d(2+\gamma) + \delta\left(r + 2\delta + \gamma(r+\delta) - \sqrt{4d + (r+2\delta)^2}\right)\right)}.$$

Easy computations lead to the following equivalence:

$$x_{SS}^c < x_{SS}^{nc} \Leftrightarrow Num/Den < 0,$$

where

$$\begin{aligned} Num &= (2+\gamma)(d + \gamma\delta(r+\delta))\left(2(r+2\delta) - \gamma r - (2+\gamma)\sqrt{4d + (r+2\delta)^2}\right), \\ Den &= \left(d(2+\gamma)^2 + \gamma(4+\gamma)\delta(r+\delta)\right) \times \\ &\quad \left(d(2+\gamma) + \delta\left(r + 2\delta + \gamma(r+\delta) - \sqrt{4d + (r+2\delta)^2}\right)\right). \end{aligned}$$

Den is positive, because the second factor can be proved to be positive:

$$
\begin{aligned}
&d(2+\gamma)+\delta\left(r+2\delta+\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)>0 \;\Leftrightarrow\\
&d(2+\gamma)+\delta\left(r+2\delta+\gamma(r+\delta)\right)-\delta\sqrt{4d+(r+2\delta)^2}>0 \;\Leftrightarrow\\
&\left[d(2+\gamma)+\delta\left(r+2\delta+\gamma(r+\delta)\right)\right]^2-\delta^2(4d+(r+2\delta)^2)>0 \;\Leftrightarrow\\
&(d+\delta(r+\delta))(d(2+\gamma)^2+\gamma\delta(r(2+\gamma)+(4+\gamma)\delta))>0.
\end{aligned}
$$

The sign of Num is the same as the sign of the following expression:

$$
2(r+2\delta)-\gamma r-(2+\gamma)\sqrt{4d+(r+2\delta)^2}. \tag{66}
$$

The expression above is always negative. If $2(r+2\delta)-\gamma r<0$, then the expression in (66) is negative. If $2(r+2\delta)-\gamma r>0$,then

$$
\begin{aligned}
&2(r+2\delta)-\gamma r-(2+\gamma)\sqrt{4d+(r+2\delta)^2}<0 \;\Leftrightarrow\\
&2(r+2\delta)-\gamma r<(2+\gamma)\sqrt{4d+(r+2\delta)^2} \;\Leftrightarrow\\
&(2(r+2\delta)-\gamma r)^2<(2+\gamma)^2(4d+(r+2\delta)^2) \;\Leftrightarrow\\
&4d(2+\gamma)^2+4\gamma(r+\delta)(2r+(4+\gamma)\delta)>0.
\end{aligned}
$$

Therefore, $Num/Den<0$, and $x^c_{SS}<x^{nc}_{SS}$. At the steady state, $s^c_{SS}=\delta x^c_{SS}$ and $s^{nc}_{SS}=\delta x^{nc}_{SS}$ that establishes that $s^c_{SS}<s^{nc}_{SS}$.

Proof of Proposition 8

First, we calculate the steady-state tax rate substituting the steady-state value of the pollution stock in (42):

$$
\tau^c_{SS}=\tau^c(x^c_{SS})=-\frac{(a-c)\gamma(\gamma\delta F_2F_3F_4-dF_1(rF_2+2\delta))}{F_2F_4(F_1^2d+\gamma\delta F_2F_3)},
$$

where

$$
F_1=\gamma+2,\;\; F_2=\gamma+4,\;\; F_3=r+\delta \text{ and } F_4=r+2\delta.
$$

Next using this tax, we derive the value of the stock of pollution, $\tilde{x}$, for which the optimal tax rule with limited commitment yields the steady-state tax rate of the SFSE. Thus this value of the pollution stock must satisfy $\tau^{nc}(\tilde{x}) = \tau^{c}_{SS}$.

$$
\begin{aligned}
\tilde{x} &= (a-c)\left(\frac{F_2F_4(\gamma^2F_3^2 - \gamma F_3F_2A_1^{nc} + 4(A_1^{nc})^2)(F_1^2d + \gamma\delta F_2F_3)}{\gamma F_3F_2(F_3 - A_1^{nc})(F_1^2d + \gamma\delta F_2F_3)(2F_1(A_1^{nc})^2 - 2\gamma F_4A_1^{nc})}\right.\\
&- \left.\frac{\gamma^3F_3(F_3 - A_1^{nc})(\gamma\delta F_2F_3F_4 - dF_1(rF_2 + 2\delta))}{\gamma F_3F_2(F_3 - A_1^{nc})(F_1^2d + \gamma\delta F_2F_3)(2F_1(A_1^{nc})^2 - 2\gamma F_4A_1^{nc})}\right).
\end{aligned}
$$

