On OPEC's evaporating market power and climate policies*

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Abstract

We develop an oligopoly-fringe model to understand the consequences of recent developments on the global energy market: the collapse of OPEC as a cartel and the shale oil revolution in the US and Canada. Our model is able to explain simultaneous supply of conventional and shale oil, despite differing extraction costs. We show that a final limit-pricing regime will occur, during which the OPEC oligopolists just undercut the price of renewable energy, if the stock of the shale fringe is not too large. The break-down of OPEC has an ambiguous effect on climate damages. On the one hand, aggregate oil extraction will become less conservative, which increases damages from climate change. On the other hand, extraction of the fringe is postponed, which lowers climate damage if the fringe's resource is relatively dirty (e.g., tar sands). The recent shale oil revolution not only leads to increased climate damages, but may even lower global welfare. We also show that renewables subsidies do not cause a Weak Green Paradox if the relative stock of the oligopolists is large enough, compared to the fringe.

JEL codes: Q31, Q42, Q54, Q58

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1 Introduction

Recent empirical evidence suggests that the influence of the Organization of Petroleum Exporting Countries (OPEC) on the global oil market is waning. One reason for this is that OPEC does not seem to act collusively anymore (Almoguera et al., 2011; Brémond et al., 2012; Kisswani, 2016; Okullo and Reyns, 2016). Furthermore, the deployment of new hydraulic fracturing technologies has enabled the supply of huge amounts of shale oil and gas in the US (McJeon et al., 2014; Behar and Ritz, 2016). Production of shale gas increased by more than a factor 6 during the period 2008-2014, whereas the production of shale oil even increased more than eightfold over these years. According to the latest estimates, this “shale revolution” has increased the technically recoverable global reserves of natural gas and oil with 47 and 11 percent, respectively (EIA, 2014). Moreover, US President Donald Trump recently ordered a revival of the Keystone XL and Dakota Access pipelines, which will increase transportation capacity and lower the transportation costs of tar sands oil from Alberta and North Dakota to refineries in Illinois and Texas. The aim of this paper is to understand the consequences of these recent development for the global energy market and for the effectiveness of climate policies.

There is a vast literature on resource models with imperfect competition. Important contributions were made by Stiglitz (1976) on monopoly, Lewis and Schmalensee (1980) on oligopoly, Gilbert (1978) and Newbery (1981) on dominant firms. Our model is close to the cartel-fringe model explored by, e.g., Benchekroun et al. (2009, 2010), Benchekroun and Withagen (2012) and Groot et al. (2003). See also Withagen (2013) for a survey and the references therein. In the present paper we offer new insights in three respects. First of all, we take account of OPEC still being an important player on the market, but with less power than some decades ago. Almoguera et al. (2011) conclude that “OPECs behaviour is best described as Cournot competition in the face of a competitive fringe constituted by non-OPEC producers.” In line with this conclusion, we model the market as a situation with a large number of price-taking mining firms and a set of oligopolists, which reduces to the cartel-fringe model if the number of oligopolists equals unity. Second, we take account of the existence of renewables that are (perfect) substitutes for fossil fuel and that can be produced in unlimited amounts, contrary to fossil fuel that is available in a finite amount. This opens the possibility of limit pricing (see, e.g., Van der Meijden et al. (2015); Andrade de Sá and Daubanes (2016) and Van der Meijden and Withagen (2016) for recent work and Hoel (1978), Salant (l979) and Gilbert and Goldman (1978) for early contributions). Third, we investigate the effect of climate change policies on the extraction paths as well as on welfare, allowing for damages from the accumulation of greenhouse gases.

We establish the existence of a Nash-Cournot equilibrium on the energy market. We fully characterize the equilibrium and perform a sensitivity analysis for varying policy measures and competitiveness indicators. The main findings are as follows.

First, the oligopolists and the fringe start out supplying simultaneously to the market, despite their differing extraction costs. Second, if the relative initial stock of the fringe is large, the phase with simultaneous supply will be followed by a phase during which only the fringe is active (and the stocks of the oligopolists are depleted). In this case, there will be no limit-pricing behaviour. However, if the initial stock of the cartel is relatively large, the phase with simultaneous supply will be followed by a period during which only the oligopolists are supplying. During this period, the oligopolists either choose to price strictly below the price of renewables, in which case the price increases over time, or to perform a limit-pricing strategy of just undercutting the renewables price, in which case the price is constant over time. If marginal profits in a limit-pricing regime are non-positive, oligopolists will start with limit pricing as soon as the fringe's stock is depleted. However, if marginal profits in a limit-pricing regime are positive, the oligopolists will start limit pricing only after the fringe's stock is depleted and their own remaining stock is smaller than a certain threshold.

Third, the order of extraction of the different resources (e.g., conventional oil and tar sands) and the occurrence and duration of a limit-pricing phase is crucially affected by the number of oligopolists. An increase in this number increases the likelihood of the oligopolists depleting before the fringe, therefore lowers the likelihood of a limit-pricing phase occurring, and lowers its maximum duration. Fourth, the collapse of OPEC as a cartel has ambiguous effects on climate damage. On the one hand, resource extraction will become less conservative, which increases climate damage. On the other hand, extraction of the relatively expensive and dirty resource owned by the fringe will be back-loaded in time, which slows down climate change. Fifth, the shale oil revolution characterized by an increase in shale oil reserves and a decrease in shale oil extraction costs, does not only increase climate damages (due to larger cumulative emissions), but also lowers 'grey welfare' because the relatively expensive shale oil partially crowds out early extraction of cheap oil by the oligopolists. Finally, a Weak Green Paradox, i.e., the increase of current carbon emissions upon the introduction of a renewables subsidy, only occurs if the relative initial stock of the oligopolists is small.

Our work has several limitations. We employ the open-loop Nash equilibrium, whereas possibly the feedback Nash equilibrium would be more appropriate. Moreover, we establish the existence of an equilibrium, whereas there at this stage we cannot exclude the existence of multiple equilibria (see also Benchekroun and Withagen, 2012). Moreover, we use constant marginal extraction cost of all fossil fuel suppliers and constant marginal cost of the backstop, which is assumed a perfect substitute (see also Van der Meijden and Withagen, 2016). Finally, we model demand for energy and energy policy at the highest level of aggregation. We neglect the possibility of strategic interaction between energy suppliers and energy demanders (see, e.g., Liski and Tahvonen, 2004; Kagan et al., 2015).

The remainder of the paper is structured as follows. Section 2 outlines the model. Section 3 characterizes the open-loop Nash equilibrium. In Section 4, we perform a comparative statics analysis. Section 5 discusses welfare effects. Finally, Section 6 concludes.

2 The model

A non-renewable resource is jointly supplied a price-taking fringe and a group of n suppliers with market power, referred to as oligopolists. The fringe is endowed with an aggregate initial stock S_0^f and has a constant per unit extraction cost k^f. The initial stock of oligopolist i is denoted by S_{0i}^c with $i = 1, .., n$. The per unit extraction cost of oligopolist i is constant and denoted by k_i^c. Extraction rates at time $t \geq 0$ by the fringe and oligopolist i are $q^f(t)$ and $q_i^c(t)$, respectively. The time argument will be dropped when possible. Aggregate supply by the oligopolists reads $q^c \equiv \sum_i q_i^c$. Demand for energy is described at the world level. The inverse demand of the non-renewable resource is given by $p + \tau = \alpha - \beta(q^f + q^c)$, with $\alpha > 0$ and $\beta > 0$, where τ denotes a constant specific tax on resource consumption and p is the price received by suppliers of the resource. A perfect substitute for the resource can be produced, indefinitely, at marginal cost $b > 0$, by using a backstop technology. We abstract from technological progress (cf. Fischer and Salant, 2014), as well as from set-up costs. Consumption of the substitute is subsidized at a constant specific rate σ.[1] Define $\hat{b} \equiv b - \sigma - \tau$. We denote the interest rate by $r > 0$.

The fringe maximizes its discounted profits,

$$\int_0^\infty e^{-rt}(p(t) - k^f)q^f(t)dt, \tag{1}$$

taking the price path as given, subject to its resource constraint

$$\dot{S}^f(t) = -q^f(t), S^f(t) \geq 0, S^f(0) = S_0^f. \tag{2}$$

Each oligopolist i is aware of its influence on the equilibrium price and maximizes

$$\int_0^\infty e^{-rt}(\alpha - \beta(q^f(t) + q^c(t)) - k_i^c)q_i^c(t)dt, \tag{3}$$

taking the time paths of q^f and q_j^c $(j \neq i)$ as given, subject to their resource constraint

$$\dot{S}_i^c(t) = -q_i^c(t), S_i^c(t) \geq 0, S_i^c(0) = S_{0i}^c. \tag{4}$$

Moreover, the existence of the perfect substitute effectively implies an upper limit on the price oligopolists can ask, yielding the additional constraint

$$\alpha - \beta(q^f(t) + q^c(t)) - \tau \leq \hat{b}. \tag{5}$$

We make the following two assumptions.

Assumption 1 (Symmetric oligopolists) *For all $i = 1, .., n$ we have:*

(i) $k_i^c = k^c$.

(ii) $S_{0i}^c = \frac{S_0^c}{n}$ *where S_0^c represents the total stock owned by the oligopolists.*

[1] The constancy of the tax can be motivated by constant marginal damages of emissions. Constancy of the renewables subsidy is convenient for presenting the results.

Assumption 2 (Relative costs) *We impose:*

(i) $k^c + \tau < k^f + \tau < b - \sigma < \alpha$.

(ii) $k^f < (\alpha - \tau + nk^c)/(1+n)$.

Assumption 1 allows us to focus on the interaction of market power and the fringe when facing a competing backstop technology. Asymmetry of oligopolists can deliver interesting insights, but would obscure the source behind the novelty of the results of the paper. Assumption 2 enables us to restrict our attention to cases that we think are empirically relevant. Part (i) ensures that the tax-inclusive marginal production costs of the non-renewable resource are lower than the after-subsidy marginal production costs of the backstop technology, and that the after-tax and after-subsidy marginal production costs are below the choke price. Part (ii) makes sure that the marginal extraction costs of the fringe are below the profit-maximizing price of the oligopolists.[2]

3 Oligopoly-Fringe equilibrium

Our problem is a hybrid version of the cartel-fringe framework where the cartel announces a price path and the fringe chooses an extraction path, and the oligopoly framework where each player chooses an extraction strategy.

Here we assume that each oligopolist chooses an extraction strategy, taking the extraction strategies of all the other players, including the fringe, as given while the fringe takes the price path as given and chooses its extraction strategy. For tractability we focus on open-loop strategies, where the strategy of each oligopolist is an extraction path.

Definition 1 *A vector of functions* $q \equiv (q_1^c, ..., q_n^l, q^f)$ *with* $q(t) \geq 0$ *for all* $t \geq 0$ *is an Open-Loop Oligopoly-Fringe Equilibrium (OL-OFE) if*

(i) all extraction paths of the vector $(q_1^c, ..., q_n^c, q^f)$ *satisfy the corresponding resource constraint,*

[2]To see this, consider the extreme case with an infinitely large S_0^c, implying a zero scarcity rent. Instantaneous profits of the oligopolists (if $q^f = 0$) are then given by $\alpha - \tau - \beta q^c(1 + 1/n) - k^c$. Hence, the profit-maximizing price is $p^* = (\alpha - \tau)\frac{1}{1+n} + k^c \frac{n}{1+n}$. Condition (ii) in Assumption 2 implies $k^f < p^*$.

(ii) for all $i = 1, 2, ..., n$

$$\int_0^\infty e^{-rs}\left[\alpha - \beta\left(q^c(s) + q^f(s)\right) - \tau - k^c\right] q_i^c(s)ds$$
$$\geq \int_0^\infty e^{-rs}\left[\alpha - \beta\left(\sum_{j\neq i} q_j^c s + \hat{q}_i^c(s) - q^f(s)\right) - \tau - k^c\right] \hat{q}_i^c(s)ds$$

for all $\hat{q}_i^c$ satisfying the resource constraint, and

(iii)

$$\int_0^\infty e^{-rs}\left[p(s) - k^c\right] q^f(s)ds \geq \int_0^\infty e^{-rs}\left[p(s) - k^c\right] \hat{q}^f(s)ds,$$

where $p(s) = \alpha - \tau - \beta\left(q^c(s) + q^f(s)\right)$, for all $\hat{q}^f$ satisfying the resource constraint.

