Members, supporters and free-riders in public goods and common pool resource games

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Abstract

For the class of games with externalities, prominent in the literature ob international environmental agreements, this paper generalizes the standard cartel formation game by allowing for supporters. Some players may not want to join a coalition that jointly provides a public good or manages a resource. Still they might want to support the coalition to set incentives for others to join. We show the existence of stable coalition structures with support. Support increases the size of the coalition and mitigates the inefficiencies in public goods games. We also show significant impacts of support on the equilibria of common pool resource games such as fisheries games.

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1 Introduction

In games with externalities it may be profitable for single agents to coordinate their individual actions by forming coalitions. Such coalition formation will usually generate coalitional spillovers to others. In games with negative spillovers, it is always beneficial to join the coalition rather than stay out and usually the grand coalition forms, at least in open membership games. In games with positive spillovers, coalition formation increases incentives for free-riding and hampers cooperation. Depending on the specification of the game, equilibrium coalitions can be large, but will more often be small, or even empty (Bloch, 1997; Yi, 1997).

A prominent application of coalition formation in games with externalities is the formation of international environmental agreements (IEAs). Two decades of research on IEAs, with seminal papers by Hoel (1992), Carraro and Siniscalco (1993), Barrett (1994) and others, have explored various design features of agreements that can limit free-riding and set incentives to participate in a coalition for the provision of public goods or the harvest of common pool resources, see Benchekroun and Long (2012) for a recent survey. An important strand within this literature explores the role that transfers can play to incentivize participation (Carraro et al., 2006; McGinty, 2007; Weikard, 2009). The transfers considered in this literature are side payments between coalition members. The present paper, however, explores the option that players who are not participating in an agreement may still want to support the coalition to make it attractive for others to join. They are paying others 'to do the job'.

The idea of coalition support has first been put forward by Carraro and Siniscalco (1993). More recently, Ansink and Withagen (2016) have analyzed a sequential game where free-riders can revise their initial decision and can join a coalition if sufficient support becomes available. In this paper we explore the same idea but, using a different approach, we generalize it in three directions. First, we do not only consider public goods games but the more general class of games with externalities. Within this class we focus on games that are non-trivial in terms of coalition formation, that is, games with positive spillovers. This generalization allows us to also assess the implications of support for coalitions that may form in the context of common pool resources such as fisheries or rangelands, where the formation of stable coalitions is particularly hard (Pintassilgo and Lindroos, 2008; Pintassilgo et al., 2015).

Second, we use a partition function approach to determine stable coalitions (Bloch, 1996; Yi, 1997; Ray and Vohra, 1999). Doing so, we avoid modeling a sequential game. We define a support partition function which determines players' payoffs for every possible coalition structure, i.e. every possible partition of the set of players into members, supporters and free-riders. A Nash equilibrium in this game is a coalition structure where no player has an incentive to switch position. The Nash equilibrium conditions can thus be formulated as a set of internal and external stability conditions.

Third, we employ two refinements of 'coalition stability' that reflect differing views of the effect of support on the behaviour of the coalition. We do so by introducing a conditionality assumption (presented formally in the next section) which states, among other things, that members' cooperation is conditional on receiving sufficient support to internally stabilize the coalition. When support is sufficient so that this condition is satisfied, the coalition behaves in order to maximize the coalition payoff. When support is insufficient, however, we consider two possible scenario's for coalition behaviour that underlie our stability concepts. First, insufficient support may cause the complete breakdown of the supported coalition. That is, inspired by the γ-core (Chander and Tulkens, 1997), the coalition members behave like singletons if others do not play their part of an equilibrium strategy. Second, insufficient support may cause down-sizing of the coalition to a size where internal stability is satisfied without support. That is, not all but only some coalition members behave like singletons while the others behave like a (smaller) coalition, whose size is determined non-cooperatively.

The specific game that we consider is a cartel formation game with symmetric players, which has been the work-horse model for the analysis of international environmental agreements. In this game, players have a binary choice to become a coalition member or a free-rider. We modify this game to allow for supporters such that players choose to become a member, a supporter or a free-rider. In contrast to supporters and free-riders, members coordinate their actions to maximize the coalition payoff. As a result, the level of spillovers crucially depends on membership. While supporters behave like free-riders in their actions in the underlying game, their support decision may impact the size of the stable coalition and thereby indirectly affect the spillovers.

In the next section we introduce a cartel formation game with positive spillovers that allows for support. In Section 3, we offer a general lemma for this class of games. We also provide conditions for existence of equilibria with support. In Section 4 we implement these conditions in two specific games, a public goods game and a common pool resource game. In both games, support increases the equilibrium coalition size and overall welfare.

2 Cartel formation with support

In this section we introduce a cartel formation game in partition function form and spell out equilibrium (i.e. stability) conditions. Our main motivation is to examine the role of support in games with positive spillovers where coalition formation is usually hampered by free-rider incentives. We proceed as follows. First we consider a conventional cartel formation game where a unique coalition forms in order to realize gains from cooperation. Coalition formation may generate positive spillovers to non-members, the singletons. Subsequentially, we introduce support to the coalition provided by a sub-set of singleton players, the supporters. Support is meant to induce higher (positive) spillovers by larger coalitions. From the conventional *cartel partition function* we derive a *support partition function*. The main difficulty in this derivation is the specification of players' out-of-equilibrium behaviour. As there is no unique way to specify such behaviour, we will consider two options. In particular, we will consider that an internally unstable coalition (one that receives insufficient support) will (i) cease cooperation or (ii) resort to an equilibrium without support.

Consider a set N of $n \equiv |N| \geq 3$ symmetric players. Players can form a unique coalition $M \subseteq N$ (also referred to as the cartel) That is, each player is either a coalition member $i \in M$ or a non-member $i \in N \backslash M$ (also referred to as a singleton).

Definition 1 (Cartel partition function)**.** A cartel partition function is a mapping v that assigns payoffs $v_i(M)$ to every player $i \in N$ for every coalition $M \subseteq N$. Hence, $v : 2^N \rightarrow \mathbb{R}^n$.

We are interested in cartel partition functions that are determined by an underlying game with positive spillovers such as public goods games or common pool resource games. We introduce specifications of these games in Section 4. Here, though, we keep the analysis general and it is only important to note that each coalition $M \subseteq N$ is associated with a unique payoff vector $\left(v_1(M), ..., v_n(M)\right)$ that describes what players earn in the equilibrium of the underlying game. We will use $v_M(M)$ to refer to the aggregate payoff to all coalition members.