Using the steady-state value of the pollution stock for the LCMPE given by (55), $\tilde{x} > x^{nc}_{SS}$ if and only if

$$
\begin{aligned}
&\frac{F_2F_4(\gamma^2F_3^2 - \gamma F_3F_2A_1^{nc} + 4(A_1^{nc})^2)(F_1^2d + \gamma\delta F_2F_3)(F_1A_1^{nc} - \gamma\delta)}{F_3F_2(F_1^2d + \gamma\delta F_2F_3)(2F_1(A_1^{nc})^2 - 2\gamma F_4A_1^{nc})(F_1A_1^{nc} - \gamma\delta)}\\
-\ &\frac{\gamma^3F_3(F_3 - A_1^{nc})(F_1A_1^{nc} - \gamma\delta)(\gamma\delta F_2F_3F_4 - dF_1(rF_2 + 2\delta))}{F_3F_2(F_1^2d + \gamma\delta F_2F_3)(2F_1(A_1^{nc})^2 - 2\gamma F_4A_1^{nc})(F_1A_1^{nc} - \gamma\delta)}\\
+\ &\frac{F_3F_2(F_1^2d + \gamma\delta F_2F_3)(2F_1(A_1^{nc})^2 - 2\gamma F_4A_1^{nc})(\gamma(1+\gamma)F_3 - 2A_1^{nc})}{F_3F_2(F_1^2d + \gamma\delta F_2F_3)(2F_1(A_1^{nc})^2 - 2\gamma F_4A_1^{nc})(F_1A_1^{nc} - \gamma\delta)} > 0, \qquad (67)
\end{aligned}
$$

where the denominator is negative since A_1^{nc} is negative.

The development of the numerator yields

$$
\begin{aligned}
&- \ 4\gamma^4\delta^2(\gamma+4)(r+\delta)^3(r+2\delta) - 2d\gamma^3\delta(\gamma+2)(r+\delta)^2G_1(\gamma,\delta,r)\\
&+ \ A_1^{nc}\delta\gamma^2(r+\delta)(4(\gamma+2)G_2(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)(r+2\delta)G_3(\gamma,\delta,r))\\
&- \ 2(A_1^{nc})^2\gamma\delta(2(\gamma+2)^2G_4(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)G_5(\gamma,\delta,r))\\
&+ \ 4(A_1^{nc})^3\delta(\gamma+2)(\gamma+4)(\gamma\delta(\gamma+4)(r+\delta) + d(\gamma+2)^2), \qquad (68)
\end{aligned}
$$

where

$$
\begin{aligned}
G_1(\gamma,\delta,r) &= \gamma^2(r+\delta) + \gamma(5r+7\delta) + 4(r+2\delta) > 0,\\
G_2(\gamma,\delta,r) &= \gamma^2(r+3\delta) + \gamma(5r+13\delta) + 8(r+2\delta) > 0,\\
G_3(\gamma,\delta,r) &= 2\gamma^2(r+\delta) + 2\gamma(3r-\delta) - 16\delta,\\
G_4(\gamma,\delta,r) &= 2\gamma^2(r+\delta) + \gamma(8r+9\delta) + 4(2r+3\delta) > 0,\\
G_5(\gamma,\delta,r) &= \gamma^3(r+\delta)^2 + \gamma^2(3r-\delta)(r+\delta) - 2\gamma\delta(8r+9\delta) - 8\delta(2r+3\delta).
\end{aligned}
$$

$G_3(\gamma,\delta,r)$ might be positive and in this case

$$
A_1^{nc}\delta\gamma^2(r+\delta)(4(\gamma+2)G_2(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)(r+2\delta)G_3(\gamma,\delta,r))
$$

might be positive too. However, if d is larger than d^{τ}_{SS} given in (65) this possibility is eliminated. The same occurs for $G_5(\gamma,\delta,r)$. In this case d should be larger than

$$\hat{d}=\frac{\gamma(\gamma+4)(r+\delta)(\gamma^3\left(r+\delta\right)^2+\gamma^2\left(3r-\delta\right)\left(r+\delta\right)-2\gamma\delta\left(8r+9\delta\right)-8\delta\left(2r+3\delta\right))}{2(\gamma+2)^2(2\gamma^2\left(r+\delta\right)+\gamma\left(8r+9\delta\right)+4\left(2r+3\delta\right))}$$

to avoid that

$$-2(A_1^{nc})^2\gamma\delta(2(\gamma+2)^2G_4(\gamma,\delta,r)d-\gamma(\gamma+4)(r+\delta)G_5(\gamma,\delta,r))$$

be positive. As $\hat{d}$ is lower than d^{τ}_{SS}, we can conclude that if d is large enough, in particular if d is larger than d^{τ}_{SS} all the terms in (68) are negative and (67) is satisfied so that we can conclude that $\tilde{x}>x^{nc}_{SS}$. Thus, as the optimal policy is increasing with respect to the pollution stock we have that $\tau^c_{SS}=\tau^{nc}(\tilde{x})>\tau^{nc}(x^{nc}_{SS})$, i.e. the steady-state tax rate with limited commitment is lower than the steady-state tax rate with commitment.