We use an optimal control approach to characterize an OL-OFE. The Hamiltonian associated with the fringe's problem reads

$$\mathcal{H}^f = e^{-rt}(p(t) - k^f)q^f + \lambda^f[-q^f]. \tag{6}$$

The necessary conditions include

$$p(t) = \alpha - \tau - \beta(q^f(t) + q^c(t)) \leq k^f + \lambda^f e^{rt}, \tag{7a}$$
$$[k^f + \lambda^f e^{rt} - (\alpha - \tau) + \beta(q^f(t) + q^c(t))]q^f(t) = 0, \tag{7b}$$
$$\dot{\lambda}^f = 0. \tag{7c}$$

Here, λ^f is the fringe's shadow price of the resource stock. Hence, (7a)-(7c) say that in an equilibrium with positive supply of the fringe, the producer price satisfies Hotelling's rule: the net price, $p - k^f$, increases over time at the rate of interest.

The Lagrangian associated with oligopolist i's problem is given by

$$L_i^c = e^{-rt}(\alpha - \tau - \beta(q^f + q^c) - k^c)q_i^c + \lambda_i^c[-q_i^c] + \mu_i^c[b - \sigma - \alpha + \beta(q^f + q^c)]. \tag{8}$$

Due to Assumption 1 we focus on the conditions that characterize an equilibrium where the extraction paths of the oligopolists are identical, $q_i^c = q^c/n$, $\lambda_i^c = \lambda^c$ and $T_i^c = T^c$ for all $i = 1, .., n$, where T_i^c denotes the date at which the resource is depleted by oligopolist

i. The necessary conditions then include

$$\alpha - \tau - \beta(q^f(t) + \left(1 + \frac{1}{n}\right) q^c(t)) \leq k^c + \lambda^c e^{rt} - \mu^c \beta e^{rt}, \tag{9a}$$

$$[k^c + \lambda^c e^{rt} - \beta \mu^c e^{rt} - \alpha + \tau + \beta(q^f(t) + \left(1 + \frac{1}{n}\right) q^c(t))]q^c(t) = 0, \tag{9b}$$

$$\mu^c(t)[b - \sigma - \alpha + \beta(q^f(t) + q^c(t)];\ \mu^c(t) \geq 0, \tag{9c}$$

$$\dot{\lambda}^c = 0, \tag{9d}$$

where λ^c denotes the shadow price of the resource stock of the oligopolists. Hence, conditions (9a)-(9d) imply that as long as $p < \hat{b}$ (i.e., as long as restriction (5) is non-binding), marginal profit of the oligopolists increases over time at the rate of interest. Because the oligopolists are free to choose the moment of depletion of their stocks, in equilibrium the Hamiltonian vanishes at date T^c, implying

$$\left(p(T^c) - k^c - \tau - \lambda^c e^{rT^c}\right) \frac{q^c(T^c)}{n} = 0. \tag{10}$$

In the OL-OFE, different phases of resource extraction exist. By F, C, S and L we denote phases with only the fringe supplying, only the oligopolists supplying, at a price strictly below b, simultaneous supply, and supply by the oligopolists at the backstop price (limit pricing), respectively.

3.1 Preliminary results

The next section provides a preliminary analysis where we first summarize the necessary conditions that hold in each phase (Lemma 1) and then proceed by elimination of specific sequences of phases (Lemma 2). From these two important Lemmata we proceed to characterize an OL-OFE in Section 3.2.

Lemma 1 *Along F we have*

$$p(t) = \alpha - \tau - \beta q^f(t) = k^f + \lambda^f e^{rt}, \tag{11a}$$

$$p(t) = \alpha - \tau - \beta q^f(t) \leq k^c + \lambda^c e^{rt}, \tag{11b}$$

$$q^f(t) = \frac{1}{\beta}(\alpha - \tau - k^f - \lambda^f e^{rt}). \tag{11c}$$

Along S we have

$$p(t) = \alpha - \tau - \beta(q^f(t) + q^c(t)) = k^f + \lambda^f e^{rt}, \tag{12a}$$

$$p(t) = \alpha - \tau - \beta(q^f(t) + \left(1 + \frac{1}{n}\right) q^c(t)) = k^c + \lambda^c e^{rt}, \tag{12b}$$

$$q^f(t) = \frac{1}{\beta}\left(\alpha - \tau - (n+1)(k^f + \lambda^f e^{rt}) + n(k^c + \lambda^c e^{rt})\right), \tag{12c}$$

$$q^c(t) = \frac{n}{\beta}\left(k^f + \lambda^f e^{rt} - k^c - \lambda^c e^{rt}\right). \tag{12d}$$

Along C we have

$$p(t) = \alpha - \tau - \beta q^c(t)) \leq k^f + \lambda^f e^{rt}, \tag{13a}$$

$$p(t) = \alpha - \tau - \left(1 + \frac{1}{n}\right) \beta q^c(t)) = k^c + \lambda^c e^{rt}, \tag{13b}$$

$$q^c(t) = \frac{1}{\beta}\frac{n}{n+1}(\alpha - \tau - k^c - \lambda^c e^{rt}). \tag{13c}$$

Along L we have

$$p(t) = \hat{b}, \tag{14a}$$

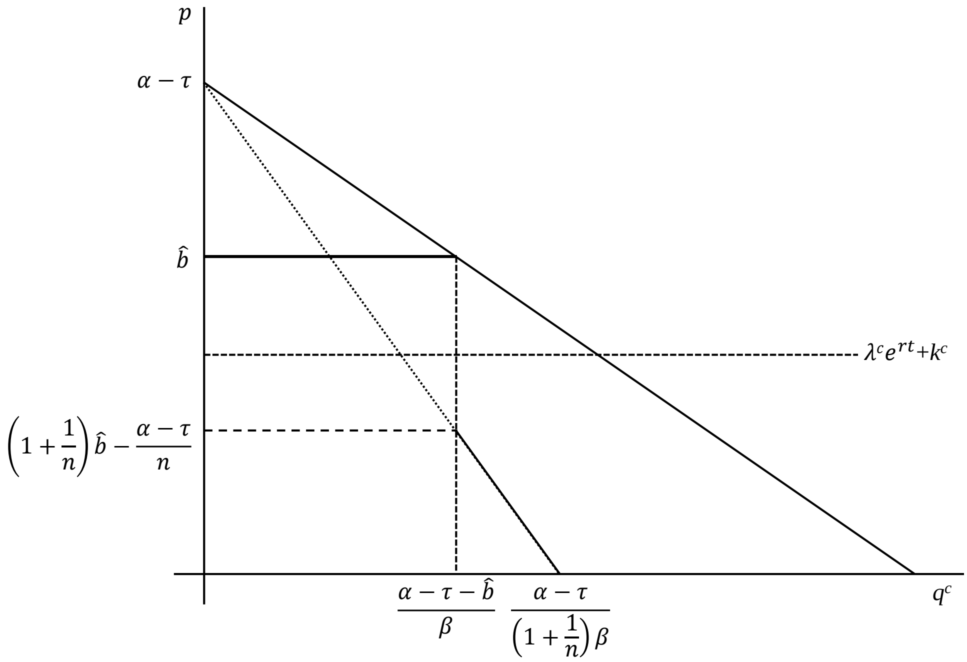
$$q^c(t) = q_L \equiv \frac{\alpha - \tau - \hat{b}}{\beta}, \tag{14b}$$

$$k^c + \lambda^c e^{rt} \geq \alpha - \tau - \left(1 + \frac{1}{n}\right)\beta q_L = \left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha - \tau}{n}. \tag{14c}$$

Proof. Straightforward from the application of the Maximum Principle to the problem of each oligopolist and the fringe and using symmetry. Rewriting conditions (7a), (7b), (9a), (9b), (9c) and (10) in each phase yields the results. Expression (14c) is obtained from (9a) with $\mu > 0$ imposed. □

During the limit-pricing phase, the price is constant and equal to $\hat{b}$ and therefore (11a) and (12a) cannot hold: the fringe's production is nil. Condition (14c) is illustrated in Figure 1. The marginal revenue jumps at $q = q_L$, and when $\hat{b} \geq k^c + \lambda^c e^{rt} \geq (1 + \frac{1}{n})\hat{b} - \frac{\alpha - \tau}{n}$ marginal revenue is not smaller (not larger) than the full marginal cost for $q < (>) q_L$ implying that the profit maximizing quantity is q_L. When $\hat{b} = k^c + \lambda^c e^{rt}$, marginal revenue equals full marginal cost for any $q \in [0, q_L]$ however the symmetric oligopolistic outcome yields q_L because of discounting.

Figure 1: Discontinuous marginal revenue and limit pricing



We denote the oligopolists' marginal profit during limit pricing by[3]

$$\hat{\pi} \equiv \left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha - \tau}{n} - k^c. \tag{15}$$

If $\hat{\pi} \leq 0$, condition (14c) always holds. Therefore, as soon as the stock of the fringe is exhausted the equilibrium will be limit pricing. Intuitively, if marginal profits remain non-positive for all $p \leq \hat{b}$, once the fringe's stock is depleted the oligopolists will set the highest possible price (of course, given that they still have a positive remaining stock). If $\hat{\pi} > 0$, we get from (9b) and (10) that the duration of the limit-pricing phase can be at most

$$\hat{T}_{LM} \equiv \frac{1}{r}\ln\left(\frac{\hat{b} - k^c}{\hat{\pi}}\right), \tag{16}$$

where the term in between brackets equals average profits over marginal profits during limit pricing. We denote a limit-pricing phase of duration $\hat{T}_{LM}$ by $\hat{L}$. A limit-pricing

[3]Define marginal profits of oligopolist i as $\pi(q_i^c; q^c, q^f) \equiv \alpha - \beta(q^c + q^f) - k^c - \tau - \beta q_i^c$. Evaluate at $q^f = 0$ and $q_i^c = q^c = q_L = \frac{\alpha-\tau-\hat{b}}{\beta}$ to get $\hat{\pi} = \pi(q_L, q_L, 0) = \left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha-\tau}{n} - k^c$.

phase with a duration different from $\hat{T}_{LM}$ is denoted by $\tilde{L}$.

We proceed by investigating which sequences of phases are possible in equilibrium. Lemma 2 lists all sequences of phases that can be ruled out because they violate the necessary conditions.

Lemma 2 *In a OL-OFE*

(i) A direct transition from C to F or vice versa is excluded.

(ii) It is not optimal to have $F \to \hat{L}$ nor $F \to \tilde{L}$.

(iii) It is not optimal to have F before S.

(iv) The initial regime is not C.

Proof. See Appendix A.2. □

PROVIDE INTUITION

3.2 Characterization of an OL-OFE

The strategy to characterize an OL-OFE is to consider for a given stock S_0^f which phases occur in equilibrium depending on the stock S_0^c. To this end it will be helpful to identify two threshold stocks S_{0S}^c and $\hat{S}_0^c$. We first identify the conditions to obtain an OL-OFE that consists of only S.

Lemma 3 *Suppose the OL-OFE consists of only S, with final time T. Then*

$$r\beta S_0^f = (\hat{b} + nk^c - (n+1)k^f)(rT - 1 + e^{-rT}) + (\alpha - \tau - \hat{b})rT, \tag{17a}$$

$$r\beta S_0^c = n(k^f - k^c)(rT - 1 + e^{-rT}). \tag{17b}$$

Proof. See Appendix A.2. □

This system defines a one to one relationship between S_0^f and S_0^c that yield an equilibrium S. Given S_0^f this system defines a unique $S_{0S}^c = \Phi\left(S_0^f\right)$ such that the equilibrium is S when the initial stocks are $\left(S_0^f, S_{0S}^c\right)$. The function Φ is strictly increasing. So for each S_0^c we have an equilibrium that reads S when $S_{0S}^f = \Phi^{-1}\left(S_0^c\right)$. Hence, we have established the following result.

Lemma 4 *For each S_0^f there exists a unique S_{0S}^c such that the equilibrium reads S.*

The unique duration of this S-phase is denoted by T_S.