A Nash equilibrium of a cartel formation game requires that no member can gain from quitting membership and becoming a singleton and no singleton can gain by joining the coalition. These conditions are, as usual (D'Aspremont et al., 1983), referred to as internal and external stability, respectively. In the following we will use short hand notation $M_{+i} \equiv M \cup i$ with $i \in N \backslash M$ to refer to a coalition that is enlarged with singleton player i. Similarly, we will use short hand notation $M_{-i} \equiv M \backslash \{i\}$ with $i \in M$ to refer to a coalition that is trimmed with member i 'leaving' the coalition.

Definition 2 (Cartel-stability)**.** Coalition M is cartel-stable if and only if it is internally and externally stable:

- Coalition M is internally stable if and only if $v_i(M) \geq v_i(M_{-i})\ \forall i \in M$;
- Coalition M is externally stable if and only if $v_i(M) > v_i(M_{+i})\ \forall i \in N \backslash M$.

Notice that our formal definition of cartel stability is slightly non-standard as we require a strict inequality for external stability. Implicitly we assume that a player who is indifferent between joining and not joining would join the coalition; cf. Weikard (2009) for a discussion of this refinement.

It follows immediately from Definition 2 that internal stability of M can be guaranteed if and only if

$$v_M(M) \geq \sum_{i \in M} v_i(M_{-i}). \tag{1}$$

The requirement in (1) has been dubbed 'potential internal stability' (Carraro et al., 2006).

Next, consider singleton players $i \in N \backslash M$ who offer support to the coalition by making payments $p_i > 0$ that are used to augment coalition members' payoffs. We call these players 'supporters'. The set of supporters is denoted by $S \subseteq N \backslash M$. We call the remaining players, who are neither members nor supporters, free-riders. The set of free-riders is denoted by F. Note that we ignore the possibility that payments $p_i = 0$ and treat such trivial supporters as free-riders. Essentially, we consider a tri-partition of players, i.e. $M \cup S \cup F = N$ and $M \cap S = M \cap F = S \cap F = \varnothing$. We refer to each tri-partition as a coalition structure $\langle M, S, F \rangle$ and, below, we derive the relevant support partition function $V(\langle M, S, F \rangle)$ from the cartel partition function $v(M)$. The rationale of supporting payments $p(M,S)$ is that support may induce a larger stable coalition which, if beneficial for the singletons, makes support worthwhile. Similar to $v_M(M)$, we will use $v_S(M)$ and $v_F(M)$ to refer to the aggregate payoff to all supporters and free-riders, respectively.

Our assumption is that an internally unstable coalition M would not effectively cooperate. Instead, members of M will resort to a default strategy θ. As discussed, we consider two options for this default strategy that we will formally specify below. Now we first define the required support to internally stabilize M. This required support is equal to the payoff shortfall of M given by

$$r(M) = \max \left[0, \sum_{i \in M} v_i(M_{-i}) - v_M(M) \right]. \tag{2}$$

The aggregate payment by supporters S to coalition members M is denoted $p(M,S) =$

$\sum_{i \in S} p_i$. By player symmetry we require $p(M,S) > 0 \Longleftrightarrow p_i(M,S) > 0$ for all $i \in S$, ruling out that there are supporters who in fact free-ride if a positive payment is made.

We can now construct the support partition function V of our game with support. It is derived from the cartel partition function v of Definition 1 as follows.

Definition 3 (Support partition function)**.** A support partition function is a mapping V that assigns payoffs $V_i\big(\langle M,S,F\rangle\big)$ to every player $i \in N$ for every coalition structure $\langle M,S,F\rangle$. Hence, $V : 3^N \to \mathbb{R}^n$, such that:[1]

$$\begin{aligned}
\text{if } v_S(M) - v_S(\theta) &\geq r(M): \begin{cases} V_M\big(\langle M,S,F\rangle\big) = v_M(M) + r(M); \\ V_S\big(\langle M,S,F\rangle\big) = v_S(M) - r(M); \\ V_F\big(\langle M,S,F\rangle\big) = v_F(M), \end{cases} \\
\text{if } v_S(M) - v_S(\theta) &< r(M): \quad V_i\big(\langle M,S,F\rangle\big) = v_i(\theta) \;\; \forall i \in N.
\end{aligned}$$

The next definition provides the equilibrium conditions for the game with support, extending the requirements for cartel-stability in Definition 2.

Definition 4 (Support-stability)**.** A coalition structure $\langle M,S,F\rangle$ is support-stable if and only if it satisfies:

- Internal support-stability of M:
  $V_i(\langle M,S,F\rangle) \geq V_i^O(\langle M,S,F\rangle) \equiv \max\big[V_i(\langle M_{-i},S_{+i},F\rangle), V_i(\langle M_{-i},S,F_{+i}\rangle)\big] \; \forall i \in M$;
- Internal support-stability of S:
  $V_i(\langle M,S,F\rangle) \geq V_i^O(\langle M,S,F\rangle) \equiv \max\big[V_i(\langle M_{+i},S_{-i},F\rangle), V_i(\langle M,S_{-i},F_{+i}\rangle)\big] \; \forall i \in S$;
- External support-stability of M and S:
  $V_i(\langle M,S,F\rangle) > V_i^O(\langle M,S,F\rangle) \equiv \max\big[V_i(\langle M_{+i},S,F_{-i}\rangle), V_i(\langle M,S_{+i},F_{-i}\rangle)\big] \; \forall i \in F$;

Definition 4 gives the equilibrium conditions as a set of internal and external stability conditions. It simply requires that no member wants to leave the coalition, no supporter wants to become a member nor a free-rider and no free-rider wants to become a member nor a supporter. Similar to the external stability condition of Definition 2, the strict inequality sign in the external stability condition implies that a player who is indifferent between joining and not joining (the coalition or the set of supporters), would join.

We close this section by specifying the default strategy θ of coalition members when support is insufficient and the internal support-stability requirement of Definition 4 is violated. Assuming that internal support-stability is a precondition for effective coalitional cooperation, we rule out that the coalition cooperates in case of insufficient support. We consider two options.

[1] By player symmetry, we write payoffs to sets of players M, S, and F, rather than individual payoffs.

(i) The first and most extreme option is that cooperation breaks down completely, resulting in payoffs equivalent to the All Singletons structure. We write $\theta = \varnothing$. A similar option has been used in models of cartel stability with minimum participation rules (Carraro et al., 2009; Weikard et al., 2015), and it is also similar in spirit to the γ-core where, upon a deviation, all other players adopt singleton's behaviour.

(ii) A less extreme option is that insufficient support is rejected and subsequently the coalition behaves like in a game without support. We write $\theta = M^*$, where M^* refers to a cartel-stable coalition (i.e. a coalition that is stable in the conventional cartel game without support). By player symmetry, we know that $|M^*|$ is unique. This option is inspired by criticism of the γ-core assumptions (cf. Finus, 2008). When support is insufficient, the coalition will not be effective and, in turn, an ineffective coalition cannot request support.