Proof of Lemma 2

Substituting in the intersection point of the optimal policy defined by (64) A_1^{nc} given by (52) we obtain that $\tau^{nc}(0)$ is equal to

$$-(a-c)\frac{2\left(r+2\delta-\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)^2-\gamma^2(r+\delta)\left(r+2\delta-\sqrt{4d+(r+2\delta)^2}\right)}{\gamma^2(r+\delta)(r+\sqrt{4d+(r+2\delta)^2})}$$

and then using (42) in Prop. 5, the difference $\tau^c(0)-\tau^{nc}(0)$ can be written as follows

$$\begin{aligned}&-\frac{(a-c)\gamma}{\gamma+4}\\&+(a-c)\frac{2\left(r+2\delta-\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)^2-\gamma^2(r+\delta)\left(r+2\delta-\sqrt{4d+(r+2\delta)^2}\right)}{\gamma^2(r+\delta)(r+\sqrt{4d+(r+2\delta)^2})},\end{aligned}$$

that taking common factor yields

$$\begin{aligned}=-\frac{(a-c)}{(r+\sqrt{4d+(r+2\delta)^2})}&\left(\frac{\gamma^3(r+\delta)(r+\sqrt{4d+(r+2\delta)^2})}{\gamma^2(r+\delta)(\gamma+4)}\right.\\-\frac{2(\gamma+4)\left(r+2\delta-\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)^2}{\gamma^2(r+\delta)(\gamma+4)}&\left.+\frac{\gamma^2(r+\delta)(\gamma+4)(r+2\delta-\sqrt{4d+(r+2\delta)^2})}{\gamma^2(r+\delta)(\gamma+4)}\right),\end{aligned}$$

that developing the numerator results in the following expression

$$= -\frac{(a-c)\gamma}{\gamma(r+\sqrt{4d+(r+2\delta)^2})}\frac{4H_2+4H_1\sqrt{4d+(r+2\delta)^2}-8d\,(\gamma+4)}{\gamma^2(r+\delta)(\gamma+4)}, \tag{69}$$

where

$$\begin{aligned}
4H_1 &= 4(4(r+2\delta)-\gamma(3r+2\delta)-2\gamma^2(r+\delta)) < 0 \text{ for } \gamma > 2,\\
4H_2 &= 4(2\gamma^2\delta\,(r+\delta)+\gamma\,(3r+2\delta)\,(r+2\delta)-4\,(r+2\delta)^2) > 0 \text{ for } \gamma > 2.
\end{aligned}$$

As

$$4H_2+4H_1\sqrt{4d+(r+2\delta)^2}-8d\,(\gamma+4) = -8\gamma^2\,(r+\delta)^2 \ \text{ for } d=0,$$

and the expression decreases with d, we can conclude that

$$4H_2+4H_1\sqrt{4d+(r+2\delta)^2}-8d\,(\gamma+4) < 0 \text{ for } d > 0,$$

and hence that (69) is positive establishing that $\tau^c(0)-\tau^{nc}(0)$ is positive.

Next, we compare the slope of the optimal policies. According to the first Riccati's equation: $(A_1^{nc})^2 = (2\delta+r)A_1^{nc}+d$ which allows to write the slope of the optimal policy for LCMPE given by (64) as follows

$$m^{nc} = \frac{2(2(r+2\delta)A_1^{nc}+d(2+\gamma))}{\gamma(r+2\delta)},$$

that substituting A_1^{nc} by (52) yields

$$m^{nc} = \frac{2[d(2+\gamma)+(2\delta+r)^2-(r+2\delta)\sqrt{(r+2\delta)^2+4d}]}{\gamma(r+2\delta)},$$

so that the difference between the slopes is

$$m^{nc}-m^c = \frac{2}{r+2\delta}\left(\frac{4(2+\gamma)d+(\gamma+4)(2\delta+r)^2-(\gamma+4)(r+2\delta)\sqrt{(r+2\delta)^2+4d}}{\gamma(\gamma+4)}\right). \tag{70}$$

This difference is positive provided that

$$4(2+\gamma)d+(\gamma+4)(2\delta+r)^2-(\gamma+4)(r+2\delta)\sqrt{(r+2\delta)^2+4d} > 0.$$

Reordering terms and taking square, we get

$$16(2+\gamma)^2d^2+4\gamma d(r+2\delta)^2(\gamma+4) > 0.$$

Therefore (70) is positive and consequently m^{nc} is larger than m^c.

Proof of Proposition 9

Notice that according to the expressions of the value functions (25) and (48), $W^c(0)$ is larger than $W^{nc}(0)$ provided that C_1^c is larger than C_1^{nc}. Using the expressions for these parameters given respectively by (31) and (54), C_1^c is larger than C_1^{nc} if and only if

$$\begin{aligned} & \Sigma_1(\gamma,\delta,d,r) + r\sqrt{4d+(r+2\delta)^2}\Sigma_2(\gamma,\delta,d,r) \\ + \quad & r\sqrt{\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)}\Sigma_3(\gamma,\delta,d,r) > 0, \end{aligned} \tag{71}$$