3.2.1 When fringe depletes last

We investigate the equilibrium outcome when we have $S_0^c < S_{0S}^c$. We establish that the sequence of regimes reads $S \rightarrow F$.

Lemma 5 *Given S_0^f, the equilibrium reads $S \rightarrow F$ when $S_0^c < S_{0S}^c$. When $S_0^c \rightarrow S_{0S}^c$ the duration of the F-phase tends to zero.*

Proof. See Appendix A.2. □

3.2.2 When fringe depletes first

Here we examine the possible outcomes when, given S_0^f, we have $S_0^c > S_{0S}^c$. We establish that there will then be a final limit-pricing phase. We start by considering the special case of the equilibrium $S \rightarrow \hat{L}$. The duration of the S-phase is denoted by $\hat{T}_S$ and the duration limit pricing is $\hat{T}_{LM}$ (see (16)). We show in the next lemma that for any given initial stock of the fringe, the existence of this equilibrium requires a unique initial stock of the oligopolists. Moreover, the length of the S-phase is larger than in the equilibrium where there is only an S-phase (Lemma 4). We will distinguish cases with positive (Lemma 6-8) and negative (Lemma 9) marginal profits of the oligopolists during limit-pricing.

Lemma 6 *Suppose marginal profits during limit pricing are positive, i.e., $\hat{\pi} > 0$. Then:*

(i) For each S_0^f, there exists a unique $\hat{S}_0^c$ such that the equilibrium reads $S \rightarrow \hat{L}$.

(ii) $\hat{T}_S > T_S$ and $\hat{S}_0^c > S_{0S}^c + S_{LM}$.

Proof. See Appendix A.2. □

Note that the duration of the limit-pricing phase depends on the marginal profit of the oligopolists at the end of the S-phase. This can be seen by combining (9b) and (10), yielding

$$e^{r\Delta} = \frac{\hat{b} - k^c}{\hat{b} - \beta \frac{q^c(T^-)}{n} - k^c}, \tag{18}$$

where Δ denotes the duration of limit pricing and $T^- \equiv \lim_{t \uparrow T} t$ denotes the end of the S-phase. It follows from Lemma 6 that if S_0^c equals the threshold $\hat{S}_0^c$, we have $\Delta = \hat{T}_{LM}$. Substitution into (16) then makes clear that the oligopolists serve the entire market at the end of the S-phase, i.e., $q^c(T^-) = q_L$ and $q^f(T^-) = 0$.

The next lemma shows what the equilibrium is if the initial stock of the oligopolists exceeds the threshold $\hat{S}_0^c$.

Lemma 7 *Suppose marginal profits during limit pricing are positive, i.e.,* $\hat{\pi} > 0$. *Given* S_0^f *then for any* $S_0^c \geq \hat{S}_0^c$ *the equilibrium reads* $S \rightarrow C \rightarrow \hat{L}$.

Proof. See Appendix A.2. □

Intuitively, compared to the equilibrium in Lemma 6, the duration of the limit-pricing phase cannot increase, as is clear from (18), because the oligopolists are already serving the entire market at the end of the S-phase. As a result, the increase in the initial stock of the oligopolists gives rise to the occurrence of an intermediate C-phase before limit-pricing starts.

Lemma 8 characterizes the equilibrium in case the initial stock of the oligopolists falls short of the threshold $\hat{S}_0^c$, but still exceeds S_{0S}^c.

Lemma 8 *Suppose marginal profits during limit pricing are positive, i.e.,* $\hat{\pi} > 0$. *Given* S_0^f *then for any* $S_0^c \in \left(S_{0S}^c, \hat{S}_0^c\right)$ *the equilibrium reads* $S \rightarrow \tilde{L}$.

Proof. See Appendix A.2. □

In this case, the oligopolists still have a positive remaining stock at the end of the S-phase, but the remaining stock size is insufficient to have a final limit-pricing phase $\hat{L}$ of duration $\hat{T}_{LM}$. As a result, there will be limit pricing for a shorter period of time.

Therefore, the market share of the oligopolists at the end of the S-phase, q^f/q_L, in this equilibrium will be smaller than that in the equilibria described in the previous two lemmata. Moreover, this market share will converge to zero if S_0^c converges to S_{0S}^c (and therefore Δ converges to zero), as can be noticed from (18).

We now consider the case in which marginal profits of the oligopolists are non-positive as long as the fringe does not supply.

Lemma 9 *Suppose marginal profits during limit pricing are non-positive, i.e.,* $\hat{\pi} \leq 0$. *Then for any* $S_0^c \geq S_{0S}^c$ *the equilibrium reads* $S \rightarrow \tilde{L}$.

Proof. See Appendix A.2. □

Intuitively, if the oligopolists have an initial stock large enough to end up with a positive remaining stock at the moment when the fringe's stock is depleted, non-positive marginal profits imply that they will maximize profits by choosing for a limit-pricing strategy from that moment onwards until depletion, irrespective of the size of their remaining resource stock.

3.2.3 The OL-OFE

We are now ready to give a full characterization of the OL-OFE. The results from Lemma 1-7 can be collected into Proposition 1.

Proposition 1 (Characterization of the equilibrium)

(i) Suppose marginal profits during limit pricing are non-positive, i.e., $\hat{\pi} \leq 0$. *Then for any given* $S_0^f \geq 0$, *there exists a unique* S_{0S}^c *such that:*

- *(a) If* $S_0^c < S_{0S}^c$ *the equilibrium reads* $S \rightarrow F$.
- *(b) If* $S_0^c = S_{0S}^c$ *the equilibrium reads* S.
- *(c) If* $S_0^c > S_{0S}^c$ *the equilibrium reads* $S \rightarrow \tilde{L}$.

(ii) Suppose marginal profits during limit pricing are positive, i.e., $\hat{\pi} > 0$. *Then for any given* $S_0^f \geq 0$, *there exists a unique* S_{0S}^c *and a unique* $\hat{S}_0^c$, *such that:*

- *(a) If* $S_0^c < S_{0S}^c$ *the equilibrium reads* $S \rightarrow F$.

(b) If $S_0^c = S_{0S}^c$ the equilibrium reads S.

(c) If $S_0^c \in (S_{0S}^c, \hat{S}_0^c)$ the equilibrium reads $S \to \tilde{L}$.

(d) If $S_0^c = \hat{S}_0^c$ the equilibrium reads $S \to \hat{L}$.

(e) If $S_0^c > \hat{S}_0^c$ then the equilibrium reads $S \to C \to \hat{L}$.

Figure 2 illustrates the equilibrium sequence for different combinations of the initial resource stock of the oligopolists (horizontal axis) and the fringe (vertical axis). Panel (a) shows the case with positive marginal profits during limit pricing (part (i) of the proposition), whereas panel (b) shows the case with non-positive marginal profits during limit pricing (part (ii) of the proposition).

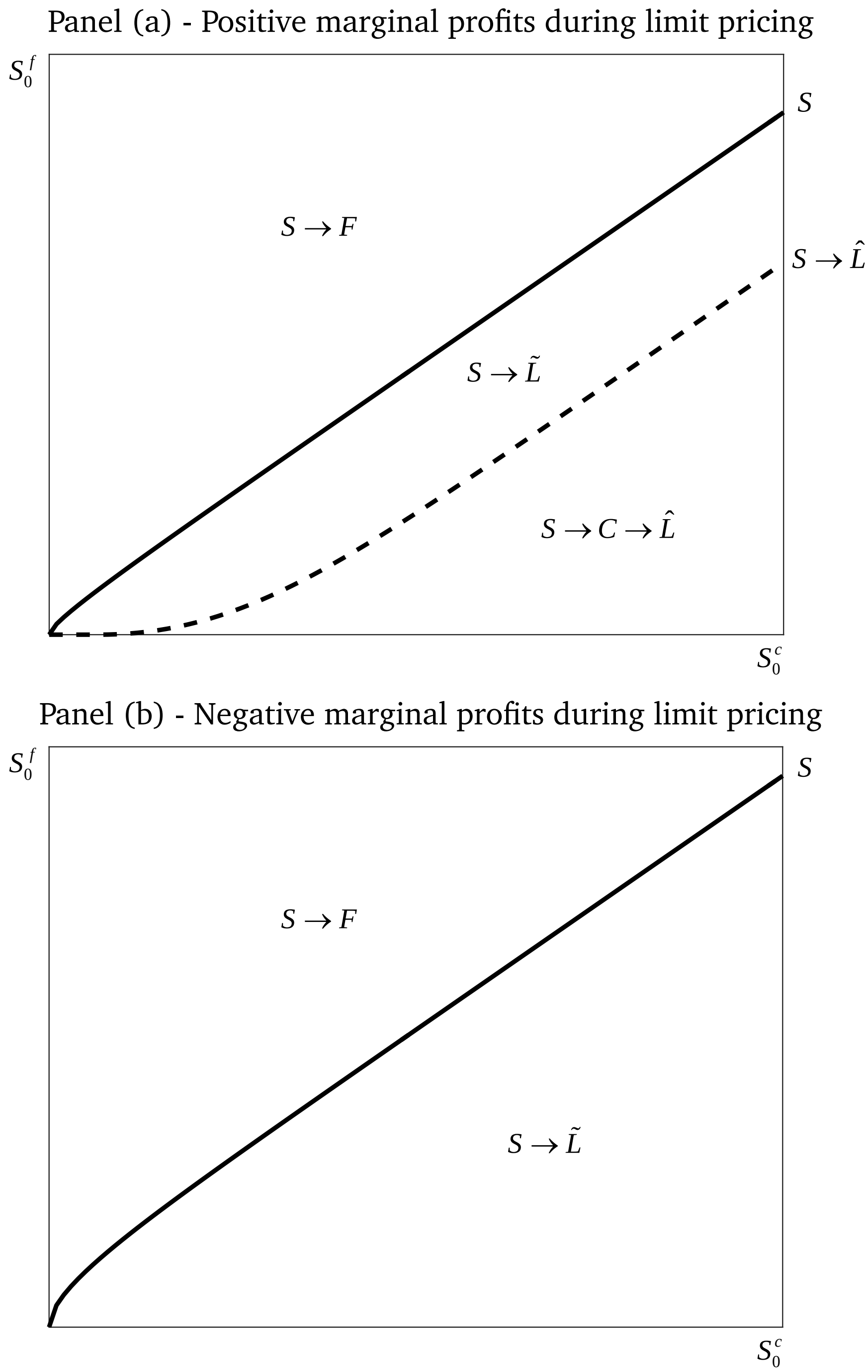
The OL-OFE of our model clearly shows what happens if a resource cartel gets confronted with the existence of a fringe and with a smaller degree of coordination between its members. Starting from the horizontal axis in panel (a) (where the S-phase is degenerate, as $S_0^f = 0$), the equilibrium reads $C \to \hat{L}$.

The entrance of a fringe implies that the equilibrium changes to $S \to C \to \hat{L}$. Moreover, the larger the initial stock of the fringe (move upwards in the figure) the smaller the duration of the intermediate C-phase becomes, until it entirely disappears when the $S \to \hat{L}$ locus is reached. Further increasing the initial stock of the fringe implies that the duration of the limit-pricing phase will go down as well, until it vanishes completely when the S-locus is reached. If the fringe's initial stock becomes even larger, the stock of the resource cartel is exhausted before that of the fringe.

The effect of an increase in n, i.e., of moving from a cartel to an oligopoly, will be discussed in the next section. However, it is clear from (16) already that the duration of the $\hat{L}$-phase will go down if n increases (while keeping S_0^c fixed).

Now moving to the horizontal axis in panel (b) (where again the S-phase is degenerate), the equilibrium reads $\tilde{L}$, meaning that there is limit-pricing throughout. The entrance of a fringe changes the equilibrium to $S \to \hat{L}$. Furthermore, the duration of S-phase increases at the cost of the duration of the $\tilde{L}$-phase as the initial resource stock of the fringe goes up. Limit-pricing disappears once the S-locus is reached. Moreover, the fringe will again survive the cartel once the locus is crossed.

