Stable Partial Cooperation in Managing Ecological Systems with Tipping Points

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Abstract

This paper combines two strands of literature. In the literature on managing ecological systems with tipping points, such as the well-known lake system, an important conclusion is that non-cooperation may lead to low welfare, because initial conditions may yield a Nash equilibrium in the bad state of the system. In the literature on cartels or international environmental agreements, an important conclusion is that stable coalitions are small, where stable means balancing the incentives to cooperate and the incentives to free ride. In this paper it is shown that larger stable coalitions exist in managing ecological systems with tipping points. The reason is that the incentives to cooperate increase, either in order to prevent tipping to a bad state of the system, or in order to induce tipping to a good state of the system. This implies that if the ecological system is initially in a bad state, the policy advice is to first form a stable coalition to shift to a good state, and then to form a stable coalition to prevent shifting back to a bad state.

JEL classification: C70; Q20

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1 Introduction

It is quite common in ecosystems that tipping points are observed which means that suddenly a non-marginal shift occurs that is not easy to restore (e.g. Scheffer *et al.* 2001). In the seminal paper by Ludwig *et al.* (1978), an insect outbreak was modelled as the interaction between the insect, its predator and the tree foliage that provides food and cover. A slow increase in the foliage has a marginal effect on the insect density first, but at some point the coverage from the predator and an abundance of food leads to a large and persistent jump in the density. A slow decrease in the foliage has the opposite effect, and at some point the insect density jumps back to a low level. This natural interacting dynamics causes tipping of the insect density back and forth.

Human activities may also be involved. A famous example is the lake (Scheffer 1998, Carpenter 2003). Agricultural activities yield the release of phosphorus on the lake. At first, this only has a marginal effect, but at some point the lake flips to a so-called turbid state, with a much lower level of water quality, availability of fish and amenity values. The release of phosphorus has to be substantially reduced to have the lake flip back to its original state, and in some cases this flip may even be irreversible. This means that at certain levels of the release of phosphorus, the lake has two stable steady states: one in the so-called oligotrophic regime of the lake, with a high level of ecosystem services, and one in the so-called eutrophic regime of the lake, with a low level of ecosystem services. This human-ecological interaction puts an interesting challenge to the management of these ecosystems. First, a trade-off has to be formulated between the benefits of using the lake as a waste sink for the agricultural activities, on the one hand, and the costs of affecting the ecosystem services, on the other hand. Second, optimal management of this trade-off has to take account of the possibility of tipping (e.g. Brock & Starrett 2003, Mäler *et al.* 2003, Wagener 2003, Dechert & O'Donnell 2006, Kossioris *et al.* 2008, Kiseleva & Wagener 2010, Kossioris *et al.* 2011, Heijdra & Heijnen 2013, Wagener 2013). The lake system can be seen as a metaphor for many of the environmental issues that humanity is facing, such as the coral reef system (e.g. Crépin 2007) and the climate system (e.g. Lenton & Ciscar 2013). Therefore, the results are also applicable to these issues, but we will develop the results for the lake system.

Mäler *et al.* (2003) consider the case that the lake is a common-property resource that is used by a number of communities around the lake. They compare cooperation and non-cooperation in managing the lake. Non-cooperation will lead to lower welfare, as usual, but in this case a considerable drop in welfare may occur. The reason is that two Nash equilibria may exist, one in the oligotrophic regime and one in the eutrophic regime of the lake.

The oligotrophic Nash equilibrium is close to the optimal management outcome, and has therefore only slightly lower welfare, but the eutrophic Nash equilibrium has much lower welfare. The communities may be trapped in the bad Nash equilibrium, depending on the initial condition. An important question is how to get out of the bad Nash equilibrium.

Cooperation is the answer but cooperation is not stable in the sense that free-rider incentives usually dominate the incentives to cooperate. This was extensively investigated in the literature on International Environmental Agreements. Based on the stability concept in cartel theory (d'Aspremont *et al.* 1983), it was concluded that the size of the stable coalition is basically small, especially if the stakes are high (Hoel 1992, Carraro & Siniscalco 1993, Barrett 1994, Finus 2003). Many extensions and alternative approaches have been developed (Finus & Caparros 2015), but in this paper we only use the basic stability concept and investigate stability in the lake model. Barrett (2013) investigates climate treaties in the presence of tipping points, but his model is different from the lake model.

The usual grim mechanisms that free-rider incentives dominate the incentives to cooperate still apply, but a new possibility occurs in the lake model. Partial cooperation in a coalition effectively lowers the number of players, and this may imply that the bad Nash equilibrium disappears and that the lake can be shifted to the oligotrophic regime. In this paper we first characterise the Nash equilibria of the game, with an arbitrary number of players. We will perform a quasi-static analysis in the sense that the players each choose a fixed level of loading of phosphorus on the lake and that the lake quickly reaches the steady state, so that we only consider the steady states of the lake. The initial condition still matters, of course. Then we consider partial cooperation. It may happen that the incentive to cooperate is stronger than the incentive to free ride, if more cooperation will shift the lake from the eutrophic regime to the oligotrophic regime. After the shift, partial cooperation may be needed to prevent shifting back to the eutrophic regime. This implies that if we allow some changes in membership over time, there is a way to get out of the bad Nash equilibrium with partial cooperation in a coalition that has the usual stability properties.

Section 2 presents the lake model and characterises the Nash equilibria of the quasi-static game. Section 3 analyses stability of coalitions. Section 4 concludes the paper.