Remark 1. A third option, not considered here, was analyzed by Ansink and Withagen (2016) for public goods in a sequential game. They assumed that insufficient support would lead to a reduction in coalition size up to the point where the coalition becomes internally stable. This much more positive perspective on cooperation within the coalition cannot be assessed using the partition function approach that we use here, since it requires the possibility of players switching from F to M and vice versa in response to the offered level of support.

3 Equilibrium conditions

To state our first result, we introduce the notion of a pivotal supporter.

Definition 5 (Pivotal supporter)**.** Consider a support-stable coalition structure $\langle M, S, F\rangle$ with $S \neq \varnothing$. A supporter $i \in S$ is pivotal for $\langle M, S, F\rangle$ if and only if M is not internally support-stable under $\langle M, S_{-i}, F_{+i}\rangle$.

The following holds.

Lemma 1. *In a support-stable coalition structure, all supporters are pivotal.*

Proof. Consider a support-stable coalition structure $\langle M, S, F\rangle$ and suppose there is a non-pivotal supporter $i \in S$. By Definition 5, the presence of a non-pivotal supporter implies that M is internally support-stable under $\langle M, S_{-i}, F_{+i}\rangle$. By construction of the support partition function, we have $V_i(\langle M, S_{-i}, F_{+i}\rangle) \geq V_i(\langle M, S, F\rangle)$. By player symmetry, this inequality violates internal stability of S in coalition structure $\langle M, S, F\rangle$, a contradiction. Hence there can be no non-pivotal player in a support-stable coalition structure. □

Lemma 1 will help us to identify equilibria. If, in a setting with symmetric players, we have identified the minimum number of supporters needed to stabilise coalition M, then there cannot be more than this number of supporters in equilibrium.

We are interested in identifying equilibria with (non-trivial) support, which, by our treatment of trivial supporters (i.e. $p_i(M,S)=0$) as free-riders, implies that such equilibria are characterized by the presence of one or more supporters. If so, potential internal stability of M as given by (1), is violated, that is $\sum_{i\in M} v_i(M_{-i}) - v_M(M) = r(M) > 0$. Notice that a cartel-stable coalition does not require and receive support.

The existence of equilibria with support requires positive spillovers in the coalition formation game. In a game without spillovers, a characteristic function form game, there is no room for support since for the singleton players $i \notin M$ we always have $v_i(M) - v_i(\theta) = 0$. Similarly, negative spillovers cannot induce support, hence our focus is on positive spillovers. We now define the concept of positive spillovers formally.

Definition 6 (Positive spillovers)**.** A support partition function V exhibits positive spillovers if and only if, for all coalition structures $\langle M,S,F\rangle$, all sets of players $J \subseteq (N \setminus M)$ with $J = J_1 \cup J_2$ and $J_1 \cap J_2 = \emptyset$, and all players $i \in N \setminus (M \cup J)$ it holds that $V_i(\langle M \cup J, S \setminus J_1, F \setminus J_2\rangle) \geq V_i(\langle M,S,F\rangle)$ with a strict inequality for at least one coalition structure $\langle M,S,F\rangle$, one set J and some $i \in (S \cup F) \setminus J$.

Stable support equilibria require a.o. internal support-stability of both M and S. Internal stability of M can be guaranteed if there is sufficient support:

Condition 1. (Sufficient support) $p(M,S) \geq r(M)$.

Internal stability of S can be guaranteed if support is profitable. We derive the related condition starting from the premise that supporters maximize payoffs. This has two implications. First, total support will not exceed the amount required to internally stabilize M. That is, we have $p(M,S) \leq r(M)$, which, joint with Condition (1), implies that if support is offered, we have:

$$p(M,S) = r(M). \tag{3}$$

Second, if support is not offered, this is because the benefits from supporting M do not at least compensate for their supporting payments. Paying support, if profitable, induces effective cooperation of an internally stabilized coalition M such that supporters are better of than with an internally unstable coalition M playing its default strategy θ. Hence, for all $i \in S$ we must have that $v_i(M) - v_i(\theta) \geq p_i(M,S)$. By player symmetry, this condition can be stated as:

Condition 2. (Profitable support) $v_S(M) - v_S(\theta) \geq p(M,S)$.

We will refer to the LHS as supporters' willingness to support. Define

$$w_i(M) \equiv v_i(M) - v_i(\theta). \tag{4}$$

If Condition 2 is not met, support is not profitable and we have $p(M,S) = 0$.

Jointly, Conditions 1 and 2 imply that support equilibria are more likely to occur if $r(M)$ is small, and if positive spillovers are large. Notice that Conditions 1 and 2, based on internal support-stability of, respectively, M and S, provide necessary but not sufficient conditions for existence of support equilibria, since external support-stability of M might be violated. To ease the identification of equilibria we introduce the following lemma.

Lemma 2. *Coalition M in coalition structure $\langle M,S,F\rangle$ is not externally support-stable if coalition M_{+i} in structure $\langle M_{+i},S,F_{-i}\rangle$ is internally support stable.*

Proof. Consider an internally support-stable coalition structure $\langle M_{+j},S,F_{-j}\rangle$. Internal stability of M_{+j} implies that j receives (at least) his outside option payoff $v_j(M)$. Because we require a strict inequality in the external stability condition and it does not hold for $j \in M$ that $V_j(\langle M_{+j},S,F_{-j}\rangle) < V_i(\langle M,S,F\rangle)$. Hence j prefers to join coalition M and makes it externally unstable. □

Lemma 2 allows us to focus on internal stability conditions when we determine the equilibria in games with spillovers. If the internal stability of M_{+i} in structure $\langle M_{+i},S,F_{-i}\rangle$ can be established, we can rule out that M in structure $\langle M,S,F\rangle$ is an equilibrium.

4 Two games

In this section we explore the implications of support for coalition formation for two specific games with externalities and positive spillovers, a public goods game and a common pool resource game. Each has important applications (cf. Andreoni, 1989; Ostrom, 1990). Our approach is to specify the cartel partition function for each game and subsequently apply Conditions 1 and 2. For each game specification introduced below, we follow the setting of Section 3 by assuming a game with positive spillovers and symmetric players, and we adopt the conventional assumption that members maximize their joint payoff while singletons (i.e. supporters and free-riders) maximize their individual payoff. We write $m \equiv |M|$, $s \equiv |S|$, and $f \equiv |F|$ to denote, respectively, the number of members, supporters and free-riders.