Figure 2: Characterization of the equilibrium



3.3 Limit pricing throughout?

Recently, Andrade de Sá and Daubanes (2016) have shown that under monopolistic oil supply, limit pricing will occur from the beginning if the price elasticity of oil demand is smaller than unity, which has often found to be the case empirically (cf. Hamilton, 2009c,a). Intuitively, as long as the price elasticity of resource demand is below unity, the monopolist maximizes its profits by setting the oil price as high as possible at each moment in time, i.e., just below the price of renewable energy. However, if the oil market is not characterized by monopolistic supply, but by an oligopoly-fringe structure instead, this 'limit pricing throughout' result may break down for two reasons.

First, as long as the fringe starts out with a positive stock, there will be an initial regime of simultaneous use during which the oil price is lower than the renewables price (see Proposition 1). The oligopolists cannot perform a limit-pricing strategy from the beginning, because the fringe would drive down the price by increasing supply.

Second, by defining the price elasticity of demand as $\varepsilon(q) \equiv -\frac{dq(p+\tau)}{dp} \cdot (p+\tau)/q$ we can rewrite condition (14c) as

$$\varepsilon(q_L) \leq \frac{1}{n} + \frac{k^c + \lambda^c e^{rt} + \tau}{\beta q_L}. \tag{19}$$

As long as this condition is satisfied, marginal profits of the oligopolists during limit pricing are non-positive, implying that they will choose a limit-pricing strategy as soon as the fringe's stock is exhausted (indeed, from the beginning if $S_0^f = 0$). For $n = 1$ (the monopoly case considered in Andrade de Sá and Daubanes (2016)), the right-hand side of (19) exceeds unity. Hence, under monopolistic oil supply this condition is always satisfied if $\varepsilon(q_L) < 1$. In that case, there will be limit pricing throughout if $S_0^f = 0$. However, if $n > 1$, the inequality $\varepsilon(q_L) < 1$ is no longer a sufficient condition for (19) to hold. Hence, even with inelastic oil demand and without the existence of a fringe, limit pricing does not necessarily occur from the beginning. The reason is that an oligopolist's *individual* price elasticity of demand during limit pricing, $\varepsilon_i(q_L)$, exceeds the *aggregate* price elasticity of demand.[4] If the individual price elasticity of demand exceeds unity, limit pricing throughout is no longer necessarily the profit maximizing

[4]From the demand function in a limit pricing phase with $q = q_L$, we get $\varepsilon(q_L) = \frac{\alpha - \beta q_L}{\beta q_L}$ and $\varepsilon_i(q_L) = \frac{\alpha - \beta q_L}{\beta q_L / n} = n\varepsilon(q_L)$. Hence $n > 1$ implies $\varepsilon_i(q_L) > \varepsilon(q_L)$.

strategy.

Hence, both recent developments on the oil market, the shale oil revolution in the US and the partial collapse of the OPEC cartel, make a strategy of limit pricing throughout less likely.

4 Comparative statics

In an oligopoly-fringe market, the effects of climate change policies differ markedly from those under the extreme circumstances of perfect competition and pure monopoly. In this section, we investigate these effects. Furthermore, we discuss the consequences of the (partial) collapse of OPEC for the resource extraction paths and the effectiveness of climate policies, by increasing the number of oligopolists above unity.

Proposition 2 discusses the effect of a renewables subsidy on initial oil extraction.

Proposition 2 (Renewables subsidy and initial extraction)

(i) If the equilibrium reads $S \rightarrow F$, a marginal increase in the renewables subsidy increases initial extraction.

(ii) If the equilibrium reads $S \rightarrow \tilde{L}$, then

(a) if marginal profits during limit pricing are positive (i.e., if $\hat{\pi} > 0$), and $n = 1$, then for initial stocks close to the S locus ($S \rightarrow \hat{L}$ locus) a marginal increase in the renewables subsidy increases (decreases) initial extraction.

(b) if marginal profits during limit pricing are non-positive (i.e., if $\hat{\pi} \leq 0$), a marginal increase in the renewables subsidy increases initial extraction.

(iii) If the equilibrium reads $S \rightarrow C \rightarrow \hat{L}$ and $n = 1$, a marginal increase in the renewables subsidy decreases initial extraction.

To understand the results in Proposition 2, it is helpful to consider the extreme cases of perfect competition and pure monopoly on the resource market. Under perfect competition, a subsidy for renewables increases initial extraction. This is the standard Green Paradox effect discussed by Sinn (2008, 2012). The reason is that by making renewables cheaper, the subsidy lowers the future market price of oil. As a result, resource owners respond by depleting their stock more rapidly, which increases initial extraction. With monopolistic resource supply, on the contrary, the resource owner responds to a renewables subsidy by increasing the initial price and thereby lowering extraction (as long as he does not choose for limit pricing from the beginning). So doing, the monopolist effectively postpones entry of renewables producers (cf. Gilbert and Goldman, 1978; Hoel, 1983; Van der Meijden and Withagen, 2016).

As long as the initial aggregate stock of the oligopolists is small relative to that of the fringe, the equilibrium reads $S \rightarrow F$ (Proposition 1, parts (ia) and (iia)). In this case, the perfectly competitive mechanism dominates, implying that initial supply goes up in response to an increase in the renewables subsidy (part (i) of Proposition 2). However, if the initial aggregate stock of the oligopolists is relatively large and if marginal profits during limit pricing are positive, the equilibrium sequence reads $S \rightarrow C \rightarrow \hat{L}$ (Proposition 1, part (iie)). In that case, for $n = 1$ the monopolistic mechanism dominates and initial extraction decreases upon a rise in the renewables subsidy (part (iii) of Proposition 2). For $n > 1$, imperfect competition may be too small to generate a decrease in initial extraction.

Finally, for the intermediate case in which the equilibrium reads $S \rightarrow \tilde{L}$ (see part (ic) and (iid) of Proposition 1), we distinguish two cases. First, if marginal profits during limit pricing are positive (part (iia) of Proposition 2) the effect of a renewables subsidy on initial extraction is ambiguous. The competitive mechanism dominates (so that initial extraction goes up) close to the S locus, whereas if $n = 1$ the monopolistic mechanism is dominant (so that initial extraction goes down) close to the $S \rightarrow \hat{L}$ locus. Second, if marginal profits during limit pricing are negative (part (iib) of Proposition 2), a renewables subsidy increases initial extraction, just like in the case under perfect competition and under pure monopoly with negative marginal profits during limit pricing, which would be characterized by limit pricing from the beginning (Van der Meijden and Withagen, 2016).

Proposition 3 considers the effects of climate policy on the duration of limit pricing and the time of depletion.

Proposition 3 (Policy and extraction duration)

(i) If the equilibrium reads $S \rightarrow C \rightarrow \hat{L}$, a marginal increase in the renewables subsidy and/or the carbon tax decrease the duration of the $\hat{L}$-phase.

(ii) A marginal increase in the renewables subsidy (carbon tax) decreases (postpones) the time of depletion.

The result in part (i) can be understood by noting from (16) that the duration of the limit-pricing phase depends on the proportional difference between average profits and marginal profits at the after-tax-and-subsidy renewables price $\hat{b}$. Both a renewables subsidy and a carbon tax lower this proportional difference and thus shorten the duration of the limit-pricing phase.

Part (ii) says that irrespective of the relative initial stocks of the oligopolists and the fringe, an increase in the renewables subsidy brings forward, whereas a carbon tax postpones the time of depletion. In the perfectly competitive case, the renewables subsidy shifts down (up) the entire resource price (extraction) path, which implies that depletion occurs sooner. Under pure monopoly, although initial extraction goes down, supply during the limit pricing phase goes up. On balance, the depletion time goes down. To understand the increase in the time of depletion upon an increase of the carbon tax, note that the carbon tax effectively increases marginal extraction costs, implying more conservative extraction.

Climate policies also affect the borders of the equilibrium sequence regions shown in Figure 2, as described in the following proposition.

Proposition 4 (Policy, number of oligopolists, and equilibrium loci)

(i) Upon an increase in the renewables subsidy (carbon tax), the S locus shift to the left (right);

(ii) Upon an increase in the renewables subsidy and/or the carbon tax, the $S \to \hat{L}$ locus shifts to the right.[5]

(iii) Upon an increase in n, both the S locus and the $S \to \hat{L}$ shift to the right.

Proof. See Appendix A.2. □

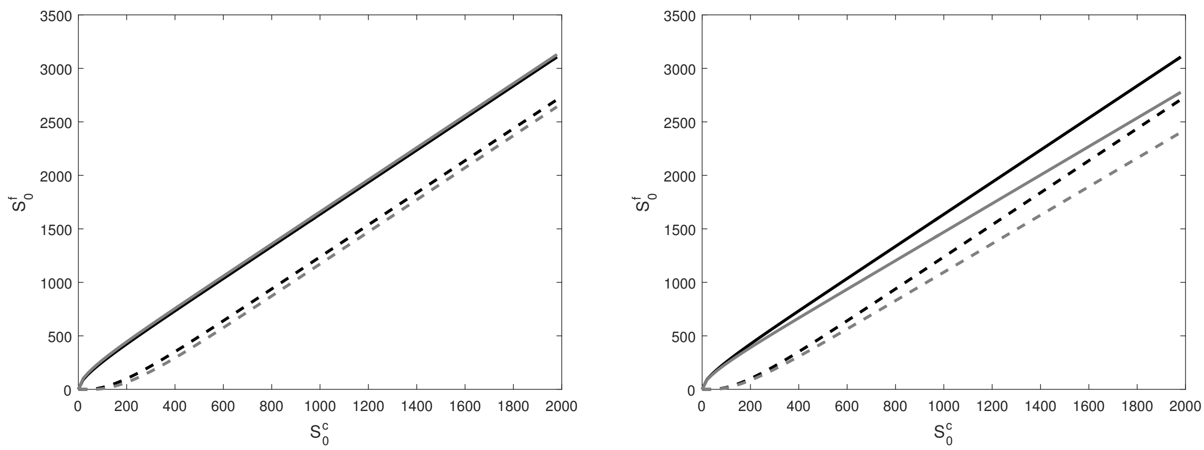
Intuitively, the renewables subsidy increases initial extraction of the fringe. Hence, a larger minimum initial resource stock of the fringe is needed to get $S \to F$. The carbon

[5]Note that the $S \to \hat{L}$ locus does not exist if $\hat{\pi} \leq 0$.

tax, however, mainly lowers extraction of the fringe during the simultaneous phase. This can be noticed from (12c) and (12d), which show that the carbon tax has a direct negative effect on q^f, and not on q^c. Therefore, a lower minimum initial resource stock of the fringe is needed to get $S \rightarrow F$. This explains part (i) of the proposition. Part (ii) follows by noting that an increase in the renewables subsidy increases the extraction rate during the $\hat{L}$ phase. This effect dominates the increase in the duration of the $\hat{L}$ phase and the increase of the fringe's initial extraction. Hence, a larger S_0^c is required to obtain the equilibrium outcome $S \rightarrow C \rightarrow \hat{L}$. The carbon tax, however, leaves the extraction rate during the $\hat{L}$ phase unaffected, but mainly decreases extraction of the fringe during the simultaneous phase. Hence, an increase in the carbon tax also pushes up the S_0^c that is required to obtain the equilibrium outcome $S \rightarrow C \rightarrow \hat{L}$. The intuition for part (iii) is simple: extraction per oligopolist during the S-phase increases if the number of oligopolists goes up. As a result, the threshold values for S_0^f to obtain $S \rightarrow \tilde{L}$ and $S \rightarrow C \rightarrow \hat{L}$ go up, implying that both loci shift to the right.

Figure 3: Effects on equilibrium sequences

Panel (a) - Renewables subsidy   Panel (b) - Carbon tax



Notes: The black lines correspond to the case of $\sigma = \tau = 0$. In panel (a), the grey lines correspond to $\sigma = 10$ (and $\tau = 0$). In panel (b), the grey lines correspond to $\tau = 10$ (and $\sigma = 0$). We have chosen the following parameter values: $\alpha = 210$, $\beta = 75/17$, $b = 157.5$, $k^c = 30$, $k^f = 60$, $n = 2$, $r = 0.1$, and $\tau = 0$.