2 Nash Equilibria in the Lake Model

The lake model with tipping points was developed by Carpenter & Cottingham (1997) and Scheffer (1998). It was shown by Carpenter *et al.* (1999) that the most important dynamics is given by

$$\dot{x}(t) = a(t) - bx(t) + \frac{x^2(t)}{x^2(t)+1}, \quad a = \sum_{i=1}^{n} a_i, \quad x(0) = x_0, \tag{2.1}$$

where x represents the phosphorus accumulated in algae in the water of the lake, a the phosphorus loadings on the lake and n the number of players. The parameter b is a characteristic for each lake, and is composed of a number of parameters from the underlying lake model. The non-linear term in (2.1) originates from the recycling of phosphorus from the bottom of the lake into the water. The full model incorporates the slow dynamics of sedimentation and recycling in the bottom of the lake (Carpenter *et al.* 1999, Grass *et al.* 2016), but we abstract from that slow dynamics here. The steady-state curve

$$a = f(x) := bx - \frac{x^2}{x^2+1}, \quad a = \sum_{i=1}^{n} a_i, \tag{2.2}$$

has different shapes, depending on the value of the parameter b. If $b \geqslant 3\sqrt{3}/8 \approx 0.6495$, each level of total loading a corresponds to one steady state x, but if $\frac{1}{2} < b < 3\sqrt{3}/8$, a range of loadings exist with two stable steady states and one unstable steady state in between. This case has two tipping points: one with the property that a higher loading will cause a shift to a steady state in the eutrophic regime of the lake, and one with the property that a lower loading will cause a shift to a steady state in the oligotrophic regime of the lake. If $b \leqslant \frac{1}{2}$, the lower tipping point moves below the x-axis. This means that once the lake is eutrophic, it is not possible to return to the oligotrophic regime of the lake, because loading cannot be negative (irreversibility). If $\frac{1}{2} < b < 3\sqrt{3}/8$, it is possible to return, but this requires a non-marginal decrease in loading (hysteresis). In the sequel we take $b = 0.52$, and then we get Figure 1. The tipping points are denoted by (a_1, x_1) and (a_2, x_2), respectively. The solid curves denote the stable steady states and the dashed curve denotes the unstable steady states. For total loadings between a_1 and a_2, there are two stable steady states, one in the oligotrophic regime to the left and one in the eutrophic regime to the right. The shaded area shows for which initial conditions x_0 and levels of total loading a, the lake will end up in the oligotrophic regime.

Following Mäler *et al.* (2003), the objectives of the players are given by

$$\max_{a_i} w_i = \log a_i - cx^2, \quad i = 1, 2, ..., n, \tag{2.3}$$

subject to (2.2), where a_i denotes the loading of phosphorus by player i and x the resulting steady-state level of accumulated phosphorus in the water of the lake. The parameter c

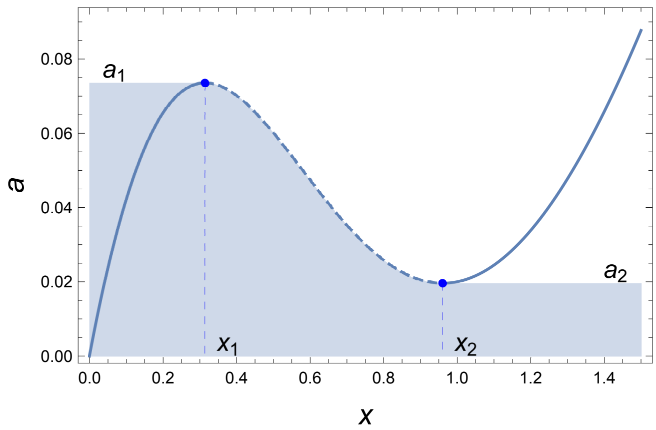


Figure 1: *Steady states of the shallow lake dynamics for* $b = 0.52$*: stable steady states are on the solid lines, unstable ones on the dashed line.*

weighs the benefits, $\log a_i$, of using the lake as a waste sink, on the one hand, and the loss of ecosystem services due to the accumulated phosphorus, x^2, on the other hand. The Lagrangians become

$$L_i = \log a_i - cx^2 + \lambda_i(a - f(x)), \quad i = 1, 2, ..., n. \tag{2.4}$$

The first-order conditions for the Nash equilibria become

$$\frac{1}{a_i} = -\lambda_i, \quad -\lambda_i f'(x) = 2cx, \quad a = f(x), \quad i = 1, 2, ..., n. \tag{2.5}$$

As λ_i is the shadow value of pollution, the first condition says that in equilibrium marginal benefits from loading equal marginal — shadow — costs; the second that marginal benefits from polluting the lake should equal marginal costs, and the third equation postulates dynamic equilibrium of the lake dynamics. It follows that the — symmetric — candidate Nash equilibria are given by

$$f(x) = g(x), \tag{2.6}$$

$$f(x) = bx - \frac{x^2}{x^2+1}, \quad g(x) = \frac{n}{2cx} f'(x), \tag{2.7}$$

$$a_i = \frac{f'(x)}{2cx}, \quad i = 1, 2, ..., n. \tag{2.8}$$

That is, the candidate Nash equilibria are characterised by the intersection points of the graphs of $f(x)$ and $g(x)$.

Figure 1 shows that for $b = 0.52$ the curve $f(x)$ has two tipping points, respectively at

x_1 and x_2. At a tipping point, the function $f'(x)$, and hence also $g(x)$, changes sign. As $g(x) \to \infty$ as $x \to 0$ and $g(x) \to 0$ as $x \to \infty$, it follows that the equation $f(x) = g(x)$ has always one solution x_l below x_1, and has either zero or, if n/c is sufficiently large, two solutions x_m and x_l above x_2. See Figure 2, where $c = 0.7$ and $n = 10$. The state x_l and the state x_h (if it exists) are the steady states of candidate Nash equilibria, but the state x_m (if it exists) cannot yield a Nash equilibrium, because the second-order conditions are not fulfilled. In the state x_l the damage cx_l^2 is low, because the level of phosphorus is low and thus the level of ecological services is high. In the state x_h (if it exists) the damage cx_h^2 is high, because the level of phosphorus is high and thus the level of ecological services is low.