4.1 A public goods game

A public goods game is associated with positive externalities and positive spillovers. Following our behavioural assumptions, members equate individual marginal costs of providing the public good with the sum of members' marginal benefits (Samuelson's rule), while singletons equate these with individual marginal benefits. The grand coalition will provide the efficient level of the public good but usually only partial coalitions will be stable and inefficiencies remain in equilibrium (e.g. Barrett, 1994). We assess the impacts of support on this conventional result for two cases. The first is a very simple setting with a discrete public good, linear costs, and linear benefits, which serves to illustrate our approach as well as the large impact that support can have on equilibrium outcomes. The second case is a setting with a perfectly divisible public good, linear benefits, and quadratic costs, as used by e.g. Barrett (1994), Botteon and Carraro (1997), Barrett (2006), and Finus and Maus (2008), which serves to illustrate the impact of support on this work-horse model specification. Importantly, in both cases all players have have dominant strategies so that there is no difference in results between assuming a Stackelberg or Cournot-Nash setting. This difference will become important when we turn to the analysis of common pool games in Section 4.2.

Case 1: Discrete, linear-linear. We start with the simplest possible specification where each player decides whether or not to provide one unit of a discrete public good at cost c. Costs and benefits are linear in the amount of public good provided. Assume m members provide the public good, then the cartel partition function v is

$$v_i = mb - \mu_i c, \tag{5}$$

where $\mu_i \in \{0, 1\}$ is an indicator function such that if i provides one unit, $\mu_i = 1$, and $\mu_i = 0$ otherwise. To avoid a trivial setting, we assume $b < c < nb$ such that $\mu_i = 0$ for all singletons, and, in case a non-trivial coalition forms, $\mu_i = 1$ for all members. This parameter combination assures that the grand coalition is efficient, but the Nash equilibrium of the game is All Singletons.

Allowing for support may change this equilibrium outcome dramatically. In order to check Conditions 1 and 2, we first calculate required support:

$$r(M) = m\big[(m-1)b - (mb - c)\big] = m(c - b). \tag{6}$$

Because there is no cartel-stable coalition without support, the default strategy of a coalition that receives insufficient support is $\theta = \emptyset$. The two different default strategies described in Section 2 coincide. We calculate supporters' willingness to support as the LHS of Condition 2. By internally stabilizing a coalition of size m each supporter would gain $v_i(M) - v_i(\theta) = w_i = mb - 0$. Using Conditions 1 and 2 as well as equation (3), support equilibria exist if and only if

$$s \cdot mb \geq p(M, S) = r(M). \tag{7}$$

Substituting (6) for $r(M)$, this inequality simplifies to $s \geq \frac{c}{b} - 1$. For a non-trivial support we also require $s \geq 1$.

Let $\lceil x \rceil$ denote the smallest integer that weakly exceeds x (also called the ceiling function). Using Lemma 1 and exploiting the strict inequality sign in the external support-stability condition of Definition 4, the unique support-stable coalition structure is given by

$$s = \left\lceil \frac{c}{b} \right\rceil - 1, \; m = n - s, \; \text{and } f = 0. \tag{8}$$

Stability of any coalition structure with free-riders is ruled out by Lemma 2. A detailed proof of the uniqueness of the support-stable equilibrium is provided in the Appendix. In this specification the number of supporters s needed to stabilize a coalition is independent of the size m of the coalition and only depends on the ratio of marginal costs to marginal benefits. It is easy to check that the game has a stable support equilibrium if the number of players is sufficiently large. Consider $b = 1$, $c = 2.5$, then 2 supporters are needed to stabilize a non-trivial coalition of size $m \geq 1$, where 1 unit of the public good is provided. This requires that the number of players is at least 3.

Case 2: Perfectly divisible, linear-quadratic. We continue with a specification where the public good is perfectly divisible, benefits are linear, and costs are quadratic. Both members and singletons may provide the public good. The associated cartel partition function v is

$$v_i = bZ - \frac{1}{2} c z_i^2, \tag{9}$$

where $b, c > 0$, z_i is player i's level of provision of the public good and $Z = \sum_{i \in N} z_i$. The coalition maximizes their joint payoff and best responses by coalition members are $z_i = m\frac{b}{c}$, while for singletons we have $z_i = \frac{b}{c}$. A well known result is that the cartel-stable coalition structure consists of $m = 3$ members (e.g. Barrett, 1994).

Allowing for support affects this outcome. In order to check Conditions 1 and 2, we first calculate required support (see Appendix):

$$r(M) = \left(\frac{b^2}{2c}\right) m(m^2 - 4m + 3). \tag{10}$$

Since the cartel partition function has a non-trivial cartel-stable coalition, we have to consider two possible default strategies by a coalition that does not receive sufficient support. (i) $\theta = \varnothing$ and (ii) $\theta = M^*$ with $m^* = 3$.

Using Conditions 1 and 2 as well as equality (3), support equilibria exist if and only if:

$$\text{if } \theta = \varnothing: \quad \left(\frac{b^2}{c}\right) s(m^2 - m) \geq p(M,S) = r(M); \tag{11}$$

$$\text{if } \theta = M^*: \quad \left(\frac{b^2}{c}\right) s(m^2 - m - 6) \geq p(M,S) = r(M). \tag{12}$$

Substituting (10) for $r(M)$, these inequalities simplify to, respectively, $s \geq (m-3)/2$ for $\theta = \varnothing$ and $s \geq m(m-1)/2(m+2)$ for $\theta = M^*$. Using Lemma 1, support-stable coalition structures are given by (see Appendix):

$$\text{if } \theta = \varnothing: \quad s \in \{1, 2, \ldots\},\ m = 2s + 3, \text{and } f = n - m - s; \tag{13}$$

$$\text{if } \theta = M^*: \quad s \in \{1, 2, \ldots\},\ m = 2s + 2, \text{and } f = n - m - s. \tag{14}$$

In this specification the number of supporters s needed to stabilize a coalition is independent of parameters b and c and only depends on m, the size of the coalition. Note that we find multiple equilibria, since the external stability requirement does not automatically lead to the grand coalition which distinguishes this case from the discrete linear-linear setting of Case 1. If we ignore Pareto-dominated equilibria, however, we find that there is at most one free-rider. A coalition structure with two free-riders cannot be an equilibrium since it would be Pareto-dominated by an equilibrium with one additional supporter and one additional member. Detailed results are reported in the Appendix.

4.2 A common pool resource game

Case 3: Common pool Cournot-Nash game. In contrast to the public goods game, the common pool resource game has negative externalities, while both have positive spillovers. In this game, members coordinate their actions as to maximize the coalition payoff while singletons maximize their individual payoff. The game that we employ here is the Nash-Cournot game commonly applied in the fisheries literature (e.g. Pintassilgo and Lindroos, 2008). In this game, costs are the effort costs of harvesting, while benefits are revenues from the sale of harvest. Negative externalities occur through the impact of harvest on the fish stock while positive spillovers occur through reduced effort by coalition members.