where

$$\begin{aligned} \Sigma_1(\gamma,\delta,d,r) &= 2[4d^2(2+\gamma^2) + 2d\left(4\gamma(2+\gamma)^2\delta^2 + 2r(2\gamma^3+7\gamma^2+4\gamma-2)(r+2\delta)\right) \\ &+ \gamma(r^2+2r\delta+2\delta^2)(2\gamma(2+\gamma)^2\delta^2+r(r+2\delta)(2\gamma^3+8\gamma^2+7\gamma-4))] > 0 \text{ for } \gamma > 2, \\ \Sigma_2(\gamma,\delta,d,r) &= 4d(2+\gamma)^2 \\ &+ \gamma[(\gamma^5+6\gamma^4+9\gamma^3+4\gamma+16)\delta^2 + r(r+2\delta)(\gamma^5+6\gamma^4+9\gamma^3+2\gamma+8)] > 0, \\ \Sigma_3(\gamma,\delta,d,r) &= 4d - \gamma(\gamma^3+2\gamma^2-3\gamma-4)\delta^2 - r(r+2\delta)(\gamma^4+2\gamma^3-3\gamma^2-4\gamma+2) \\ &+ 2r\sqrt{4d+(r+2\delta)^2}. \end{aligned}$$

The sign of $\Sigma_3(\gamma,\delta,d,r)$ depends on the value of d. For a d large enough, $\Sigma_3(\gamma,\delta,d,r)$ is positive and therefore (71) is also positive that establishes that $C_1^c > C_1^{nc}$. Suppose now that this is not the case then (71) implies the following inequality

$$\begin{aligned} & \left(\Sigma_1(\gamma,\delta,d,r) + r\sqrt{4d+(r+2\delta)^2}\Sigma_2(\gamma,\delta,d,r)\right)^2 \\ - \quad & r^2\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)\Sigma_3(\gamma,\delta,d,r)^2 > 0, \end{aligned}$$

that yields

$$\chi_1(\gamma,\delta,d,r) + \chi_2(\gamma,\delta,d,r)\sqrt{4d+(r+2\delta)^2} > 0, \tag{72}$$

with

$$\chi_1(\gamma,\delta,d,r) = \Sigma_1(\gamma,\delta,d,r)^2 + r^2\left(4d+(r+2\delta)^2\right)\Sigma_2(\gamma,\delta,d,r)^2$$
$$-r^2(\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2))\times$$
$$\{[4d-\gamma(\gamma^3+2\gamma^2-3\gamma-4)\delta^2-r(r+2\delta)(\gamma^4+2\gamma^3-3\gamma^2-4\gamma+2)]^2+4r^2(4d+(r+2\delta)^2)\},$$
$$\chi_2(\gamma,\delta,d,r) = 2r\Sigma_1(\gamma,\delta,d,r)\Sigma_2(\gamma,\delta,d,r)$$
$$-4r^3(\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2))\times$$
$$\{4d-\gamma(\gamma^3+2\gamma^2-3\gamma-4)\delta^2-r(r+2\delta)(\gamma^4+2\gamma^3-3\gamma^2-4\gamma+2)\}.$$

Once the expressions of $\Sigma_1(\gamma,\delta,d,r), \Sigma_2(\gamma,\delta,d,r)$ and $\Sigma_3(\gamma,\delta,d,r)$ have been replaced in $\chi_1(\gamma,\delta,d,r)$ and $\chi_2(\gamma,\delta,d,r)$ and after some tedious computations carried out with Mathematica 10.1, they can be rewritten as

$$\chi_1(\gamma,\delta,d,r) = 8d^4(2+\gamma)^2+16d^3p_1(r,\gamma,\delta)+4d^2p_2(r,\gamma,\delta)+2d\gamma p_3(r,\gamma,\delta)+\gamma^2(r+\delta)^2p_4(r,\gamma,\delta),$$
$$\chi_2(\gamma,\delta,d,r) = 8d^3(2+\gamma)^2+2d^2p_5(r,\gamma,\delta)+4d\gamma p_6(r,\gamma,\delta)+\gamma^2(r+\delta)^2p_7(r,\gamma,\delta),$$

where $p_i(r,\gamma,\delta), i=1,\ldots,7$ are the polynomials in terms of parameters r,γ and δ

$$p_1(r,\gamma,\delta) = r^2\gamma(4+7\gamma+2\gamma^2)+2r\delta(-2+4\gamma+7\gamma^2+2\gamma^3)+2\gamma(2+\gamma)^2\delta^2,$$

$$p_2(r,\gamma,\delta) = r^4(4-4\gamma-5\gamma^2+16\gamma^3+24\gamma^4+12\gamma^5+2\gamma^6)$$
$$+4r^3(4-4\gamma+3\gamma^2+24\gamma^3+17\gamma^4+6\gamma^5+\gamma^6)\delta$$
$$+2r^2(8-28\gamma+39\gamma^2+96\gamma^3+38\gamma^4+6\gamma^5+\gamma^6)\delta^2+4r\gamma(-12+27\gamma+40\gamma^2+11\gamma^3)\delta^3$$
$$+12\gamma^2(2+\gamma)^2\delta^4,$$