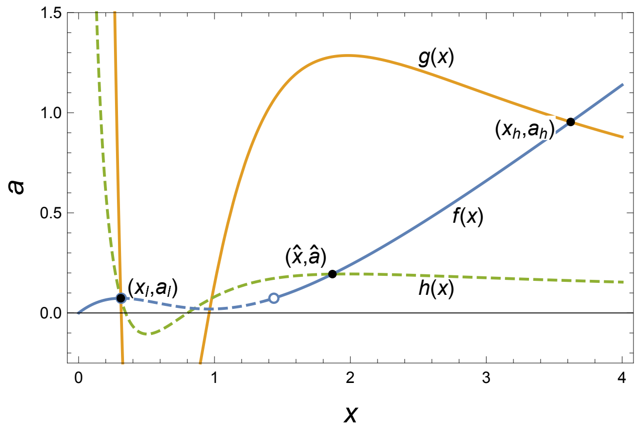


Figure 2: *Oligotrophic initial state: Graphs of $f(x)$, $g(x)$ and $h(x)$. Candidate Nash equilibria are located at intersections of $f(x)$ and $g(x)$. Intersections of $f(x)$ and $h(x)$ are critical points of the welfare of an individual player. Parameters:* $(b, c) = (0.52, 0.7)$

The steady states x_l and x_h correspond to candidate Nash equilibria, but we still have to verify that these are indeed Nash equilibria. It may happen that the loading a_i in (2.8) of player i is not a best response to the loadings a_j, $j \neq i$, of the other players. It will be a local best response, where locally the second-order conditions are satisfied, but it may not be a global best response. Consider, for example, the steady state x_l. The best response follows from maximising (2.3), subject to

$$a_i = f(x) - \sum_{j \neq 1} a_j(x_l); \tag{2.9}$$

here $a_j(x_l)$ denotes the loading (2.8) at x_l. The Lagrangian becomes

$$L_i = \log a_i - cx^2 + \lambda_i \left(a_i - f(x) + \sum_{j \neq 1} a_j(x_l) \right). \tag{2.10}$$

The first-order conditions for optimisation become

$$\frac{1}{a_i} + \lambda_i = 0, \quad \lambda_i f'(x) = -2cx, \quad a_i + \sum_{j \neq 1} a_j(x_l) = f(x). \tag{2.11}$$

The candidates for the best response solve

$$f(x) = h(x), \tag{2.12}$$

where

$$h(x) = \frac{f'(x)}{2cx} + \sum_{j \neq 1} a_j(x_l) \quad \text{and} \quad a_i = \frac{f'(x)}{2cx}. \tag{2.13}$$

One candidate is obviously x_l, but there may be another candidate $\hat{x}$ that arises as an intersection point of the curves $f(x)$ and $h(x)$, in the same way as the steady states in the analysis above: see the dotted line in Figure 2. This candidate may yield higher welfare for player i than x_l: if the other players stick to their loadings in x_l, one player may have an incentive to increase the total loading substantially and to shift the lake to the eutrophic regime.

Figure 3 shows the welfare of player i. There is a local maximum $w(a_{l,i}) = -4.9802$ for $a_{l,i} = 0.0074$, but player i can increase its welfare to $w(\hat{a}_i) = -4.4989$ by pushing up the loading to $\hat{a}_i = 0.1280$ and the total loading to $\hat{a} = \hat{a}_i + (n-1)a_{l,i} = 0.1941$, which shifts the lake to the eutrophic regime. It follows that x_l is not the steady state of a Nash equilibrium. The same can happen for the candidate Nash equilibrium x_h.

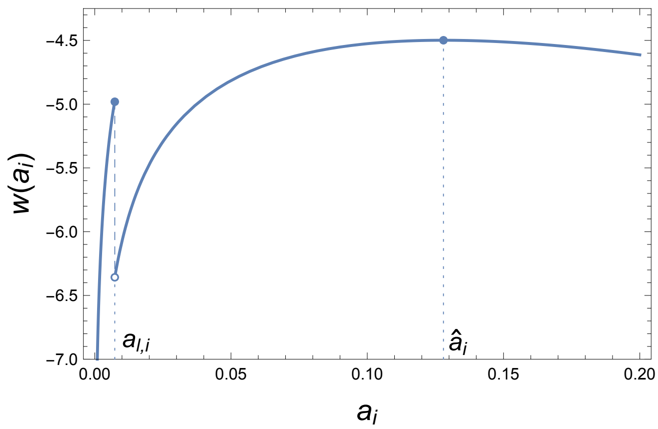


Figure 3: *Oligotrophic initial state: A candidate Nash equilibrium that is not Nash. Payoff of deviating player. Parameters:* $(b, c) = (0.52, 0.7)$

The initial condition matters as well. For example, if x_h exists — that is if the curves $f(x)$ and $g(x)$ have three intersection points — and the corresponding level of total loading a_h

lies below the tipping point a_1, the state x_h cannot be reached from the initial conditions x_0 in the shaded region of Figure 1. This region contains all initial conditions that are smaller than the unstable steady state x for a_h. Similarly, x_l cannot be reached for sufficiently high initial conditions x_0. We will refer to the domain of attraction as the initial conditions from where a certain steady state can be reached.

The parameter n/c drives the results. If n/c is small, the curves $f(x)$ and $g(x)$ intersect only once, so that there is only one Nash equilibrium, located in the oligotrophic regime of the lake. This implies that if the number of players n is small and/or the weight parameter c is large — that is, damage to the ecosystem services is relatively important — the Nash equilibrium will be in the oligotrophic regime and the loss of welfare in the absence of cooperation is relatively small. However, if n/c is large, the curves $f(x)$ and $g(x)$ have three intersection points, so that there may be two Nash equilibria, one in the oligotrophic regime and one in the eutrophic regime. In other words, if the number of players n increases or the importance of the damage to the ecosystem decreases, another candidate Nash equilibrium pops up. If this is indeed a Nash equilibrium, and if the initial condition of the lake is in the domain of attraction of this Nash equilibrium, the loss of welfare can be considerable.

This was basically the message of Mäler et al. (2003). It is interesting to note that if the number of players n decreases, the opposite may occur. The numbers of players in any problem is fixed, of course, but cooperation essentially lowers the number of players. Mäler et al. (2003) had two players and compared full cooperation with non-cooperation, that is, they compared $n = 1$ with $n = 2$. They argued that in the absence of cooperation, the loss of welfare can be substantial if the lake shifts to the eutrophic regime. In the present paper, we will take a higher number of players n and we will consider partial cooperation. In the literature on stable International Environmental Agreements, it is well known that it is usually not possible to sustain a high level of cooperation, as free-rider incentives dominate the incentives to cooperate.