Before we can specify the cartel partition function, we introduce the standard model from this literature. We denote a player's individual harvest by $h_i \equiv qe_iX$, where q is the so-called 'catchability' coefficient, e_i is individual effort and X is the resource stock. The stock grows with rate r according to a logistic growth function $\frac{dX}{dt} = rX\left(1-\frac{X}{k}\right)$, where k is the system's carrying capacity. The analysis is conducted for steady states where harvest equals growth such that $\sum_{i\in N} qe_iX = rX\left(1-\frac{X}{k}\right)$. Individual payoff is given by $v_i = ph_i - ce_i$, where p is the price for selling one's harvest and c is the marginal effort cost.

In this game, individual harvests are taken from the common stock which reduces everyone's harvest per unit of effort. The equilibrium exhibits the well-known overfishing or overgrazing result. Without support, coalition formation cannot solve this tragedy. In a symmetric game with linear effort costs that we analyze here, any coalition – internalizing within-coalition externalities – behaves like a single player. As a result, a coalition of $m \geq 1$ players generates a common pool resource game with $n-m+1$ players. Because the coalition behaves like a single player the coalition payoff is equal to the singleton payoff, but it must be shared between coalition members. As a result, free-riding incentives are higher than in the public goods game. Also, spillovers are stronger than in the public goods game since any effort reduction of the coalition triggers a rebound effect in the sense that singletons respond by increasing their efforts. As a result, for $n \geq 3$, no cartel-stable coalition exists. The cartel partition function v for this game is (see Appendix):

$$v_i = \left(\frac{1}{(n-m+2)^2}\right)\alpha \; \forall i \in N \setminus M, \tag{15}$$

where $\alpha = (kpq-c)^2 r/kpq^2$

Next, we assess the impacts of support. In order to check Conditions 1 and 2, we first calculate required support (cf. also Ansink and Bouma, 2013) (see Appendix):

$$r(M) = \left(\frac{m}{(n-m+3)^2} - \frac{1}{(n-m+2)^2}\right)\alpha. \tag{16}$$

Because there exists no cartel-stable coalition without support for $n \geq 3$, the default strategy of a coalition that receives insufficient support is $\theta = \emptyset$. The two different default strategies described in Section 2 coincide.

Using Conditions 1 and 2 as well as equality (3), support equilibria exist if and only if:

$$w(M,S) = \alpha s\left(\frac{1}{(n-m+2)^2} - \frac{1}{(n+1)^2}\right) \geq p(M,S) = r(M). \tag{17}$$

Substituting (16) for $r(M)$, and using Lemma 1, support-stable coalition structures require a number of supporters given by :

$$s = \left\lceil \frac{\frac{m}{(n-m+3)^2} - \frac{1}{(n-m+2)^2}}{\frac{1}{(n-m+2)^2} - \frac{1}{(n+1)^2}} \right\rceil. \tag{18}$$

We find that this ratio is independent of the technical, biological and economic parameters of the model (i.e. q, r, k, c, p). In other words, coalition stability only depends on the number of players and the coalition structure. We find mulitple support-stable equilibria, provided $n > 3$. For example for a game with 10 players we find an equilibrium with 2 members, 4 supporters and 4 free-riders, and an equilibrium with 5 members, 5 supporters and no free-riders. The cartel game equilibrium where $m = 1$ (a trivial coalition) with all remaining players being free-riders is also a support-stable equilibrium. We report the equilibrium results in the Appendix together with further details of the derivations.

Case 4: Common pool Stackelberg game. In a simple Nash-Cournot common pool game there are no incentives to form a coalition as the gains from cooperation will be picked up by others. This is different in a game where the coalition, if formed, immediately determines its effort level and therefore can behave as a Stackelberg leader. A common pool coalition formation game where the coalition acts as a Stackelberg leader has been described by (Long and Flaaten, 2011) in the context of a fisheries game. Here we extend the analysis to consider the role of support in a Stackelberg common pool game. The set up of the game is as in Case 3. The only difference is the first mover advantage of the coalition. For the coalition, acting as the Stackelberg leader we find that the effort is independent of n and m. It is given by (see Appendix) $e_M = \frac{r}{2q}(1-\beta)$.

Free-riders and supporters can be characterized by the following response function (see Appendix) $e_i = \frac{r}{2q}(1-\beta)\frac{1}{n-m+1}$ for $i \in S \cup F$.

Using the equilibrium effort levels we can calculate payoffs of members and free-riders for all coalition structures and find the required support

$$r(m) = \alpha\left(\frac{m}{4(n-m+2)^2} - \frac{1}{4(n-m+1)}\right). \tag{19}$$

Using the support profitability condition we can also find the amounts supporters are willing to pay $w_i(M,S)$ (see Appendix).

$$\text{If } \theta = \emptyset: \qquad w = s\alpha\left(\frac{1}{4(n-m+1)^2} - \frac{1}{4n^2}\right), \tag{20}$$

$$\text{if } \theta = M^*: \qquad w = s\alpha\left(\frac{1}{4(n-m+1)^2} - \frac{1}{4(n-m^*+1)^2}\right). \tag{21}$$

In the latter case ($\theta = M^*$), we find:

$$m^* = \left\lfloor \frac{1}{4}(5+3n-\sqrt{-7-2n+n^2}) \right\rfloor. \tag{22}$$

This means that for large n a coalition that comprises about $\frac{1}{2}$ of the players is stable without support, see Table 5. The coalition as a Stackelberg leader can reap a larger share of the harvest which facilitates internal stability.

The option of support can further increase the size of the coalition and reduce inefficient overexploitation of resources. We check the condition $w(m,s) \geq r(m)$ and find

$$\text{if } \theta = \emptyset: \qquad s = \left\lfloor \frac{\frac{m}{(n-m+2)^2} - \frac{1}{(n-m+1)}}{\frac{1}{(n-m+1)^2} - \frac{1}{n^2}} \right\rfloor, \tag{23}$$

$$\text{if } \theta = M^*: \qquad s = \left\lfloor \frac{\frac{m}{(n-m+2)^2} - \frac{1}{(n-m+1)}}{\frac{1}{(n-m+1)^2} - \frac{1}{(n-m^*+1)^2}} \right\rfloor. \tag{24}$$

Again in both cases we find multiple equilibria. A detailed account of the equilibria is provided in the Appendix.

5 Conclusion

In this paper we explore the idea that players who do not join a coalition may still have incentives to support others who form a coalition. To study the role of such incentives we introduce the notion of support stability in cartel games in partition function form. In particular we consider games with positive spillovers where coalition formation is hampered by free-rider incentives. We find generally larger coalitions and support-stable equilibria generate a higher level of welfare. Depending on the partition function, the difference between conventional equilibria and support stable equilibria can be striking.