4.1 Calibration

We calibrate our model by using data on proven crude oil reserves, global crude oil consumption, extraction costs, the oil price, and the price elasticity of oil demand. Proven reserves owned by OPEC and the rest of the world amount to 1220 and 433 billion barrels, respectively (EIA, 2017). We choose $\alpha = 210$ and $\beta = \frac{150}{34}$ to get initial oil demand equal to the 33.6 billion barrels of average yearly global oil consumption in 2012-2014 (EIA, 2017), an initial price elasticity of oil demand equal to 0.4, which is within the range of long-run price elasticities reported by Hamilton (2009b), if the initial oil price would be 61.5 US dollars per barrel, close to the current oil price of about 55 US dollars per barrel. We use the break even estimates of around 30 and 60 US dollars per barrel as values for the marginal extraction costs of the OPEC oligopolists and the shale fringe, respectively (Rystad Energy, 2014).[6] Following Brémond et al. (2012), we devide OPEC within two subgroups that act cohesively by imposing $n = 2$. Finally, we have picked $b = 157.5$ to indeed get an initial equilibrium price equal to 61.5 US dollars per barrel. An overview of our benchmark calibration and the implied equilibrium values is provided in Table 1.[7]

Table 1: Benchmark calibration

parameters	**description**	**value**	**unit**
α	choke price	210	US$/bbl
β	slope inverse demand function	150/34	US$/bbl
b	renewables price	157.5	US$/BOE
k^c	marginal extraction cost oligopolists	30	US$/bbl
k^f	marginal extraction cost fringe	60	US$/bbl
n	number of oligopolists	2	number
S_0^c	initial (aggregate) stock oligopolists	1220	billion bbl
S_0^f	initial stock fringe	433	billion bbl
implied values	**description**	**value**	**unit**
$q^c(0) + q^f(0)$	initial oil consumption	33.6	billion bbl
$q^c(0)/(q^c(0) + q^f(0))$	initial OPEC market share	0.4	billion bbl
$p(0)$	initial oil price	61.5	US$/bbl
$\varepsilon(q^c(0) + q^f(0))$	initial price elasticity of demand	0.4	elasticity

4.2 Numerical illustration

In this section, we simulate the model to quantify the effects of changes in policy and market power in our benchmark calibration.

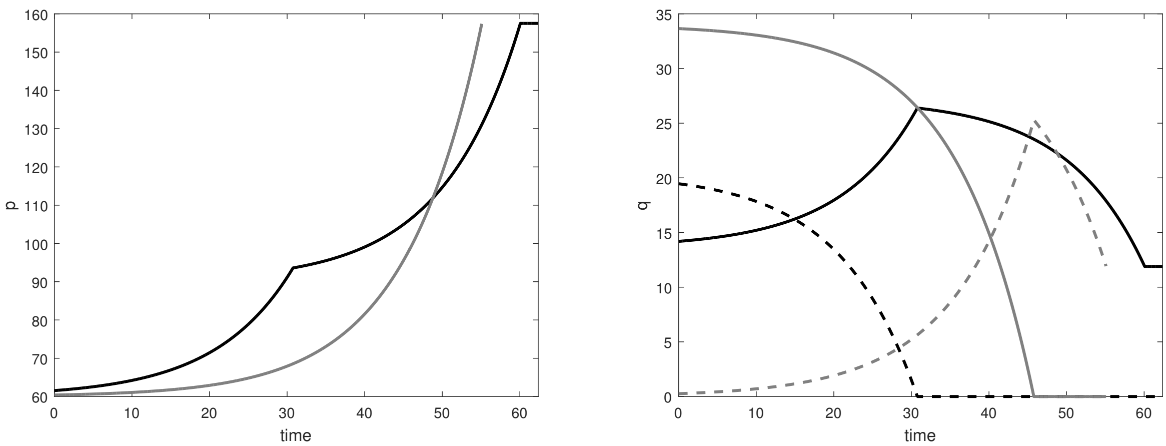
[6]Rystad Energy (2014) defines the break even price as the "Brent oil price at which NPV equals zero, and considers all future cash flows using a real discount rate of 7.5 percent".

[7]We use tC to denote 'metric tonnes of carbon', bbl for 'barrels of oil' (one barrel contains about 159 litres) and BOE for 'barrels of oil equivalent'.

Figure 3 illustrates the results of Proposition 4 by showing the shifts in the two loci upon an increase of the renewables subsidy (panel (a)) and the carbon tax (panel (b)), both from 0 to 10 (US$/bbl). The black lines correspond to the benchmark equilibrium with $\sigma = \tau = 0$. Recall from Figure 2 that in the area below the dashed line, the equilibrium sequence reads $S \to C \to \hat{L}$, in between the solid and the dashed line, we have $S \to \tilde{L}$ and above the dashed line $S \to F$.

Figure 4: Effect of cartel cohesion on price and extraction paths

Panel (a) - Fossil price   Panel (b) - Fossil use

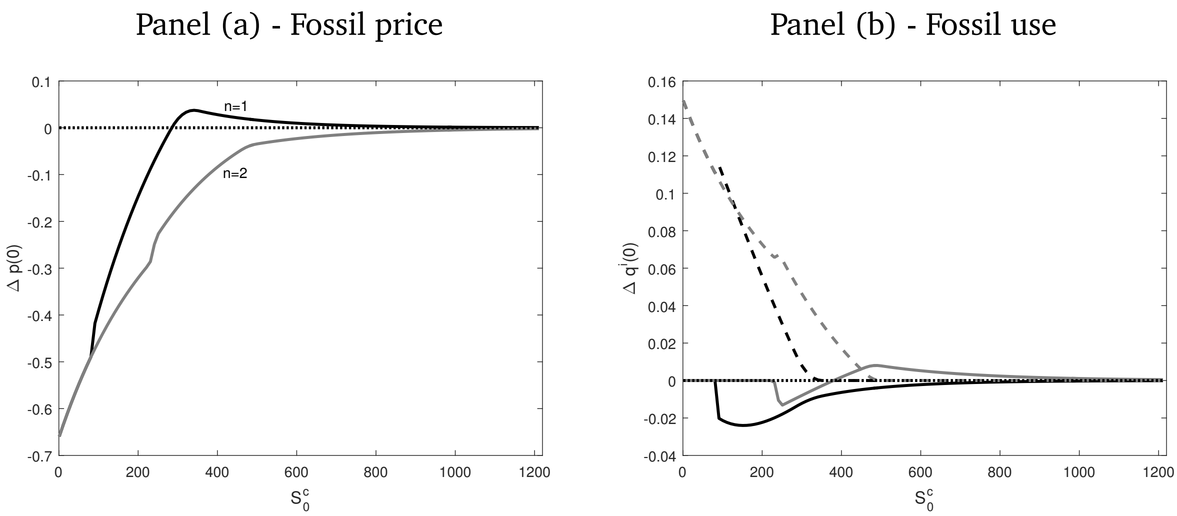


Notes: The black lines correspond to the benchmark case with $n = 2$. The grey lines correspond to the case with $n = 5$. In panel (b), the solid line represents extraction of the oligopolists and the dashed line represents extraction of the fringe. We have chosen the following parameter values: $\alpha = 210$, $\beta = 175/17$, $b = 157.5$, $k^c = 30$, $k^f = 60$, $r = 0.1$, $\sigma = 0$, $\tau = 0$, $S_0^c = 1220$, and $S_0^f = 433$.

Figure 4 examines the effect of the number of oligopolists (keeping the aggregate initial stock S_0^c constant) on the price path (panel (a)) and the extraction paths (panel (b)). The solid lines correspond to the benchmark case with $n = 2$, and the dashed lines represent the equilibrium with $n = 5$. Increasing the number of oligopolists, i.e., increasing competition, yields the intuitive result that the initial price drops and depletion occurs sooner (panel (a)). Furthermore, it is clear from the two price paths that in this particular example the equilibrium sequence changes from $S \to C \to \hat{L}$ to $S \to F$ (as the limit-pricing phase disappears), in line with our discussion of Figure 3. Panel (b) shows the accompanying extraction paths, where the black lines correspond to the extraction paths of the oligopolists and the grey lines to the extraction paths of the fringe. Increasing the number of oligopolists markedly boosts their aggregate extraction initially, whereas the fringe responds by postponing its extraction.

The solid line in panel (a) of Figure 5 shows that in the cartel-fringe case (with

Figure 5: Relative stock and the Green Paradox - renewables subsidy



Notes: The figure shows the effect of an increase in σ from 0 to 10 on the initial price (panel (a)) and the initial extraction rates (panel (b)). The black lines correspond to the case with $n = 1$ and the grey lines to the case with $n = 2$. In panel (b), the solid lines represent extraction of the oligopolist(s) and the dashed lines represent extraction of the fringe. We have chosen the following parameter values: $\alpha = 210$, $\beta = 75/17$, $b = 157.5$, $k^c = 30$, $k^f = 60$, $r = 0.1$, $\tau = 0$, $S_0^f = 433$.

$n = 1$), increasing the renewables subsidy from $\sigma = 0$ to $\sigma = 10$ causes a Weak Green Paradox, i.e., an increase in current extraction, only if the initial resource stock of the cartel is small compared to that of the fringe. If the stock of the cartel is above a certain threshold, the initial resource price rises and aggregate initial extraction goes down upon the introduction of a renewables subsidy (see parts (iia) and (iii) of Proposition 2). This effect does no longer occur if the cartel breaks down: the dashed line in panel (a) represents our benchmark case with $n = 2$, in which the initial price decreases over the entire range of the oligopolist's initial resource stock. Panel (b) contains the effect on the extraction paths. The solid (dashed) line represents extraction of the oligopolists (fringe). The black (grey) lines correspond to the case with $n = 1$ ($n = 2$). Although for $n = 2$ the extraction rate of the oligopolists still goes down upon the introduction of a renewables subsidy, this decrease is dominated by the boost in the fringe's extraction.

5 Welfare analysis

This section examines the effects of climate policies and market structure on climate damages and social welfare. We define 'grey' welfare, W^G, as the discounted sum of consumer surplus, producer surplus and tax revenue, minus subsidy costs:[8]

$$W^G \equiv \int_0^{\bar{T}} e^{-rt}\left\{\frac{1}{2}[\alpha-(p+\tau)](q^c+q^f)+(p-k^c)q^c+(p-k^f)q^f+\tau(q^c+q^f)\right\}dt$$
$$+\frac{e^{-r\bar{T}}}{r}\left\{\frac{1}{2}[\alpha-(b-\sigma)]q_L-\sigma q_L\right\},$$

where $\bar{T}$ denotes the moment at which the last resource stock is depleted. Rearranging this expression by using the resource demand function yields

$$W^G \equiv \int_0^{\bar{T}} e^{-rt}\left[(\alpha-k^c)q^c+(\alpha-k^f)q^f-\frac{1}{2}\beta(q^c+q^f)^2\right]dt$$
$$+\frac{e^{-r\bar{T}}}{r}\left[\frac{\alpha-b}{\beta}(\alpha-\tau-\hat{b})-\frac{1}{2}\frac{1}{\beta}(\alpha-\tau-\hat{b})^2\right].$$