In the present context, this is different. First, if the lake is initially in the oligotrophic regime, it may happen that non-cooperation induces a costly shift to the eutrophic regime, while cooperation keeps the lake in the oligotrophic regime. Second, if the lake is initially eutrophic, a certain level of cooperation can shift the lake to the oligotrophic regime, so that the incentives to cooperate can be sufficiently high for a large stable coalition to occur. After the shift, the size of the coalition may change again. The question is whether a temporary large stable coalition can get the lake out of the Nash equilibrium in the eutrophic regime and can decrease the welfare loss substantially, in case cooperation is difficult because it is not stable. This is the subject of the next section.

3 Coalition formation

The literature on International Environmental Agreements (IEAs) usually concludes that cooperation is hard to sustain, because free-rider incentives dominate the incentives to cooperate (e.g., Hoel, 1992, Carraro and Siniscalco, 1993, Barrett, 1994, Finus, 2003). It uses a stability concept that was developed in cartel theory (d'Aspremont et al., 1983). The best way to formalise this is to construct a two-stage membership game. In the first stage, the players choose to be a member of the coalition or not; in the second stage, a non-cooperative game is played between the coalition and the individual outsiders. The Nash equilibrium of this two-stage game has the so-called internal and external stability properties: a stable coalition has the property that a member of the coalition does not have an incentive to leave the coalition, and an outsider does not have an incentive to join the coalition. In order to investigate stability of a coalition in the lake game with n players, we first have to derive the Nash equilibrium between a coalition of size $k < n$, with $n - k$ individual outsiders.

The coalition maximises the sum of the objectives (2.3) of the members of the coalition, so that the Lagrangian for the coalition becomes

$$L = \sum_{i=1}^{k} \log a_i - kcx^2 + \lambda(a - f(x)), \tag{3.1}$$

and the Lagrangians for the individual outsiders become

$$L_i = \log a_i - cx^2 + \lambda_i(a - f(x)), \quad i = k+1, ..., n. \tag{3.2}$$

The first-order conditions for the Nash equilibria between the coalition and $n - k$ outsiders are

$$\frac{1}{a_i} + \lambda = 0, \quad \lambda f'(x) = -2kcx, \quad a = f(x), \quad i = 1, 2, ..., k, \tag{3.3}$$

$$\frac{1}{a_i} + \lambda_i = 0, \quad \lambda_i f'(x) = -2cx, \quad a = f(x), \quad i = k+1, ..., n. \tag{3.4}$$

These are solved by

$$a_i = \frac{f'(x)}{2kcx}, \quad i = 1, 2, ..., k, \tag{3.5}$$

$$a_i = \frac{f'(x)}{2cx}, \quad i = k+1, ..., n. \tag{3.6}$$

It follows that the candidate Nash equilibria between the coalition and $n-k$ outsiders are given by

$$f(x) = g(x,k) \tag{3.7}$$

where

$$g(x,k) := \frac{n-k+1}{2c}\frac{f'(x)}{x}. \tag{3.8}$$

We have moreover

$$a_i^m = \frac{1}{k(n-k+1)}f(x), \quad a_i^o = \frac{1}{n-k+1}f(x), \tag{3.9}$$

where m denotes coalition member and o denotes outsider. Note that effectively the coalition as a whole acts like an individual outsider, so that the total number of players becomes $n-k+1$.

Before we continue, it is interesting to consider the linear case, with $f(x) = bx$, because it confirms the usual grim result of small stable coalitions. In this case the second stage has one Nash equilibrium, given by

$$a_i^m = \frac{b}{k\sqrt{(n-k+1)2c}}, \quad a_i^o = \frac{b}{\sqrt{(n-k+1)2c}}, \tag{3.10}$$

$$x = \sqrt{\frac{n-k+1}{2c}}. \tag{3.11}$$

Welfare becomes

$$w_i^m(k) = \log\frac{b}{k\sqrt{(n-k+1)2c}} - \frac{1}{2}(n-k+1), \tag{3.12}$$

$$w_i^o(k) = \log\frac{b}{\sqrt{(n-k+1)2c}} - \frac{1}{2}(n-k+1). \tag{3.13}$$

We only have to check internal stability, because external stability will follow. Internal stability requires that a member of the coalition does not have an incentive to leave the coalition, that is

$$w_i^m(k) > w_i^o(k-1). \tag{3.14}$$

This implies

$$\frac{1}{2} > \log\frac{k\sqrt{n-k+1}}{\sqrt{n-k+2}} \Rightarrow \frac{k^2(n-k+1)}{n-k+2} < e. \tag{3.15}$$

It is easy to see that k cannot be larger than 2, regardless of the total number of players n, so that this gives the same grim picture as always.

However, in case of the lake, where $f(x) = bx - x^2/(x^2+1)$, it is possible to have a larger stable coalition, because a shift can be made from the Nash equilibrium x_h, with a high level of phosphorus, to a Nash equilibrium with a low level of phosphorus. By increasing the size of the coalition k, the curve of $g(x,k)$ is pulled down, so that the bad Nash equilibrium x_h may disappear. This may very well change the usual grim picture for the level of cooperation.

To investigate this further, we have to distinguish between the situation that the lake is initially in the oligotrophic regime, or in the eutrophic regime, or in between. Of course, in situations where there is only one steady state, the initial conditions do not matter. But if the total loading a is between the tipping values $a_1 < a < a_2$, then there are two attracting steady states, and it depends on the initial state x_0 which one is selected. From Figure 1 we can see that, given the total loading a, any initial state in the region $0 \leqslant x_0 \leqslant x_1$ will lead to the steady state in the oligotrophic regime. Likewise, given the loading a, any initial state $x_2 \leqslant x_0$ will lead to the steady state in the eutrophic regime. The former region we will call the 'oligotrophic' initial states, and the latter region the 'eutrophic' initial states. Only in the 'intermediate' region $x_1 < x_0 < x_2$, the outcome depends on the initial state. First, we will consider the 'oligotrophic' initial states and the 'eutrophic' initial states separately, and then we will connect the results in the last section.