$$p_3(r,\gamma,\delta) = r^6\gamma^4(2+\gamma)(3+\gamma)^2+2r^5(8-14\gamma+33\gamma^3+50\gamma^4+43\gamma^5+16\gamma^6+2\gamma^7)\delta$$
$$+r^4(80-140\gamma+32\gamma^2+329\gamma^3+242\gamma^4+135\gamma^5+48\gamma^6+6\gamma^7)\delta^2$$
$$+4r^3(32-72\gamma+44\gamma^2+159\gamma^3+76\gamma^4+24\gamma^5+8\gamma^6+\gamma^7)\delta^3$$
$$+r^2(64-272\gamma+320\gamma^2+613\gamma^3+208\gamma^4+26\gamma^5+8\gamma^6+\gamma^7)\delta^4$$
$$+16r\gamma(-6+15\gamma+19\gamma^2+5\gamma^3)\delta^5+16\gamma^2(2+\gamma)^2\delta^6,$$

$$p_4(r,\gamma,\delta) = r^6\gamma^4(3+\gamma)^2 + 6r^5\gamma^4(3+\gamma)^2\delta + r^4(16-40\gamma+29\gamma^2+38\gamma^3+126\gamma^4+78\gamma^5+13\gamma^6)\delta^2$$
$$+ 4r^3(16-40\gamma+29\gamma^2+38\gamma^3+36\gamma^4+18\gamma^5+3\gamma^6)\delta^3$$
$$+ 4r^2(16-48\gamma+43\gamma^2+54\gamma^3+22\gamma^4+6\gamma^5+\gamma^6)\delta^4$$
$$+ 16r\gamma(-4+7\gamma+8\gamma^2+2\gamma^3)\delta^5 + 8\gamma^2(2+\gamma)^2\delta^6,$$

$$p_5(r,\gamma,\delta) = r^2(-8+8\gamma+26\gamma^2+8\gamma^3+9\gamma^4+6\gamma^5+\gamma^6)$$
$$+ 2r(-8+24\gamma+30\gamma^2+8\gamma^3+9\gamma^4+6\gamma^5+\gamma^6)\delta$$
$$+ \gamma(2+\gamma)^2(12-3\gamma+2\gamma^2+\gamma^3)\delta^2,$$

$$p_6(r,\gamma,\delta) = r^4\gamma^4(3+\gamma)^2 + r^3(-8+14\gamma+16\gamma^2-5\gamma^3+30\gamma^4+23\gamma^5+4\gamma^6)\delta$$
$$+ 2r^2(-12+25\gamma+22\gamma^2-4\gamma^3+21\gamma^4+17\gamma^5+3\gamma^6)\delta^2$$
$$+ r(-16+60\gamma+40\gamma^2-\gamma^3+30\gamma^4+23\gamma^5+4\gamma^6)\delta^3 + \gamma(2+\gamma)^2(6-3\gamma+2\gamma^2+\gamma^3)\delta^4,$$

$$p_7(r,\gamma,\delta) = r^4\gamma^4(3+\gamma)^2 + 4r^3\gamma^4(3+\gamma)^2\delta + r^2(-16+40\gamma+3\gamma^2-6\gamma^3+12\gamma^4+42\gamma^5+7\gamma^6)\delta^2$$
$$+ 2r(-16+40\gamma+3\gamma^2-6\gamma^3+26\gamma^4+18\gamma^5+3\gamma^6)\delta^3 + 2\gamma(16+4\gamma+9\gamma^3+6\gamma^4+\gamma^5)\delta^4.$$

It is straightforward to check that the polynomials $p_i(r,\gamma,\delta), i = 1,\ldots,7$ always take positive values for any value of the parameters r, δ and $\gamma > 2$. Therefore, $\chi_1(\gamma,\delta,d,r)$ is positive too ($\chi_2(\gamma,\delta,d,r)$ is also positive), and in consequence condition (72) is fulfilled, and inequality $C_1^c > C_1^{nc}$ is always satisfied.

Next we compare the present value of net profits for $x_0 = 0$. First, we show that A_1^{nc} is lower than A_1^c. Suppose that this is not the case. Then using (29) and (52) the following inequality must hold

$$\frac{\gamma(4+\gamma)(r+2\delta) - \sqrt{\gamma(4+\gamma)(4d(2+\gamma)^2 + \gamma(4+\gamma)(r+2\delta)^2)}}{2(2+\gamma)^2}$$
$$\leq \frac{1}{2}(r+2\delta - \sqrt{(r+2\delta)^2+4d}),$$

that simplifying terms gives

$$(2+\gamma)^2\sqrt{(r+2\delta)^2+4d} - 4(r+2\delta)$$
$$\leq \sqrt{\gamma(4+\gamma)(4d(2+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)},$$

where the left-hand side is positive for $d \geq 0$. Notice that the expression is positive for $d = 0$ and increasing with d. Taking square in both sides of the inequality yields

$$(r + 2\delta)^2 + 8d \leq (r + 2\delta)\sqrt{(r + 2\delta)^2 + 4d},$$

that taking square again leads to the following contradiction

$$3\,(r + 2\delta)^2\, d + 16d^2 \leq 0.$$

Thus, we can conclude that A_1^{nc} is lower than A_1^c.