Furthermore, we impose a climate damage function $Z=\psi E^{\phi}$, with $\psi>0$ and $\phi\geq 1$, where E denotes the atmospheric stock of carbon, which evolves according to

$$\dot{E}(t)=\omega^c q^c(t)+\omega^f q^f(t),$$

where ω^c and ω^f denote the emission factors of the oligopolists and the fringe, respectively. The discounted value of climate damage is given by:

$$D=\int_0^{\infty} e^{-rt}\psi[E(t)]^{\phi}dt.$$

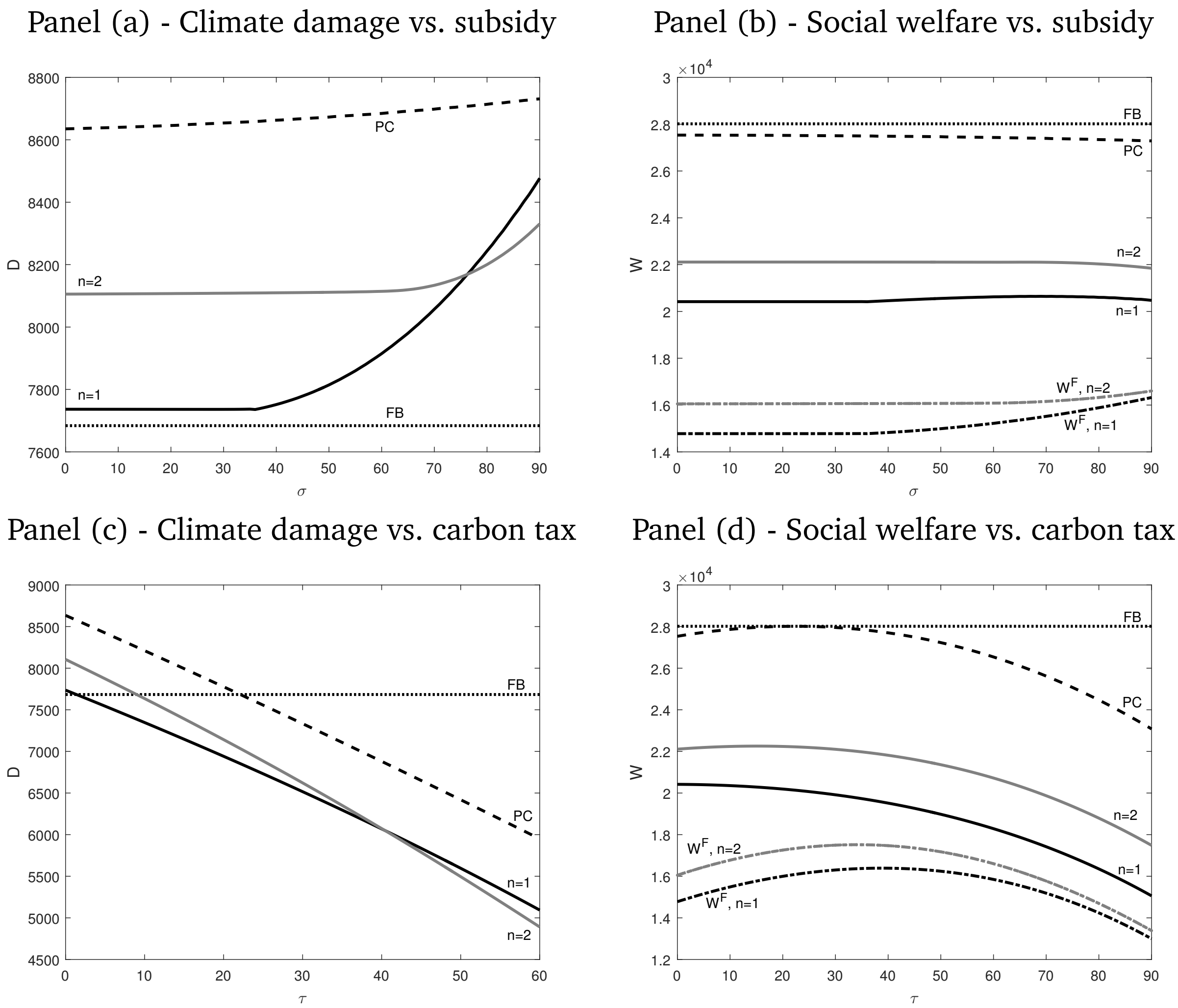
Social welfare, W, is defined as the difference between grey welfare and climate damage: $W\equiv W^G-D$.

In our benchmark calibration, we impose $\phi=1$ and $\phi=20$. This implies a constant social cost of carbon (SCC) equal to $\frac{\psi}{r}=200$ US$ per metric ton carbon, in line with the estimates reported by Pindyck (2016) based on expert opinions. The carbon content of conventional crude oil, ω^c, equals 0.1108 tC/bbl (EPA, 2017). We assume that the emission factor of the fringe is 25 percent larger than the emission factor of the oligopolists (i.e., $\omega^f=0.1385$), in order to take into account that producing oil from,

[8]Alternatively, we could define a the quasi-linear utility function $U(q^c+q^f+x)=\alpha(q^c+q^f+x)-\frac{1}{2}\beta(q^c+q^f+x)^2+M$ (which gives rise to our linear demand function), where x denotes renewables consumption and M expenditure on a *numeraire* good. This results in the same expression for W^G.

e.g., tar sands, is more polluting than extracting conventional oil (cf. Brandt, 2008).

Figure 6: Welfare effects of climate policies



Notes: The figure shows climate damage (panels (a) and (c)) and social welfare (panels (b) and (d)) for various values of the renewables subsidy (panels (a) and (b)) and the carbon tax (panels (c) and (d)). The solid black (grey) lines correspond to the oligopoly-fringe equilibrium with $n = 1$ ($n = 3$). The dashed lines indicate the perfectly competitive equilibrium (indicated with PC). The dotted lines correspond to the first-best outcome (indicated with FB). The dashed-dotted lines in panels (b) and (d) correspond to social welfare excluding the oligopolists' profits. We have chosen the following parameter values: $\alpha = 210$, $\beta = 75/17$, $b = 157.5$, $k^c = 30$, $k^f = 60$, $r = 0.1$, $S_0^c = 1220$, $S_0^f = 433$, $SCC = 200$, $\omega^f = 1$, $\omega^c = 1.25$.

Figure 6 shows the effects of a renewables subsidy (upper two panels) and a carbon tax (lower two panels) on climate damage (left panels) and social welfare (right panels). The dashed (dotted) lines in the figure shows the first-best and the equilibrium under perfect competition, respectively, as reference points.[9] In each panel, the black line corresponds to the cartel-fringe case ($n = 1$) and the grey line to the benchmark oligopoly-fringe case with $n = 2$. The dashed-dotted lines in panels (b) and (d) depict

[9]The first-best and the equilibrium under perfect competition are derived in Appendix A.3.

the social welfare excluding profits of the oligopolists.[10]

Panel (a) shows that climate damage is much lower in the cartel-fringe and oligopoly cases than under perfect competition, because of relatively conservative extraction. Still, climate damage is larger than in the first-best. Nevertheless, as indicated in panel (b), social welfare for the imperfectly competitive cases is lower than under perfect competition. There are two reasons for this. First, in terms of grey welfare, extraction is too conservative in the cartel-fringe and oligopoly-fringe equilibria (conservation effect). Second, in these cases the Herfindahl rule is violated, meaning that extraction of the relatively expensive resource starts before the cheap resource is exhausted (sequence effect). An increase in the subsidy increases climate change in all three equilibria, as show in panel (a). Furthermore, panel (b) shows that under perfect competition the subsidy reduces social welfare by speeding up extraction. With imperfect competition however, extraction is too conservative in the benchmark equilibrium, implying that the welfare effect of the subsidy is smaller. For $n = 1$, social welfare even increases in the renewable subsidy over a certain range of subsidy rates. Social welfare excluding the oligopolists' profits increases in the subsidy, as the subsidy causes a reallocation from the oligopolists to the consumers of oil.

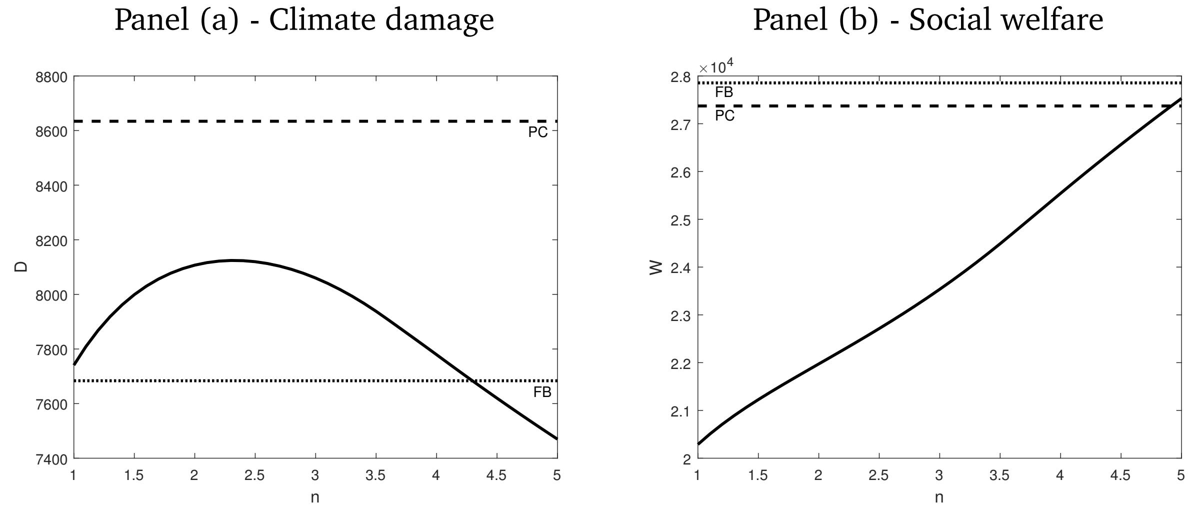
Panels (c) and (d) show the effects of the carbon tax. A constant carbon tax slows down resource extraction. If the tax rate is large enough, climate damage in all three equilibria will be lower than in the first-best. Panel (d) shows that the perfectly competitive equilibrium almost coincides with the first-best if the carbon tax equals the social cost of carbon.[11] The second-best tax rate is lower in the imperfectly competitive cases (i.e., the top of the solid grey and black lines in panel (d) is located to the left of the top of the dashed line), because of relatively conservative extraction by the cartel and the oligopolists. On the contrary, the second-best tax rate in case profits of the oligopolists are excluded is higher, because the tax causes a reallocation from the oligopolists to the consumers.

In Figure 7, we show the effect of cartel cohesion (in terms of the number of oligopolists) on climate damage and social welfare. Panel (a) shows that the effect

[10]Social welfare excluding profits of the oligopolists is defined as $W^F \equiv W - \int_0^\infty e^{-rt}(p(t) - k^c)q^c(t)dt$.

[11]Because extraction by the oligopolists and by the fringe are taxed at the same rate (so, strictly speaking, τ denotes a *resource* tax instead of a *carbon* tax), the perfectly competitive equilibrium only coincides with the first-best if emission factors of the cartel/oligopolists and the fringe are the same and the tax rate equals the SCC multiplied by the common emission factor.