3.1 Oligotrophic initial conditions

We remain in the context of Section 2, i.e. we retain $b = 0.52$ and $c = 0.7$, and we consider the payoffs of the players in the local Nash equilibria (a_l, x_l) and (a_h, x_h). These are

$$w_i^o(a_l) \approx -4.98 \quad \text{and} \quad w_i^o(a_h) \approx -11.53.$$

The welfare in the eutrophic local Nash equilibrium (a_h, x_h) is much lower than in the oligotrophic local equilibrium (a_l, x_l), as the lake has tipped.

Especially in the case of low welfare, it is rational for the players to start exploring the benefits of forming a coalition. Under full cooperation, the individual outcomes even improve on the Nash outcome $w_i^o(a_l)$. We will however see that the grand coalition is not stable, as players have incentives to leave the coalition.

At this point the issue arises how these incentive should be evaluated. A coalition member has to compare its present payoff to the payoff it would receive being an outsider to a coalition whose size is one less than the present coalition. However, the game between the remaining, smaller, coalition and the outsiders may have more than one Nash equilibrium, and it is not clear at which of these the outsider's payoff has to be evaluated.

If a coalition member leaves and the resulting game ends up in the Nash equilibrium with a lower welfare for the outsiders, the incentives of leaving the coalition are smaller than if the game ends up in the equilibrium with higher welfare for outsiders. In the former situation, the chances of stabilising the coalition are therefore larger. A game of this kind has been investigated by Brock & de Zeeuw (2002).

For oligotrophic initial states, if the game has two Nash equilibria, the oligotrophic one will be selected.

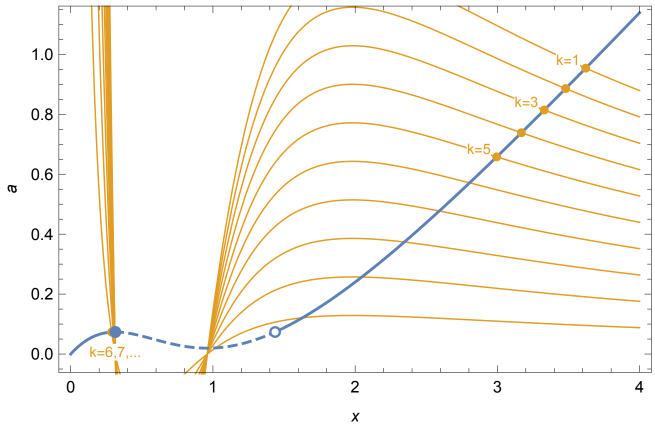


Figure 4: *Oligotrophic initial states: Functions $f(x)$ and $g(x,k)$ intersect in local Nash equilibria. Selected Nash equilibria for various coalition sizes are indicated by solid dots. Parameters:* $(b, c) = (0.52, 0.7)$

Figure 4 shows $f(x)$ and the $g(x,k)$ for $k = 1, 2, \ldots, 10$. For all values of k, there are two local Nash equilibria. A stability check shows that for the lower coalition sizes, $k = 1, \ldots, 5$, the oligotrophic local Nash equilibrium is not Nash: players have an incentive to choose larger deviating loadings, as illustrated in Figure 3. For the other values of k, both local equilibria are in fact Nash, and, the oligotrophic one is selected. The resulting Nash equilibria are indicated in the figure. The large open dot indicates the infimum of the eutrophic steady states that can be reached from an oligotrophic initial condition.

Table 1 shows the resulting payoffs $w_i^m(k)$ and $w_i^o(k)$ of coalition members and outsiders. To evaluate the incentive to leave or join a coalition, we introduce the quantity $\Delta(k)$, which is the difference between the equilibrium payoffs of a coalition member to a size-k coalition and an outsider to a size-$(k-1)$ coalition

$$\Delta(k) = w_i^m(k) - w_i^o(k-1).$$

k	x^*	w_i^m	w_i^o	Δ
1	3.6213	−11.5296	−11.5296	
2	3.4799	−11.4883	−10.7951	−0.0414
3	3.3297	−11.1444	−10.0458	0.3493
4	3.1690	−10.6652	−9.2789	0.6194
5	2.9953	−10.1002	−8.4907	0.8213
6	0.3078	−6.0778	−4.2860	−2.4129
7	0.3064	−6.0083	−4.0624	1.7223
8	0.3041	−5.8535	−3.7740	1.7911
9	0.2996	−5.5647	−3.3675	1.7907
10	0.2875	−4.9756		1.6082

Table 1: *Oligotrophic initial state: Coalition size k, Nash equilibrium states x^*, member and outsider payoffs w_i^m and w_i^o and leaving incentive Δ. Parameters: $(b, c) = (0.52, 0.7)$*

If this difference is positive, a coalition member has an incentive to leave; conversely, if it is negative, an outsider of a size-$(k-1)$ coalition has an incentive to join. In the terminology of d'Aspremont *et al.* (1983), a size-k coalition is 'internally stable' if $\Delta(k) \leq 0$ — coalitions members have no incentive to leave —, and it is 'externally stable' if $\Delta(k+1) \geqslant 0$ — outsiders have no incentive to join.

The critical coalition size is $k = 6$, as the system will tip to an eutrophic state if a member leaves the coalition. The resulting free-rider benefits are

$$\log a_i^o(5) - \log a_i^m(6) = -2.2106 - (-6.0115) = 3.8009,$$

but these are dominated by the large increase in damages that are the consequence of shifting to the eutrophic regime

$$cx^*(5)^2 - cx^*(6)^2 = 6.2802 - 0.0663 = 6.2139.$$

That is, the coalition of size 6 is internally stable. From the table, we learn that it is also externally stable, since $\Delta(7) = 1.7223 > 0$.

On the other hand, the coalition of size $k = 5$ is neither internally nor externally stable. What will happen if such a coalition is present depends on how the membership game is played. An outsider may join, which is the case we consider here, or a coalition member may leave, which will ultimately lead to the usual small-size stable coalition.