Appendix

Proof of uniqueness Case 1

For a support stable coalition the willingness to support must (weakly) exceed required support. We have

$$smb \geq m(c-b) \Leftrightarrow s \geq \frac{c}{b} - 1. \tag{25}$$

Satisfying (25) guarantees the the internal stability of a coalition M of any size m. Applying Lemma 1 we can write the number of supporters as $s = \lceil \frac{c}{b} \rceil - 1$ and we know that all supporters are pivotal. Hence, no supporter wants to leave to become a free-rider and no free-rider wants to join S. From the fact that the required number of supporters is independent of m we know that coalition $M_{+j}, j \in S$ needs the same of support as M that no supporter will leave S to join M. It remains to be shown whether any of the internally support stable coalitions M are externally stable. From Lemma 2 we know that if a coalition of size $m+1$ is internally support stable, then a coalition of size m cannot be internally support stable. Thus in equilibrium m will take its largest possible value $m = n - \lceil \frac{c}{b} \rceil + 1$ and there are no free-riders.

Equilibria for Case 2

The total willingness to support must exceed the required support. We obtain from (11)

$$\text{if } \theta = \emptyset\text{:} \quad \left(\frac{b^2}{c}\right) s(m^2 - m) \geq \left(\frac{b^2}{2c}\right) m(m^2 - 4m + 3); \tag{26}$$

$$\text{if } \theta = M^*\text{:} \quad \left(\frac{b^2}{c}\right) s(m^2 - m - 6) \geq \left(\frac{b^2}{2c}\right) m(m^2 - 4m + 3). \tag{27}$$

This implies

$$\text{if } \theta = \emptyset\text{:} \quad s = \left\lceil \frac{1}{2}(m-3) \right\rceil, \; m = 2s + 3; \tag{28}$$

$$\text{if } \theta = M^*\text{:} \quad s = \left\lceil \frac{m(m-1)}{2(m+2)} \right\rceil, \; m = \left\lfloor \frac{1}{2}(1 + 2s + \sqrt{1 + 20s + 4s^2} \right\rfloor. \tag{29}$$

As the derivative of $\frac{1}{2}(1 + 2s + \sqrt{1 + 20s + 4s^2}$ approaches 2, we find that each additional supporter provides sufficient support for two additional members and we instead write $m = 2s + 2$. We have the following support stable structures:

$$\text{if } \theta = \varnothing: \quad \text{for } s = \{1,2,...\} \quad (m,s,f) = (2s+3, s, n-3s-3); \tag{30}$$

$$\text{if } \theta = M^*: \quad \text{for } s = \{1,2,...\} \quad (m,s,f) = (2s+2, s, n-3s-2). \tag{31}$$

We find multiple support-stable equilibria. The numbers in Table 1 indicate the minimum number of supporters needed to stabilise a coalition of size m.

Table 1: Number of supporters in support-stable coalitions (Case 2)

n														
3	0	**0**												
4	0	**0**	n											
5	0	**0**	**1**	n										
6	0	**0**	1	**1**	n									
7	0	**0**	1	**1**	n	n								
8	0	**0**	1	**1**	**2**	n	n							
9	0	**0**	1	**1**	2	**2**	n	n						
10	0	**0**	1	**1**	2	**2**	n	n	n					
11	0	**0**	1	**1**	2	**2**	**3**	n	n	n				
12	0	**0**	1	**1**	2	**2**	3	**3**	n	n	n			
13	0	**0**	1	**1**	2	**2**	3	**3**	n	n	n	n		
14	0	**0**	1	**1**	2	**2**	3	**3**	4	n	n	n	n	
15	0	**0**	1	**1**	2	**2**	3	**3**	4	**4**	n	n	n	n
m	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*

Bold numbers indicate that the respective coalitoin structure is support-stable.

A bold number indicates that the respective structure is a support-stable coalition structure for $\theta = \varnothing$. For example for $n = 10$ we have the following stable structures: $(3,0,7), (5,1,4), (7,2,1)$. Structures with a larger number of members cannot be stable for the lack of a sufficient number of supporters, indicated by "n". Structures like, for example, $(6,2,2)$ satisfy the internal stability condition for M and S, but violate the external stability of M. For such cases we know from Lemma 2 that one of the free riders will (weakly) prefer to join M.

We obtain a similar pattern with exactly one member less for $\theta = M^*$.

Equilibria for Case 3

The canonical version of a common pool resource game (Mesterton-Gibbons, 1993; Ruseski, 1998; Pintassilgo and Lindroos, 2008) is described by a logistic growth function

$$g = rX\left(1-\frac{X}{k}\right), \tag{32}$$

where the stock X grows with rate $r > 0$ and $k > 0$ is the system's carrying capacity. Payoff functions are derived for the system's steady state, i.e. where harvest equals growth $\frac{dX}{dt} \equiv g = h \equiv \sum h_i$. [2]

Individual harvest depends on efforts e_i and stock $h_i \equiv qe_iX$. Thus in the steady state we have

$$\sum qe_iX = rX\left(1-\frac{X}{k}\right). \tag{33}$$

Solving for X we obtain

$$X = \frac{k}{r}(r - q\sum e_i) \tag{34}$$

as the steady state stock. Individual payoff is given by $\pi_i = ph_i - ce_i$, where p is the resource price and c is the marginal effort cost and we assume that an individual's share of the harvest is equal to his share of efforts $h_i/h = e_i/\sum e_j = e_i/e$.

Player i's best response function is obtained by solving $\max_{e_i}\left(p\frac{e_i}{\sum e_j}h - ce_i\right)$. Taking derivatives the first order condition gives $\frac{c}{pqk} = 1 - \frac{qe}{r} - \frac{qe_i}{r}$. Define $\frac{c}{pqk} \equiv \beta$ and $\sum_{j\in N\setminus\{i\}} e_j \equiv e_{-i}$, then we can write the response function

$$e_i^* = \frac{r(1-\beta)}{2q} - \frac{1}{2}e_{-i}. \tag{35}$$

Note that for symmetric players, in a symmetric equilibrium $e_{-i} = (n-1)e_i$. Further, note that since a coalition behaves like a single player, if m players form a coalition, we have $n-m+1$ players harvesting the resource. The coalition's and each singleton player's effort response function is

$$e_i^* = \frac{r(1-\beta)}{2q} - \frac{1}{2}(n-m)e_j^*. \tag{36}$$

Assuming symmetry $e_j^* = e_i^*$ and solving for e_i^* we obtain the equilibrium effort level.

$$e_i^* = \frac{r(1-\beta)}{q} \cdot \frac{1}{n-m+2}. \tag{37}$$

[2] If a sum runs over the set of all players we will use simplified notation $\sum h_i$ for $\sum_{i\in N} h_i$.