Next, we compare C_2^c and C_2^{nc}. Notice that for $x_0 = 0$, $V^c(0) = C_2^c$ and $V^{nc}(0) = C_2^{nc}$. Taking into account the expressions of C_2^c and C_2^{nc} given by (41) and (63) $C_2^c < C_2^{nc}$ if and only if

$$\begin{aligned}
&- \left[(4 - 2\gamma + \gamma^2)\left(A_1^{nc}\right)^2 - 2\gamma(2+\gamma)(r+\delta)A_1^{nc} + \gamma^2(1+2\gamma)(r+\delta)^2\right] \\
&\qquad \times \left[(2+\gamma)^2 A_1^c - \gamma(4+\gamma)(r+\delta)\right]^2 \\
&+ \left[\left(A_1^c\right)^2(2+\gamma)^3 - 2\gamma(8+6\gamma+\gamma^2)(r+\delta)A_1^c + \gamma^2(8+9\gamma+2\gamma^2)(r+\delta)^2\right]\gamma^3(r+\delta-A_1^{nc})^2 < 0. \quad (73)
\end{aligned}$$

With the help of Mathematica 10.1 this expression can be written as the following product

$$\begin{aligned}
&\left[-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma) - 2\gamma(r+\delta)\right) + A_1^{nc}\left(2(2+\gamma)A_1^c + \gamma(-4+\gamma+\gamma^2)(r+\delta)\right)\right] \times \\
&\qquad \left\{A_1^{nc}\left[2(2+\gamma)^2 A_1^c - \gamma(8+\gamma(1+\gamma)(2+\gamma))(r+\delta)\right]\right. \\
&\qquad \left. +\gamma(r+\delta)\left[-(1+\gamma)(2+\gamma)^2 A_1^c + 2\gamma(2+4\gamma+\gamma^2)(r+\delta)\right]\right\}
\end{aligned}$$

where the factor in curly brackets is positive provided that both A_1^c and A_1^{nc} are negative values. Then (73) is negative if and only if

$$-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma) - 2\gamma(r+\delta)\right) + A_1^{nc}\left(2(2+\gamma)A_1^c + \gamma(-4+\gamma+\gamma^2)(r+\delta)\right) > 0,$$

that can be rewritten as

$$-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma) - 2\gamma(r+\delta)\right) + \gamma(-4+\gamma+\gamma^2)(r+\delta)A_1^{nc} + 2(2+\gamma)A_1^{nc}A_1^c > 0.$$

Taking into account that $A_1^c > A_1^{nc}$, a sufficient condition that ensures the fulfillment of the last inequality is given by

$$-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma) - 2\gamma(r+\delta)\right) + \gamma(-4+\gamma+\gamma^2)(r+\delta)A_1^{nc} + 2(2+\gamma)\left(A_1^{nc}\right)^2 > 0.$$

Substituting the expressions of A_1^c and A_1^{nc} and rearranging terms, the inequality above reads

$$
\begin{aligned}
&2(2d(2+\gamma)^2+r^2(4+2\gamma^2+\gamma^3)+r(16+4\gamma+3\gamma^2+\gamma^3)\delta+2(8+4\gamma+\gamma^2)\delta^2)\\
&\quad+\gamma(1+\gamma)(r+\delta)\sqrt{\gamma(4+\gamma)[4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}\\
&\quad-(2+\gamma)\left(r(4-2\gamma+\gamma^2+\gamma^3)+(8+\gamma^2+\gamma^3)\delta\right)\sqrt{4d+(r+2\delta)^2}>0.
\end{aligned}
$$

The first and second lines in the inequality above are positive, while the third one is negative. Therefore, the inequality above is equivalent to the following inequality

$$
\begin{aligned}
&\big[2(2d(2+\gamma)^2+r^2(4+2\gamma^2+\gamma^3)+r(16+4\gamma+3\gamma^2+\gamma^3)\delta+2(8+4\gamma+\gamma^2)\delta^2)\\
&\quad+\gamma(1+\gamma)(r+\delta)\sqrt{\gamma(4+\gamma)[4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}\big]^2\\
&\quad-\left[(2+\gamma)\left(r(4-2\gamma+\gamma^2+\gamma^3)+(8+\gamma^2+\gamma^3)\delta\right)\right]^2(4d+(r+2\delta)^2)>0.
\end{aligned}
$$