When we assume that — say on the basis of cheap talk — the players start by considering the grand coalition, we see that the first few players have an incentive to leave, as the Nash equilibrium state $x^*(k)$ of the game between coalition and outsiders does hardly change, and consequently the damage $c(x^*(k))^2$ remains almost constant. If by successive leaving the coalition, the size of the coalition has diminished to $k = 6$, any other member leaving the coalition will trigger a regime shift to the eutrophic regime, resulting in much larger damages and lower payoffs. Effectively the threat of a regime shift stabilises the coalition.

3.2 Effects of decreasing pollution costs

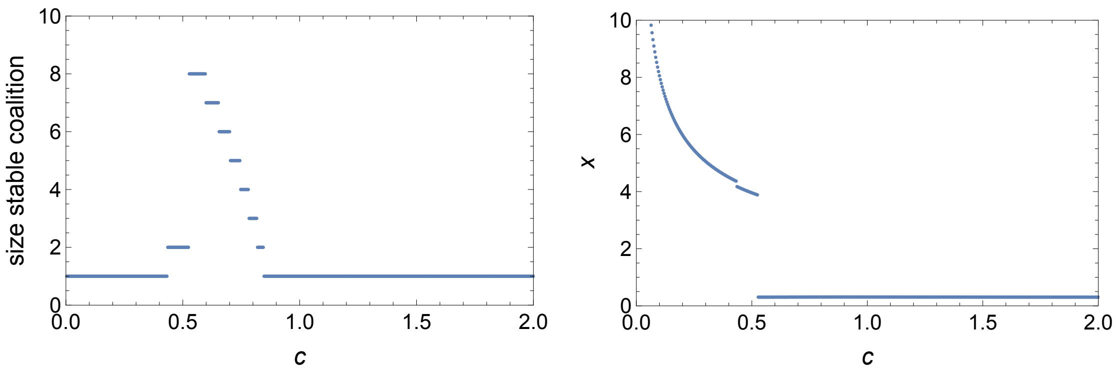
To better understand this stabilisation mechanism, we investigate the effect of changes in the weight parameter c. It could be argued that if c is larger, then a shift to the eutrophic regime is costlier, which should stabilise even larger coalitions.

The direct opposite is the case. Figures 5a and 6 show, respectively, the largest stable coalition size and the total welfare of the agents, defined as the sum of the individual welfares, in Nash equilibria for three coalition sizes: the individual Nash equilibrium $k = 1$, the full-cooperative outcome $k = n$, and the Nash equilibrium with the largest stable coalition. Throughout, the 'oligotrophic' selection rule is applied, if there is more than one Nash equilibrium.

The figures show that at high values of c, for instance at $c = 1$, no (non-trivial) coalition is stable. At the same time, the welfare difference between the individual Nash and the cooperative equilibria is negligible. As pollution costs are high, a regime shift is too costly, and the total loading a can therefore not exceed the critical value a_1. However, for all coalition sizes, total loading will be close to the critical value, which explains the small payoff differences.

If c is lowered, at a certain point around $c \approx 0.85$ the oligotrophic local Nash equilibrium ceases to be Nash through the mechanism illustrated in Figure 3: one player has an incentive to increase its loading. Other players will follow and the lake will shift to the eutrophic regime, so that the game ends up in the eutrophic local Nash equilibrium, which is the unique individual Nash equilibrium in this situation. The resulting drop in total welfare is clearly visible in Figure 6. The shift and the ensuing welfare loss can be avoided if two agents join in a coalition.

This mechanism repeats: at some point around $c \approx 0.82$, even a size-2 coalition cannot prevent tipping, but a size-3 coalition can. Continuing in this way, the coalition size keeps growing until it reaches the size $k = 8$. The total welfare reached by the coalition is always close to the full-cooperative value. On a scale where the total individual Nash welfare is 0 and the full-cooperative welfare is 1, it never drops below 0.9.



(a) *Size of largest stable coalition*

(b) *Steady states*

Figure 5: *Oligotrophic initial state: Coalition size and steady state as functions of c. Parameter* $b = 0.52$

Around $c \approx 0.53$, the mechanism stops working: the restrictions on the loadings of the members of a size-9 coalition, in order to prevent the regime shift, are too costly to compensate for the benefits. As a result, the lake shifts to the eutrophic regime and the largest stable coalition size falls, first to $k = 2$, and then for even smaller values of c back to $k = 1$. Finally, at around $c \approx 0.31$, it is even for the grand coalition optimal to let the system tip to the eutrophic regime.

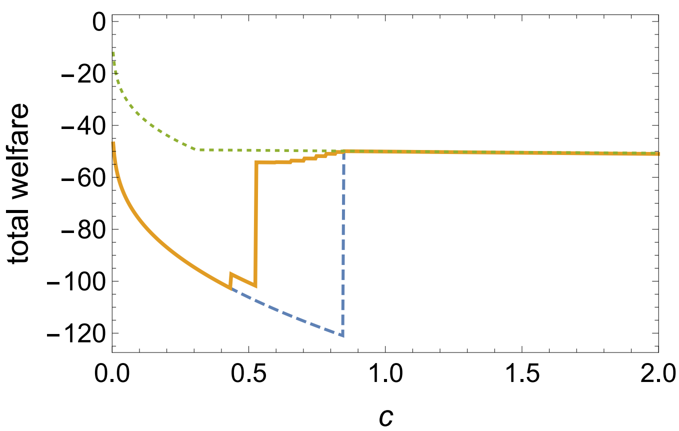


Figure 6: *Oligotrophic initial state: Total welfare as a function of the cost parameter c. Shown are Nash equilibria if* $k = 1$ *(dashed), Nash equilibria for the largest stable coalitions (solid), and the full-cooperative solution (dotted). Parameter* $b = 0.52$

3.3 Eutrophic initial conditions

We now turn to eutrophic initial conditions, i.e. the initial state x_0 is larger than the second tipping point, $x_0 > x_2$. As noted, all eutrophic initial conditions will yield the same result, so there is no need to further distinguish between them.