Note that the coaliton's effort level, and hence its share of the harvest is the same as the effort level of each individual singleton. Substituting equilibrium effort levels into the individual payoff

$$\pi_i = p\frac{1}{n-m+1}h^* - ce_i^*, \tag{38}$$

where harvest depends on effort and stock, we can determine the cartel partition function. Substituting appropriately and simplifying we obtain the the partition function, i.e. payoff for a singleton and the coalition as a function of coalition size m:

$$v_M(m) = v_i(m) = \frac{\alpha}{(n-m+2)^2}, i \in N\backslash M, \tag{39}$$

where $\alpha \equiv \frac{(kpq-c)^2 r}{kpq^2}$ summarises the bio-economic parameters of the model. It is clear from (39) that payoffs are increasing in m. A member must share this payoff with other members and earns only $\frac{1}{m}$ of what a free-rider earns.

To obtain the outside option payoff we must consider the free-rider payoff under a coalition of size $(m-1)$, which equals $v_i(m-1) = \frac{\alpha}{(n-m+3)^2}$. We can now determine the required support:

$$\begin{aligned} r(m) &= m \cdot v_i(m-1) - v_M(m) \\ &= \alpha\left(\frac{m}{(n-m+3)^2} - \frac{1}{(n-m+2)^2}\right). \end{aligned} \tag{40}$$

The next step is to determine the payment that would be offered by supporters. The maximum willingness to support w is the difference between the free-rider payoff if a coalition of size m is stable $v_i(m), i \in N\backslash M$, and the payoff obtained in All Singletons $(m = 1)$, i.e. the situation when M is unstable and thus ineffective.

$$\begin{aligned} w_i &= v_i(m) - v_i(1) \\ &= \alpha\left(\frac{1}{(n-m+2)^2} - \frac{1}{(n+1)^2}\right). \end{aligned} \tag{41}$$

A coalition of size m with s supporters will be support stable if

$$\begin{aligned}
sw_i &\geq r(m) \Leftrightarrow \\
s(v_i(m) - v_i(1)) &\geq m \cdot v_i(m-1) - v_M(m) \Leftrightarrow \\
s\alpha\left(\frac{1}{(n-m+2)^2} - \frac{1}{(n+1)^2}\right) &\geq \alpha\left(\frac{m}{(n-m+3)^2} - \frac{1}{(n-m+2)^2}\right) \Leftrightarrow \\
s &\geq \frac{\frac{m}{(n-m+3)^2} - \frac{1}{(n-m+2)^2}}{\frac{1}{(n-m+2)^2} - \frac{1}{(n+1)^2}}. \qquad (42)
\end{aligned}$$

Table 2: Number of supporters in support-stable coalitions (Case 3)

n														
3	**0**	**1**	n											
4	**0**	**1**	n	n										
5	**0**	2	**2**	n	n									
6	**0**	**2**	**3**	n	n	n								
7	**0**	3	3	**3**	n	n	n							
8	**0**	**3**	4	**4**	n	n	n	n						
9	**0**	4	**4**	**5**	n	n	n	n	n					
10	**0**	**4**	5	5	**5**	n	n	n	n	n				
11	**0**	5	**5**	6	**6**	n	n	n	n	n	n			
12	**0**	**5**	6	6	**6**	n	n	n	n	n	n	n		
13	**0**	6	**6**	7	7	**7**	n	n	n	n	n	n	n	
14	**0**	**6**	7	7	**7**	**8**	n	n	n	n	n	n	n	n
15	**0**	**7**	8	8	8	**8**	n	n	n	n	n	n	n	n
m	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*

Bold numbers indicate that the respective coalitoin structure is support-stable.

Using (42) we can see that for the common pool game described above the number of supporters needed to stabilise a coalition of a given size m is independent of the bio-economic parameters of the model and depends only on the number of players n. Solving (42) for m does not yield a convenient expression. However, it is easy to report the minimum number of supporters required to stabilise a coalition of size m in an n-player game. In Table 2 we can see that, for example, in a 5-player game 2 supporters are needed to stabilise a coalition of 3. Ther are not enough supporters for larger coalitions. A coalition of 2 is not support-stable as it violates the external stability condition.

Equilibria for Case 4

The previous section has examined a Cournot Nash game of coalition formation. In the absence of support there is no stable coalition as any benefit from conservation is reaped by other players. Thus in a common pool there is no point in forming a coalition unless the coalition can establish a strategic advantage by making its effort decision before other players. Thus we will now assume that the coalition M is a Stackelberg leader. We consider a cartel-fringe model in this setting. The play of the game is as follows. The coalition moves first a determines its effort. Then all other players, the fringe players, fix their effort levels simultaneously.

For the analysis we can employ the response function (36), where we rewrite the aggregate effort of all other players. Consider a fringe player i. Others' efforts are $e_{-i} \equiv e_M + \sum_{j \in N\setminus(M\cup i)} e_j$. Since all fringe players act simultaneously we have $e_{-i} \equiv e_L + (n-m-1)e_j$. In equilibrium

$$e_i^* = \frac{r(1-\beta)}{2q} - \frac{1}{2}(e_M + (n-m-1)e_i^*). \tag{43}$$

Solving for e_i^* and since we have $n-m$ players in the fringe, the total effort of the fringe players is $e^*_{fringe} = \frac{n-m}{(n-m+1)}(\frac{r(1-\beta)}{q} - e_M)$. Knowing how the fringe will react to their effort e_M the coalition maximises its payoff $\pi_M = p\frac{e_M}{e_M + e^*_{fringe}}h^* - ce_M$. We obtain

$$e_M^* = \frac{r(1-\beta)}{2q}. \tag{44}$$

Hence, the coalition has a dominant strategy, which is also independent of its size m. The coalitions share of the total harvest $\frac{e_M}{e_M+e^*_{fringe}} = \frac{n-m+1}{2(n-m)+1}$. This fraction is larger than $1/2$ and approaches $1/2$ as the number of fringe players is larger.

Using the equilibrium effort levels we can calculate each member's and each fringe player's payoff,

$$v_i(m) = \frac{\alpha}{4(n-m+1)m}, i \in M, \tag{45}$$

$$v_j(m) = \frac{\alpha}{4(n-m+1)^2}, j \in N\setminus M, \tag{46}$$

where $\alpha \equiv \frac{(kpq-c)^2 r}{kpq^2}$, as before. From the equilibrium payoff (45) we can determine the set of internally stable coalitions. These must satisfy for any $i \in M$ that a member's payoff (weakly) exceeds the free-rider payoff, i.e. $v_i(m) \geq v_j(m-1)$, $j \in N\setminus M$. Therefore, we can write the internal stability condition as $\frac{\alpha}{4(n-m+1)m} \geq \frac{\alpha}{4(n-m+2)^2} \Leftrightarrow \frac{1}{(n-m+1)m} \geq \frac{1}{(n-m+2)^2} \Leftrightarrow$

$(n-m+2)^2 \geq (n-m+1)m$. Notice that this condition is independent of the bio-economic parameters and depends only on n and m. Solving the inequality for m gives the largest number members that can form a stable coalition:

$$m^* = \left\lfloor \frac{1}{4}(5+3n-\sqrt{-7-2n+n^2}) \right\rfloor. \tag{47}$$

Table 3 lists the largest coalitions for the Stackelberg common pool game with different numbers of players.