After some calculus, the expression above can be rewritten as:

$$
\Omega_1(\gamma,\delta,d,r)+\Omega_2(\gamma,\delta,d,r)\sqrt{\gamma(4+\gamma)[4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}>0, \tag{74}
$$

where $\Omega_1(\gamma,\delta,d,r)$ and $\Omega_2(\gamma,\delta,d,r)$ are positive and given by

$$
\begin{aligned}
\Omega_1(\gamma,\delta,d,r) &= \gamma(1+\gamma)(r+\delta)\times\\
&\quad\left[2d(2+\gamma)^2+r^2(4+\gamma^2(2+\gamma))+r(16+\gamma(4+\gamma(3+\gamma)))\delta+2(8+\gamma(4+\gamma))\delta^2\right],\\
\Omega_2(\gamma,\delta,d,r) &= 4d^2(2+\gamma)^4\\
&\quad+4d\gamma(2+\gamma^2)(r+\delta)\left[r(4+\gamma(-1+\gamma+3\gamma^2+\gamma^3))+(2+\gamma)(4-3\gamma+\gamma^3)\delta\right]\\
&\quad+\gamma^2\left[r^4(16+\gamma(1+\gamma)(-4+\gamma(2+\gamma)(4+\gamma)))+r^3(80+\gamma(1+\gamma)(-28+\gamma(32+\gamma(31+6\gamma))))\delta\right.\\
&\quad+r^2(128+\gamma(-104+\gamma(-40+\gamma(3+\gamma)(35+13\gamma))))\delta^2\\
&\quad\left.+4r(-4+\gamma(2+\gamma))(-4+9+\gamma(10+3\gamma)))\delta^3+4\gamma(-24+\gamma(-16+\gamma(3+\gamma(5+\gamma))))\delta^4\right].
\end{aligned}
$$

$\Omega_1(\gamma,\delta,d,r)$ is always positive, and it can be easily proved that Ω_2 is positive for any value of γ greater than 2. Therefore, the inequality in (74) is always satisfied for $\gamma>2$, and consequently, $C_2^c<C_2^{nc}$ what implies that $V^c(0)$ is lower than $V^{nc}(0)$.

References

[1] Abrego, Lisandro, and Carlo Perroni (2002). "Investment Subsidies and Time-Consistent Environmental Policy." *Oxford Economic Papers*, 54, 617-635.

[2] Barnett, A. H. (1980). "The Pigouvian Tax Rule under Monopoly." *American Economic Review*, 70, 1037-1041.

[3] Benchekroun, Hassan and Ngo Van Long (1998). "Efficiency-Inducing Taxation for Polluting Oligopolists." *Journal of Public Economics*, 70, 325-342.

[4] Benchekroun, Hassan, and Ngo Van Long (2002). "On The Multiplicity of Efficiency-Inducing Tax Rules." *Economic Letters,* 76, 331-336.

[5] Bertinelli, Luisito, Luca Marchiori, Amer Tabakovic, and Benteng Zou (2017). "The Impact of Unilateral Commitment on Transboundary Pollution." *Environmental Modeling & Assessment*, forthcoming.

[6] Biglaiser, Gary, John K. Horowitz, and John Quiggin (1995). "Dynamic Pollution Regulation." *Journal of Regulatory Economics*, 8, 33-44.

[7] Brunner, Steffen, Christian Flachsland, and Robert Marschinski (2012). "Credible Commitment in Carbon Policy." *Climate Policy*, 12, 255-271.

[8] D'Amato, Alessio, and Bouwe R. Dijkstra (2015). "Technology Choice and Environmental Regulation under Asymmetric Information." *Resource and Energy Economics*, 41, 224-247.

[9] Denicolò, Vincenzo (1999). "Pollution-Reducing Innovations under Taxes or Permits." *Oxford Economic Papers*, 51, 184-199.

[10] Feenstra, Talitha, Peter M. Kort, and Aart de Zeeuw (2001). "Environmental Policy Instruments in an International Duopoly with Feedback Investment Strategies." *Journal of Economic Dynamics & Control,* 25, 1665-1687.

[11] Gerlagh, Reyer, and Matti Liski (2017). "Consistent Climate Policies." *Journal of the European Economic Association*, forthcoming.

[12] Golombek, Rolf, Mads Greaker, and Michael Hoel (2010). "Carbon Taxes and Innovation without Commitment." *B.E. Journal of Economic Analysis and Policy*, 10(1).

[13] Haurie, Alain, Jacek B. Krawczyk, and Georges Zaccour (2012). *Games and Dynamic Games.* World Scientific.

[14] Kort, Peter M. (1996). "Pollution Control and the Dynamics of the Firm: the Effects of Market-Based Instruments on Optimal Firm Investments." *Optimal Control Applications & Methods,* 17, 267-279.

[15] Kydland, Finn E., and Edward C. Prescott (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy*, 85, 473-492.

[16] Laffont, Jean-Jacques, and Jean Tirole (1996). "Pollution Permits and Environmental Innovation." *Journal of Public Economics*, 62, 127-140.

[17] Long, Ngo Van (2010). *A Survey of Dynamic Games in Economics.* World Scientific.

[18] Marsiliani, Laura, and Thomas I. Renström (2000). "Time Inconsistency in Environmental Policy: Tax Earmarking as a Commitment Solution." *Economic Journal*, 110, C123-C138.

[19] Moner-Colonques, Rafael, and Santiago J. Rubio (2016). "The Strategic Use of Innovation to Influence Environmental Policy: Taxes versus Standards." *B.E. Journal of Economic Analysis and Policy*, 16, 973-1000.