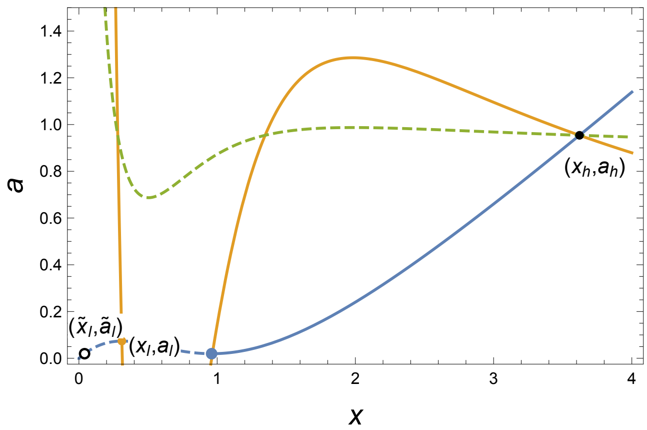


Figure 7: *Eutrophic initial state: Graphs of $f(x)$, $g(x)$ and $h(x)$. The intersection (a_l, x_l) of $f(x)$ and $g(x)$ is not a candidate Nash equilibrium, as it cannot be reached from an eutrophic initial state. At $(\tilde{a}_l, \tilde{x}_l)$, there is a generalised Nash equilibrium. Parameters $(b, c) = (0.52, 0.7)$*

We retain the parameter values $b = 0.52$ and $c = 0.7$. Figure 7 shows the resulting configuration. The right hand intersection of $f(x)$ and $g(x)$ yields, as before, the eutrophic local Nash equilibrium. The left hand intersection (x_l, a_l) does, however, not yield a local equilibrium, as it is inaccessible by the lake dynamics when starting from a eutrophic initial state: if $a = a_l$, the dynamics will get 'stuck' on the eutrophic steady state at around $x = 1.4371$.

There is nevertheless a candidate for an oligotrophic Nash equilibrium at $(\tilde{x}_l, \tilde{a}_l) \approx (0.0409, 0.0196)$. This is the supremum of the oligotrophic steady states that can be reached from an eutrophic initial state. While not a proper Nash equilibrium, it is an ε-equilibrium of the game for every $\varepsilon > 0$ (see Jackson *et al.* 2012), and therefore a Nash equilibrium in this generalised sense.

Figure 8 shows $f(x)$ and the $g(x, k)$ for $k = 1, 2, \ldots, 10$. For $k = 1, .., 9$, Nash equilibria occur in the eutrophic regime, but for $k = 10$, the game ends up in the generalised Nash equilibrium in the oligotrophic regime. This suggests that the coalition size $k = 10$ might be stable. Table 2 however shows that this is not the case. For $k = 3, \ldots, 10$, the leaving incentive $\Delta(k) = w_i^o(k-1) - w_i^m(k)$ is positive, showing that coalitions of these sizes are not stable. Only the size-2 coalition is stable, but the corresponding state is far in the eutrophic regime.

Figures 9a–10 show the effect of decreasing pollution costs. We see that the same mechanism as in the oligotrophic situation is at work, only shifted to larger values of c: for $c > 1.63$, there is no stable coalition of non-trivial size, but under the Nash equilibrium the players reduce their emissions to such a degree that the lake ends up in an oligotrophic

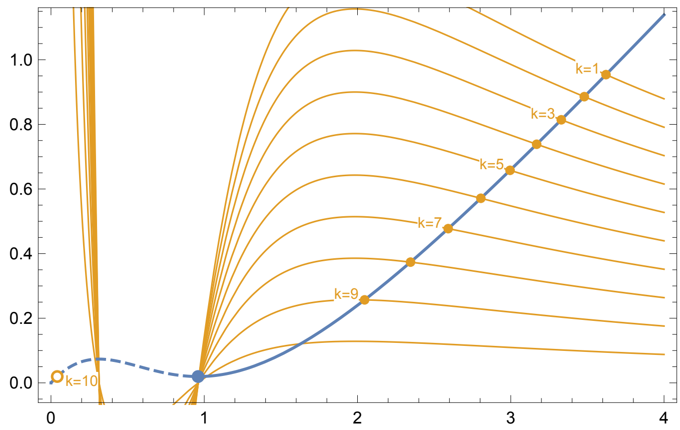


Figure 8: *Eutrophic initial states: Functions $f(x)$ and $g(x,k)$ intersect in local Nash equilibria. Selected Nash equilibria for various coalition sizes are indicated by solid dots, the only generalised Nash equilibrium with an open dot. Parameters:* $(b,c) = (0.52, 0.7)$

k	x^*	w_i^m	w_i^o	Δ
1	3.6213	−11.5296	−11.5296	
2	3.4799	−11.4883	−10.7951	−0.0414
3	3.3297	−11.1444	−10.0458	0.3493
4	3.1690	−10.6652	−9.2789	0.6194
5	2.9953	−10.1002	−8.4907	0.8213
6	2.8048	−9.4679	−7.6762	0.9772
7	2.5918	−8.7739	−6.8280	1.0977
8	2.3459	−8.0149	−5.9354	1.1869
9	2.0463	−7.1807	−4.9835	1.2453
10	0.0409	−6.2360	−3.9334	1.2525

Table 2: *Eutrophic initial state: Coalition size k, Nash equilibrium states x^*, member and outsider payoffs w_i^m and w_i^o and leaving incentive Δ. Parameters:* $(b,c) = (0.52, 0.7)$

steady state. If c drops below this value, Nash equilibrium loadings fail to steer the lake past the tipping point, and it ends up in the eutrophic regime instead, leading to a huge welfare loss: see Figure 10. A size-2 coalition can however still shift the lake back to the oligotrophic regime, and the avoided damage is sufficiently large to stabilise this coalition. For even lower values of c, the size-2 coalition cannot prevent the shift any more, but a size-3 coalition can. This continues until a size-9 coalition is reached. For $c \approx 1.05$ or lower, the welfare gains from avoided environmental damages fail to stabilise even this coalition, and the lake remains in the eutrophic regime.