Table 3: Largest coalitions for the Stackelberg common pool game

n	2	3	4	5	6	7	8	9	10	...	15	30	50	100
m^*	2	3	4	4	4	5	5	6	6	...	9	16	26	51

Coalitions of size m^* are stable without support. If the number of players is small (≤ 4) the grand coalition is stable. For larger numbers of players the size of the largest stable coalition approaches $\frac{1}{2}n$ from above. Now we turn to the question whether it is possible to stabilise larger coalitions with support.

The required support is given as

$$\begin{aligned} r(m) &= m \cdot v_j(m-1) - v_M(m) \\ &= \alpha\left(\frac{m}{4(n-m+2)^2} - \frac{1}{4(n-m+1)}\right). \end{aligned} \tag{48}$$

In order to determine the payment that would be offered by supporters we need to distinguish two cases. First, we examine support stability based on the assumption that no coalition will be formed should suppor fall short of the requirement, that is $\theta = \emptyset$. Second we assume that, in the case of lacking support the largest coalition that is stable without support will be formed, $\theta = M^*$.

Default strategy $\theta = \emptyset$. We start with the analysis of $\theta = \emptyset$. In this case the maximum willingness to support w is the difference between the free-rider payoff if a coalition of size m is stable $v_i(m), i \in N \backslash M$, and the payoff obtained when no coalition is formed and a single player is the Stackelberg leader.

$$\begin{aligned} w_j &= v_j(m) - v_j(1);\ j \in N \backslash M \\ &= \alpha\left(\frac{1}{4(n-m+1)^2} - \frac{1}{4n^2}\right). \end{aligned} \tag{49}$$

A coalition of size m with s supporters will be support stable if

$$
\begin{aligned}
sw_i &\geq r(m) \Leftrightarrow \\
s(v_j(m) - v_j(1)) &\geq m \cdot v_j(m-1) - v_M(m) \Leftrightarrow \\
s\alpha\left(\frac{1}{4(n-m+1)^2} - \frac{1}{4n^2}\right) &\geq \alpha\left(\frac{m}{4(n-m+2)^2} - \frac{1}{4(n-m+1)}\right) \Leftrightarrow \\
s &\geq \frac{\frac{m}{(n-m+2)^2} - \frac{1}{(n-m+1)}}{\frac{1}{(n-m+1)^2} - \frac{1}{n^2}}. \qquad (50)
\end{aligned}
$$

Using (50) and applying the ceiling function we can calculate the number of supporters needed to stabilise a coalition of a given size m depending on the number of players n. This is reported in Table 4. Zeros indicated that no support is needed. "n" indicates "not stable". This applies when there are not enough supporters. For example, with 5 players the grand coalition would need a supporter, however, there is no sixth player.

Table 4: Number of supporters in support-stable coalitions (Case 4, $\theta = \emptyset$)

n														
3	0	0												
4	0	0	0											
5	0	0	0	n										
6	0	0	0	1	n									
7	0	0	0	0	1	n								
8	0	0	0	0	1	n	n							
9	0	0	0	0	0	2	n	n						
10	0	0	0	0	0	1	2	n	n					
11	0	0	0	0	0	0	2	n	n	n				
12	0	0	0	0	0	0	1	2	n	n	n			
13	0	0	0	0	0	0	0	2	3	n	n	n		
14	0	0	0	0	0	0	0	1	3	n	n	n	n	
15	0	0	0	0	0	0	0	0	2	3	n	n	n	n
m	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*

Default strategy $\theta = M^*$. Now we turn to the analysis of $\theta = M^*$. In this case the maximum willingness to support w is generally lower as, when support fails, the coalition does not break down completely but a coalition that is stable without support will be established. Such coalition has size m^* as reported in Table A1. The willingness to corntribute of a free-rider is the difference between the free-rider's payoff if a coalition of size m is support stable $v_i(m), i \in N \backslash M$, and the free-rider payoff obtained under a coalition of size m^*.

$$\begin{aligned} w_j &= v_j(m) - v_j(m^*);\ j \in N \backslash M \\ &= \alpha\left(\frac{1}{4(n-m+1)^2} - \frac{1}{4(n-m^*+1)^2}\right). \end{aligned} \tag{51}$$

A coalition of size m with s supporters will be support stable if

$$\begin{aligned} sw_i &\geq r(m) \Leftrightarrow \\ s(v_j(m) - v_j(m^*)) &\geq m \cdot v_j(m-1) - v_M(m) \Leftrightarrow \\ s\alpha\left(\frac{1}{4(n-m+1)^2} - \frac{1}{4(n-m^*+1)^2}\right) &\geq \alpha\left(\frac{m}{4(n-m+2)^2} - \frac{1}{4(n-m+1)}\right) \Leftrightarrow \\ s &\geq \frac{\frac{m}{(n-m+2)^2} - \frac{1}{(n-m+1)}}{\frac{1}{(n-m+1)^2} - \frac{1}{(n-m^*+1)^2}}. \end{aligned} \tag{52}$$

Table 5: Number of supporters in support-stable coalitions (Case 4, $\theta = M^*$)

n														
3	0	0												
4	0	0	0											
5	0	0	0	n										
6	0	0	0	1	n									
7	0	0	0	0	n	n								
8	0	0	0	0	1	n	n							
9	0	0	0	0	0	n	n	n						
10	0	0	0	0	0	2	n	n	n					
11	0	0	0	0	0	0	n	n	n	n				
12	0	0	0	0	0	0	2	n	n	n	n			
13	0	0	0	0	0	0	0	n	n	n	n	n		
14	0	0	0	0	0	0	0	3	4	n	n	n	n	
15	0	0	0	0	0	0	0	0	n	n	n	n	n	n
m	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*

Using (52) and applying the ceiling function we can calculate the number of supporters needed to stabilise a coalition of a given size m depending on the number of players n. This is reported in Table 5. Zeros indicate that no support is needed. "n" indicates "not stable". This applies when there not enough supporters. For example, with 7 players a coalition of 6 would need 2 supporters, however, there is only one remaining player to support the coalition.

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