Figure 7: Welfare effect of cartel cohesion

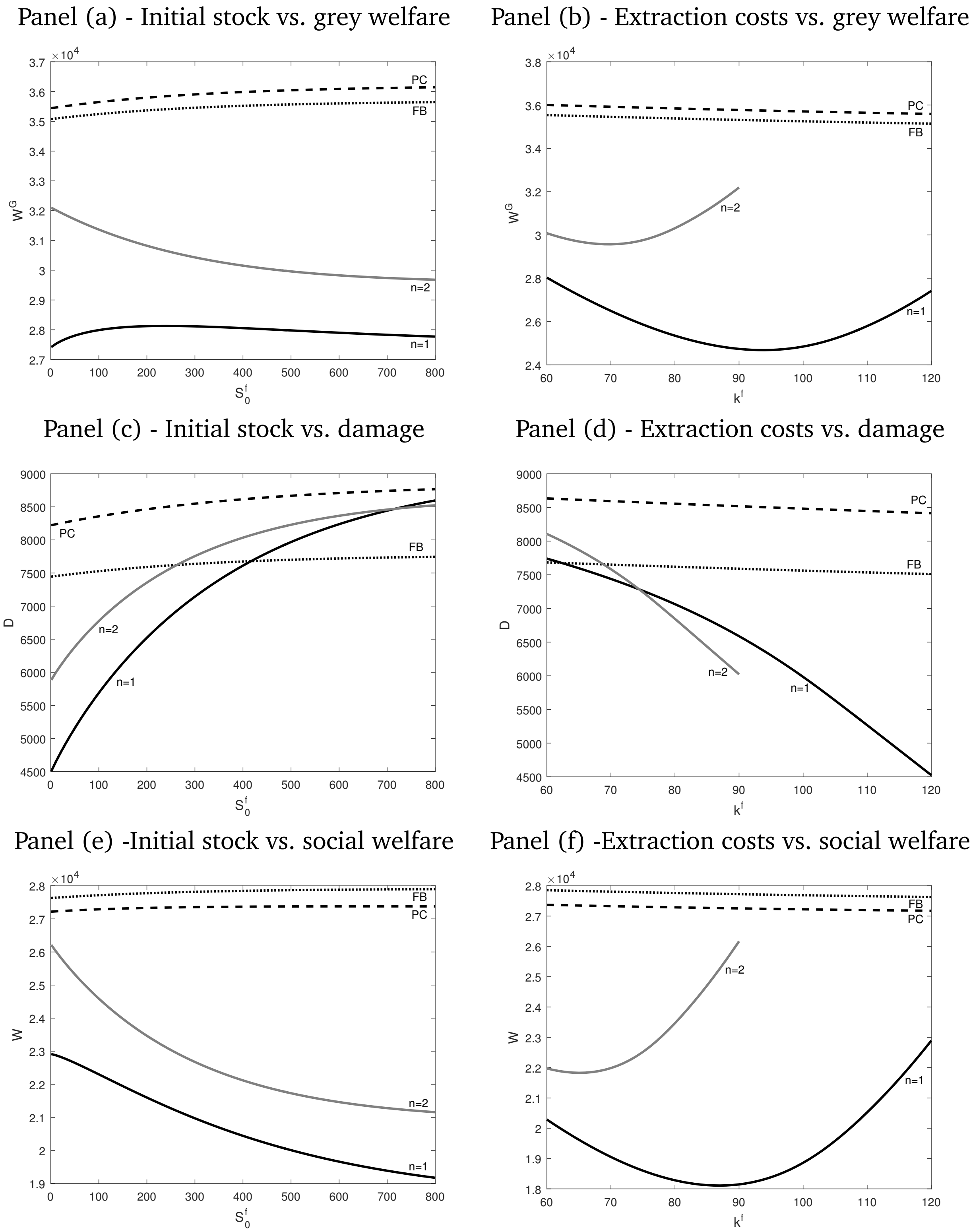


Notes: The figure shows climate damage (panels (a)) and social welfare (panel (b))) for various values of n. The solid black lines correspond to the oligopoly-fringe equilibrium. The dashed lines indicate the perfectly competitive equilibrium (indicated with PC). The dotted lines correspond to the first-best outcome (indicated with FB). We have chosen the following parameter values: $\alpha = 210$, $\beta = 75/17$, $b = 157.5$, $k^c = 30$, $k^f = 60$, $r = 0.1$, $S_0^c = 1220$, $S_0^f = 433$, $SCC = 200$, $\omega^f = 1$, $\omega^c = 1.25$.

on climate damage is non-monotonic in the number of oligopolists. This result is due to the difference in emission factors between the oligopolists and the fringe. On the one hand, increasing the number of oligopolists makes resource extraction less conservative and therefore accelerates climate change. On the other hand, increasing the number of oligopolists reduces front-loading of the relatively expensive resource. Accordingly, if this relatively expensive resource (shale oil) is also relatively more polluting than the relatively cheap resource (conventional OPEC oil), increasing competition may be beneficial for the climate after all. The figure shows that the latter effect dominates the former if the number of oligopolists becomes larger than 2. Panel (b) shows that social welfare is monotonically increasing in the number of oligopolists. Moreover, due to lower climate change, the oligopoly-fringe equilibrium may generate higher welfare than under perfect competition, if the number of oligopolists is large enough. On the basis of Figure 7, we can conclude that the recent collapse of the OPEC cartel has increased social welfare. The effect on climate damage, however, depends on the remaining level of collusion.

Figure 8 examines the welfare and climate effects of the recent shale oil revolution, characterized by an increase in the proven reserves to 433 billion barrels (left panels) and a decrease in marginal extraction costs to 60 US$/bbl (right panels). Panels (a) and (b) show that the increase in resource reserves and a decrease in extraction costs

Figure 8: Welfare effects of the 'shale revolution'



Notes: The figure shows grey welfare (panels (a) and (b)), climate damage (panels (c) and (d)) and social welfare (panels (e) and (f)) for various values of the fringe's initial resource stock and marginal extraction costs of the fringe. The solid black (grey) lines correspond to the oligopoly-fringe equilibrium with $n = 1$ ($n = 3$). The dashed lines indicate the perfectly competitive equilibrium (indicated with PC). The dotted lines correspond to the first-best outcome (indicated with FB). We have chosen the following parameter values: $\alpha = 210$, $\beta = 75/17$, $b = 157.5$, $k^c = 30$, $k^f = 60$ (in panels (a), (c) and (e)), $r = 0.1$, $\sigma = 0$, $\tau = 0$, $S_0^c = 1220$, $S_0^f = 433$ (in panels (b), (d) and (f)), $SCC = 200$, $\omega^f = 1$, $\omega^c = 1.25$.

would have lead to higher grey welfare under perfect competition and in the first best. However, in the cartel/oligopoly-fringe equilibrium, these relationships are reversed or become non-monotonic, due to the conservation and sequence effects. Panels (c) and (d) show that both the increase in reserves and the decrease in extraction costs have caused higher climate damages. In our benchmark calibration, it turns out that the increase in the fringe's oil reserve has lowered social welfare, although it would have enhanced social welfare under perfect competition and, obviously, in the first-best, as shown in panel (e). Panel (f) shows that the relationship between marginal extraction costs of the fringe and social welfare is non-monotonic in the two imperfectly competitive equilibria. In our benchmark case with $n = 2$, the decrease in extraction costs from 90 to 60 US$/bbl has lowered social welfare.

6 Conclusion

We have developed an oligopoly-fringe model to examine the implications of recent developments on the global energy market, the collapse of OPEC as a cartel and the shale revolution in the US and Canada, for the time paths of oil prices and extraction, for the effectiveness of climate policies, and for climate damage as well as social welfare. We have taken account of the existence of renewables that are (perfect) substitutes for fossil fuel and that can be produced in unlimited amounts by using backstop technologies.

By establishing the existence of and by fully characterizing a Nash-Cournot equilibrium on the energy market, we were able to show that the oligopolists and the fringe start out supplying simultaneously to the market, despite their differing extraction costs. If the relative initial stock of the fringe is large, the phase with simultaneous supply will be followed by a phase during which only the fringe is extracting. However, if the initial stock of the cartel is relatively large, the phase with simultaneous supply will be followed by a period during which only the oligopolists are active. During this period, depending on their remaining resource stock and on the marginal profits at the limit price, the oligopolists either choose to price strictly below the price of renewables, or to perform a limit-pricing strategy of just undercutting the renewables price. The collapse of OPEC as a cartel decreases the likelihood of a limit-pricing phase occurring, and lowers its maximum duration.

Furthermore, we have shown that the break-down of OPEC has an ambiguous effect on climate damages. On the one hand, climate change is accelerated as extraction becomes less conservative. On the other hand, extraction of the fringe is back-loaded, which slows down climate change if the fringe's resource is relatively dirty. Oil from tar sands in Canada that may soon be transported by the Keystone XL pipeline to refineries in the US might qualify for an even dirtier resource supplied by the fringe. As a result, climate damage in the oligopoly-fringe equilibrium may even become larger than under perfect competition.

Our results also show that the the recent shale oil revolution not only increases climate damage due to larger cumulative emissions, but also lowers grey welfare as it crowds out (postpones) the extraction of relatively cheap OPEC oil. Finally, we have demonstrated that a Weak Green Paradox does not occur upon a renewables subsidy if the initial relative stock of the oligopolists is large enough, compared to the fringe.

In future work, it would be interesting to determine the feedback Nash equilibrium for our model. Moreover, the existence of multiple equilibria should be investigated. Furthermore, it seems worthwhile to characterize optimal and second-best policies. Another promising direction for future research would be the introduction of strategic interaction between energy suppliers and energy demanders.

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Appendix

A.1 Equilibrium sequences

Lemma A.1 *Suppose the equilibrium reads $S \to F$ with transition at T^c and final time T. Then*

$$\begin{aligned} r\beta S_0^f = &-n(k^f - k^c)(rT^c - 1 + e^{-rT^c}) + (\hat{b} - k^f)(rT - 1 + e^{-rT}) \\ &+ (\alpha - \tau - \hat{b})rT, \end{aligned} \tag{A.1a}$$

$$r\beta S_0^c = n(k^f - k^c)(rT^c - 1 + e^{-rT^c}). \tag{A.1b}$$

Proof. Along S we have (12c) and (12d). Along F we have (11c). Furthermore $\lambda^f = (\hat{b} - k^f)e^{-rT}$. Also $\lambda^c = (k^f - k^c)e^{-rT^c} + \lambda^f = (k^f - k^c)e^{-rT^c} + (\hat{b} - k^f)e^{-rT}$. Then taking the time integrals of q^f and q^c yields the result. □

Lemma A.2 *Suppose the equilibrium reads $S \to \hat{L}$ with transition at T and final time T^c. Then*

$$r\beta S_0^f = (\alpha - \tau + nk^c - (n+1)k^f)(rT - 1 + e^{-rT}), \tag{A.2a}$$

$$\begin{aligned} r\beta S_0^c = &(\hat{b} - (\alpha - \tau) + n(k^f - k^c))(rT - 1 + e^{-rT}) \\ &+ (\alpha - \tau - \hat{b})rT^c, \end{aligned} \tag{A.2b}$$

$$(\hat{b} - k^c)e^{-rT^c} = \left[\left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha - \tau}{n} - k^c\right]e^{-rT}. \tag{A.2c}$$

Proof. Along S we have (12c) and (12d). Along the $\hat{L}$-phase we have (14b). It follows from (12a) that $\lambda^f = (\hat{b} - k^f)e^{-rT}$. It follows from (12b) and continuity of the price and the Hamiltonian of the cartel that $\lambda^c = \left[\left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha-\tau}{n} - k^c\right]e^{-rT}$. Condition (A.2c) is obtained by combining (9b) and (10). Then taking the time integrals of q^f and q^c yields the result. □

Lemma A.3 *Suppose the equilibrium reads $S \to \tilde{L}$ with transition at T and final time T^c.*

Then

$$
\begin{aligned}
r\beta S_0^f = &\left(\alpha - \tau + nk^c - (n+1)k^f\right) rT - n(\hat{b} - k^c)e^{-rT^c}(1 - e^{rT}) \\
&- (n+1)(\hat{b} - k^f)(1 - e^{-rT}), \qquad \text{(A.3a)} \\
r\beta S_0^c = &\, n(k^f - k^c)rT + n(\hat{b} - k^f)(1 - e^{-rT}) + n(\hat{b} - k^c)e^{-rT^c}(1 - e^{rT}) \\
&+ (\alpha - \tau - \hat{b})r(T^c - T). \qquad \text{(A.3b)}
\end{aligned}
$$

Proof. Along S we have (12c) and (12d). Along the $\tilde{L}$-phase we have (14b). It follows from (10) that $\lambda^c = (\hat{b} - k^c)e^{-rT^c}$. It follows from (12a) together with price continuity that $\lambda^f = (\hat{b} - k^f)e^{-rT}$. Then taking the time integrals of q^f and q^c yields the result. □

Note that the Hamiltonian is discontinuous at T if the initial stocks differ from those in Lemma A.2: q^c and μ jumps upward T, while q^f jumps downward.

Lemma A.4 *Suppose the equilibrium reads $S \to C \to \hat{L}$ with transitions at T_1 and T_2 and final time T^c. Then*

$$
\begin{aligned}
r\beta S_0^f &= \left(\alpha - \tau + nk^c - (n+1)k^f\right)(rT_1 - 1 + e^{-rT_1}), \qquad \text{(A.4a)} \\
r\beta S_0^c &= \frac{n}{n+1}\left((n+1)k^f - nk^c - (\alpha - \tau)\right)(rT_1 - 1 + e^{-rT_1}) \\
&\quad + \frac{n}{n+1}\left(\frac{n+1}{n}\hat{b} - k^c - \frac{\alpha - \tau}{n}\right)(rT_2 - 1 + e^{-rT_2}) \\
&\quad + (\alpha - \tau - \hat{b})rT^c, \qquad \text{(A.4b)} \\
(\hat{b} - k^c)e^{-rT^c} &= \left[\left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha - \tau}{n} - k^c\right]e^{-rT_2}. \qquad \text{(A.4c)}
\end{aligned}
$$

Proof. Along S we have (12c) and (12d). Along the C-phase we have (13c). Along the $\hat{L}$-phase we have (14b). Moreover, we have $\lambda^c = (\hat{b} - k^c)e^{-rT^c}$. The price is continuous at T_1 and T_2 so that

$$
\begin{aligned}
k^f + \lambda^f e^{rT_1} &= \frac{1}{n+1}\left(\alpha - \tau + n(k^c + \lambda^c e^{rT_1})\right), \qquad \text{(A.5a)} \\
\frac{\alpha - \tau - \hat{b}}{\beta} &= \frac{1}{\beta}\frac{n}{n+1}(\alpha - \tau - k^c - \lambda^c e^{rT_2}). \qquad \text{(A.5b)}
\end{aligned}
$$

Condition (A.4c) is obtained by combining (9b) and (10). Then taking the time integrals of q^f and q^c yields the result. □

A.2 Proofs of Lemmata and Propositions

Proof of Lemma 2.