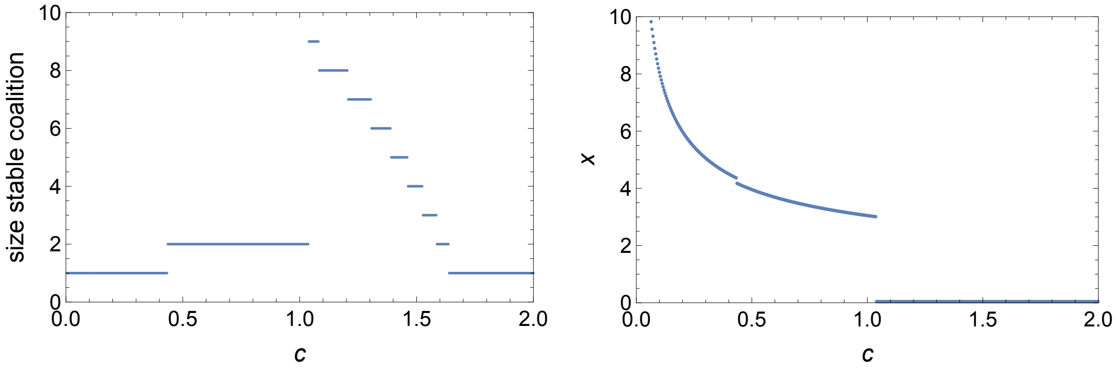
3.4 Coalition formation in steps

When the game starts in the eutrophic regime, the membership game may be played in two steps. If a stable coalition is formed that steers the lake past the lower tipping point, after some time the oligotrophic regime is reached and incentives may arise to leave the coalition again. As we have seen in Section 3.1, the players will now just aim for a stable coalition that prevents shifting back to the eutrophic regime.

Combining the previous sections, we can see that in this way the welfare outcome for the lake will be dramatically improved if the cost parameter is in the interval $1.05 < c < 1.63$. For these values of c, when the initial condition of the lake is eutrophic, the players will first partially cooperate up to the level where the lake is shifted to the oligotrophic regime, ending up in the generalised Nash equilibrium. This implies that an incentive arises to play the membership game again, because the initial condition has shifted from eutrophic to oligotrophic. Figure 5a shows that there are no non-trivial stable coalitions, but from Figure 5b we see that even for Nash equilibrium loadings, the lake remains in the oligotrophic regime. Both levels of partial cooperation are stable. In the first step, one player less in the coalition will not cause a shift to the oligotrophic regime, and in the second step, there is no incentive for any one player to shift the lake back to the eutrophic regime. If the players had not taken the option to form the larger stable coalition first, in order to move to the oligotrophic regime, but if they had formed the usual smaller stable coalition in the eutrophic regime, welfare would have been much lower.

Take for instance $c = 1.2$ and a eutrophic initial state. Without any cooperation, the game will settle on a eutrophic Nash equilibrium, where the individual payoff to each players is $w_i = -13.291$. On the other hand, starting from an oligotrophic initial state, under full cooperation the individual payoff is -5.0148. In the following, we shall express resulting welfares on a efficiency scale where the first value is the 0% baseline level, while the second value corresponds to 100%.

For a eutrophic initial state, there are two stable coalition sizes, the usual small stable coalition size $k = 2$, that reduces the pollution stock just a little bit, and the larger stable coalition size $k = 6$, that yields a shift to the oligotrophic regime. In the small stable coalition, welfare for insiders is -13.1882 and welfare for outsiders is -12.4951, or 1.24% and 9.62%, respectively, on our efficiency scale. After the two-step coalition formation, i.e. using first the larger stable coalition to shift the lake to the oligotrophic regime, and then letting the coalition unravel, welfare for each player increases to -5.02709. This is 99.85% of the result attainable under full cooperation! That is definitely worth the effort.



(a) *Size of largest stable coalition*

(b) *Steady states*

Figure 9: *Eutrophic initial state: Coalition size and steady state as functions of* c*. Parameter* $b = 0.52$

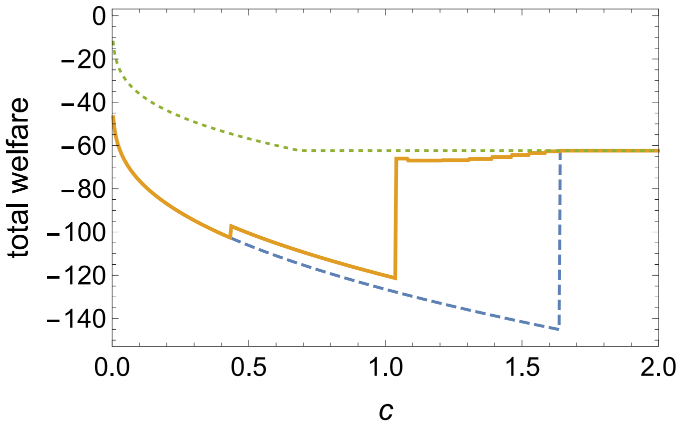


Figure 10: *Eutrophic initial state: Total welfare as a function of the cost parameter* c*. Shown are Nash equilibria if* $k = 1$ *(dashed), Nash equilibria for the largest stable coalitions (solid), and the full-cooperative solution (dotted). Parameter* $b = 0.52$

4 Conclusion

This paper considers management of ecological systems with tipping points, such as the well-known lake system. In case of many users, it may happen that the non-cooperative outcome ends up in the eutrophic region, with a low level of welfare. On the other hand, cooperation requires stability in the sense that the incentives to cooperate outweigh the incentives to free ride. This paper shows that if the lake is initially in an oligotrophic state, a stable level of partial cooperation exists that prevents the lake to shift to the eutrophic regime. Furthermore, if the lake is initially in a eutrophic state, a stable level of partial cooperation exists that shifts the lake to the oligotrophic regime. This implies that the membership game has to be played in two steps, if the lake is in the eutrophic regime. First, a stable coalition has to be formed to shift the lake to the oligotrophic regime. Second, a stable coalition has to be formed to prevent the lake shifting back to the eutrophic regime. This will ultimately give the highest welfare under the requirement that cooperation has to be stable.

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