[20] Montero, Juan Pablo (2011). "A Note on Environmental Policy and Innovation when Governments Cannot Commit." *Energy Economics*, 33, S13-S19.

[21] Petrakis, Emmanuel, and Anastasios Xepapadeas (1999). "Does Government Precommitment Promote Environmental Innovation?" In *Environmental Regulation and*

Market Power, edited by Emmanuel Petrakis, Eftichios Sartzetakis, and Anastasios Xepapadeas. Edward Elgar.

[22] Petrakis, Emmanuel, and Anastasios Xepapadeas (2003). "Location Decisions of a Polluting Firm and the Time Consistency of Environmental Policy." *Resource and Energy Economics*, 25, 197-214.

[23] Poyago-Theotoky, Joanna, and Khemarat Teerasuwannajak (2002). "The Timing of Environmental Policy: A Note on the Role of Product Differentiation." *Journal of Regulatory Economics*, 21, 305-316.

[24] Puller, Steven L. (2006). "The Strategic Use of Innovation to Influence Regulatory Standards." *Journal of Environmental Economics and Management,* 52, 690-706.

[25] Requate, Till (2005). "Timing and Commitment of Environmental Policy, Adoption of New Technology, and Repercussions on R&D." *Environmental and Resource Economics*, 31, 175-199.

[26] Rezai, Armon and Frederick van der Ploeg (2017). "Second-Best Renewable Subsidies to De-carbonize the Economy: Commitment and the Green Paradox." *Environmental and Resource Economics*, 66, 409-434.

[27] Stimming, Martina (1999). "Capital-Accumulation Games under Environmental Regulation and Duopolistic Competition." *Journal of Economics*, 69, 267-287.

[28] Tauri, Nori, and Stephen Polasky (2005). "Environmental Regulation with Technology Adoption, Learning and Strategic Behavior." *Journal of Environmental Economics and Management*, 50, 447-467.

[29] Ulph, Alistair, and David Ulph (2013). "Optimal Climate Change Policies when Governments Cannot Commit." *Environmental and Resource Economics*, 56, 161-176.

[30] Wirl, Franz (2014). "Taxes versus Permits as Incentive for the Intertemporal Supply of a Clean Technology by a Monopoly." *Resource and Energy Economics*, 36, 248-269.

[31] Xepapadeas, Anastasios P. (1992). "Environmental Policy, Adjustment Costs, and Behavior of the Firm." *Journal of Environmental Economics and Management*, 23, 258-275.

[32] Yanase, Akihiko (2009). "Global Environment and Dynamic Games of Environmental Policy in an International Duopoly." *Journal of Economics* 97, 121-140.

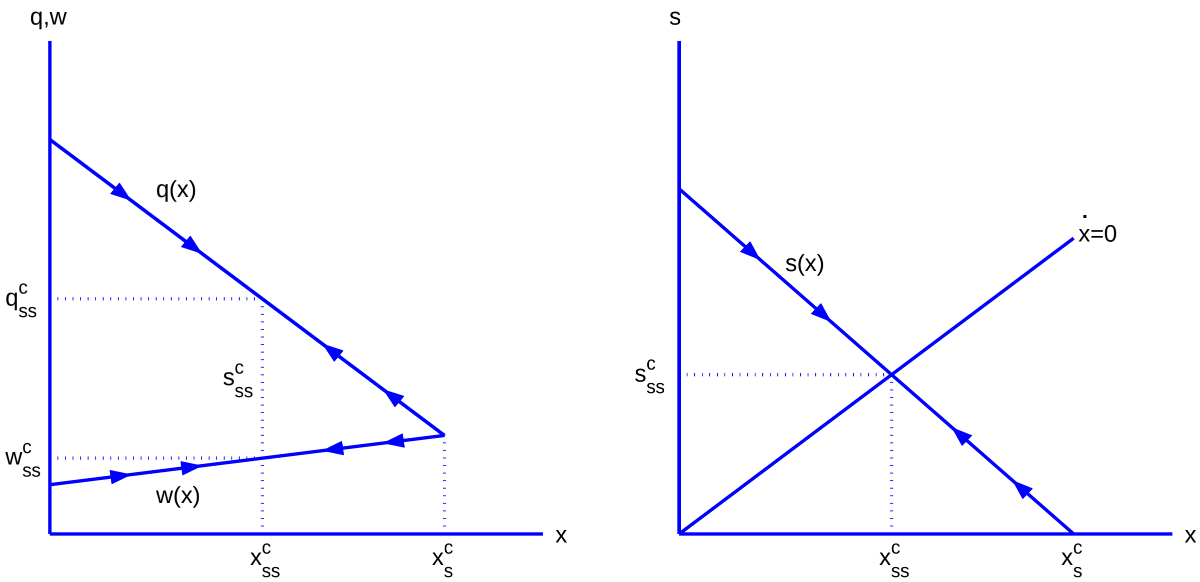

Figure 1: Optimal strategies and steady state