Part (i). Given our definition of C and F a transition can only take place at a moment T where the price is below b. Indeed, in an equilibrium the price is increasing in both phases and must therefore be smaller than b. The price is continuous at T:

$$\begin{aligned} p(T) &= \alpha - \beta(q^f(T) + q^c(T)) = k^f + \lambda^f e^{rT} \\ &= \frac{1}{n+1}(\alpha - \tau + n(k^c + \lambda^c e^{rT})). \end{aligned}$$

If we have $F \to C$ then it follows from (11a) and (11b) that $k^f + \lambda^f e^{rT} \leq k^c + \lambda^c e^{rT}$. Hence $\left[(n+1)(k^f + \lambda^f e^{rT}) - (\alpha - \tau)\right] \frac{1}{n} \geq k^f + \lambda^f e^{rT}$, implying $p(T) \geq \alpha - \tau$, a contradiction. The proof for $C \to F$ is similar.

Part (ii). Suppose it is optimal to have $F \to \hat{L}$ or $F \to \tilde{L}$ and assume the transition takes place at T. Then for $0 \leq t \leq T$ we have

$$q^f(t) = \frac{\alpha - \tau - k^f - \lambda^f e^{rt}}{\beta}, q^f(T) = \frac{\alpha - \tau - \hat{b}}{\beta} \tag{A.6}$$

because of price continuity. Hence:

$$\begin{aligned} \hat{b} - k^f &= \lambda^f e^{rT}, \\ q^f(t) &= \frac{\alpha - \tau - k^f - (\hat{b} - k^f)e^{rt - rT}}{\beta}. \end{aligned}$$

The oligopolists should not want to supply before T so that for $0 \leq t \leq T$ we have

$$\alpha - \tau - \beta[\frac{\alpha - \tau - k^f - (\hat{b} - k^f)e^{r(t-T)}}{\beta}] \leq k^c + \lambda^c e^{rt} = k^c + (\hat{b} - k^c)e^{r(t - T^c)}, \tag{A.7}$$

from (10), or $k^f(1 - e^{r(t-T)}) - k^c(1 - e^{r(t-T^c)}) \leq \hat{b}e^{rt - rT^c}(1 - e^{r(T^c - T)})$. Take the limit for t approaching T. Then the condition boils down to $(\hat{b} - k^c)(1 - e^{r(T - T^c)}) \leq 0$, a contradiction.

Part (iii). Along F we have $k^f + \lambda^f e^{rt} \leq k^c + \lambda^c e^{rt}$, which implies $k^f - k^c \leq (\lambda^c - \lambda^f)e^{rt}$. At the transition from F to S at say T we have from the continuity of the price $k^f - k^c = (\lambda^c - \lambda^f)e^{rt}$. Because we have assumed $k^f > k^c$, the left-hand sides of the latter two expressions are positive. Hence, the right-hand side of these expressions is growing over time. However, since F precedes S, $(\lambda^c - \lambda^f)e^{rt}$ is larger than $k^f - k^c$ before T and equal to $k^f - k^c$ at T. Hence, the right-hand sides must be declining, which yields a contradiction.

Part (iv). Suppose the initial regime is C. Then it follows from (13a) and (13b) that along C we have $\alpha - \tau + nk^c - (n+1)k^f \leq ((n+1)\lambda^f - n\lambda^c)e^{rT}$. There is no transition possible to F. Hence there must be a transition to S, say at T. So $\alpha - \tau + nk^c - (n+1)k^f = ((n+1)\lambda^f - n\lambda^c)e^{rT}$. Since $\alpha - \tau + nk^c - (n+1)k^f > 0$ by assumption and C starts at time 0, we have $(n+1)\lambda^f - n\lambda^c > 0$, so that $((n+1)\lambda^f - n\lambda^c)e^{rt}$ is increasing over time, yielding a contradiction. □

Proof of Lemma 3. Along S we have (12c) and (12d). Moreover, $\lambda^c = (\hat{b} - k^c)e^{-rT}$ and $p(T) = \alpha - \beta(q^f(T) + q^c(T)) = \hat{b}$ so that $\lambda^f = (\hat{b} - k^f)e^{-rT}$. Then taking the time integrals of q^f and q^c yields the result. □

Proof of Lemma 5. First rewrite the system (A.1a) and (A.1b) as

$$
\begin{aligned}
\mathbf{F}(T^c, T) = & -n(k^f - k^c)(rT^c - 1 + e^{-rT^c}) + (\hat{b} - k^f)(rT - 1 + e^{-rT}) \\
& + (\alpha - \tau - \hat{b})rT - r\beta S_0^f \\
\mathbf{H}(T^c, T) = & \, n(k^f - k^c)(rT^c - 1 + e^{-rT^c}) - r\beta S_0^c
\end{aligned}
$$

Given S_0^c, if the equilibrium reads $S \to F$ then the transition time $T^c = T_S$ denotes the duration of the equilibrium the equilibrium that reads S.

Now given $T^c = T_S$, is there a solution $T \geq T_S$ that solves $\mathbf{F}(T_S, T) = 0$? When $S_0^f = S_{0S}^f$ we have $\mathbf{F}(T_S, T_S) = 0$, so when $S_0^f > S_{0S}^f$ we get $\mathbf{F}(T_S, T_S) < 0$. We derive

$$
\begin{aligned}
\mathbf{F}_T &= r\left((\hat{b} - k^f)(1 - e^{-rT}) + (\alpha - \tau - \hat{b})\right) > 0, \\
\mathbf{F}_{TT} &= r^2(\hat{b} - k^f)e^{-rT} > 0.
\end{aligned}
$$

Hence, $\mathbf{F}$ is monotonically increasing and strictly convex in T. As a result, there exists at most one T that solves $\mathbf{F}(T_S, T) = 0$ with $T > T_S$. Such a solution exists if

$$
\lim_{T \to \infty} \mathbf{F}(T_S, T) > 0.
$$

We have $\lim_{T\to\infty} \mathbf{F}(T_S, T) = \lim_{T\to\infty} \left(\hat{b} - k^f + \alpha - \tau - b\right) rT = \infty$. $\square$

Proof of Lemma 6. Given S_0^f there exists a unique T denoted $\hat{T}_S$ that satisfies

$$r\beta S_0^f = (\alpha - \tau + nk^c - (n+1)k^f)(rT - 1 + e^{-rT}).$$

Next, we establish that $\hat{T}_S > T_S$. Note from (17a)-(17b) that T_S is the solution to

$$\begin{aligned} r\beta S_0^f &= (\hat{b} + nk^c - (n+1)k^f)(rT_S - 1 + e^{-rT_S}) + (\alpha - \tau - \hat{b})rT_S \\ &= (\alpha - \tau + nk^c - (n+1)k^f)(rT_S - 1 + e^{-rT_S}) + (\alpha - \tau - \hat{b})\left(1 - e^{-rT_S}\right). \end{aligned}$$

Let $\mathbf{f}(T) \equiv (\alpha - \tau + nk^c - (n+1)k^f)(rT - 1 + e^{-rT})$. We have $\mathbf{f}' > 0$ and $\mathbf{f}\left(\hat{T}_S\right) = \mathbf{f}(T_S) + (\alpha - \tau - \hat{b})\left(1 - e^{-rT_S}\right) > \mathbf{f}(T_S)$, implying $\hat{T}_S > T_S$.

We now argue that there exist $T^c = T_{LM} + \hat{T}_{LM}$ and $\hat{S}_0^c > S_{0S}^c + S_{LM}$ which satisfy (A.2b) and (A.2c):

$$\begin{aligned} r\beta \hat{S}_0^c &= (\hat{b} - (\alpha - \tau) + n(k^f - k^c))(rT_{LM} - 1 + e^{-r\hat{T}_S}) \\ &\quad + (\alpha - \tau - \hat{b})r\left(\hat{T}_S + \hat{T}\right), \end{aligned} \tag{A.8a}$$

$$(\hat{b} - k^c)e^{-r\hat{T}} = (1 + \frac{1}{n})\hat{b} - \frac{\alpha - \tau}{n} - k^c). \tag{A.8b}$$

Condition (A.2c) is satisfied by definition of $\hat{T}_{LM}$.

The rest of the proof consists of showing that the stock given by (A.8a), $\hat{S}_0^c$, is larger than $S_{0S} + S_{LM}$. Using the definition of S_{LM}, (A.8a) becomes

$$r\beta\left(\hat{S}_0^c - S_{LM}\right) = (\hat{b} - (\alpha - \tau) + n(k^f - k^c))(r\hat{T}_S - 1 + e^{-r\hat{T}_S}) + (\alpha - \tau - \hat{b})r\hat{T}_S. \tag{A.9}$$

Summing (A.2a) with $T = \hat{T}_S$ and (A.9) yields

$$\begin{aligned} r\beta\left(\hat{S}_0^c - S_{LM} + S_0^f\right) &= (\alpha - \tau + nk^c - (n+1)k^f)(r\hat{T}_S - 1 + e^{-r\hat{T}_S}) \\ &\quad + (\hat{b} - (\alpha - \tau) + n(k^f - k^c))(r\hat{T}_S - 1 + e^{-r\hat{T}_S}) \\ &\quad + (\alpha - \tau - \hat{b})r\hat{T}_S, \end{aligned}$$

which simplifies into

$$r\beta\left(\hat{S}_0^c - S_{LM} + S_0^f\right) = (\hat{b} - k^f)(r\hat{T}_S - 1 + e^{-r\hat{T}_S}) + (\alpha - \tau - \hat{b})r\hat{T}_S, \tag{A.10}$$

$$= (\hat{b} - k^f)\mathbf{g}\left(\hat{T}_S\right) + (\alpha - \tau - \hat{b})r\hat{T}_S, \tag{A.11}$$

where $\mathbf{g}(T) \equiv rT - 1 + e^{-rT}$. By definition of S_{0S}^c we have

$$r\beta S_0^f = (\hat{b} + nk^c - (n+1)k^f)(rT_S - 1 + e^{-rT_S}) + (\alpha - \tau - \hat{b})rT_S, \tag{A.12a}$$

$$r\beta S_{0S}^c = n(k^f - k^c)(rT_S - 1 + e^{-rT_S}). \tag{A.12b}$$

Summing (A.12a) and (A.12b) gives

$$r\beta\left(S_{0S}^c + S_0^f\right) = (\hat{b} - k^f)(rT_S - 1 + e^{-rT_S}) + (\alpha - \tau - \hat{b})rT_S, \tag{A.13}$$

$$= (\hat{b} - k^f)\; g\left(T_S\right) + (\alpha - \tau - \hat{b})rT_S. \tag{A.14}$$

Since $\mathbf{g}\left(T\right) + (\alpha - b)rT$ is increasing in T and since $\hat{T}_S > T_S$, we have from (A.11) and (A.14) $r\beta\left(S_{0S}^c + S_0^f\right) < r\beta\left(\hat{S}_0^c - S_{LM} + S_0^f\right)$, implying $\hat{S}_0^c > S_{0S}^c + S_{LM}$. $\square$

To prove Lemma 8 it will be useful to make the following two remarks.

Remark 1 *Given S_0^f, when $S_0^c \to S_{0S}^c$ we have $T^c \to T \to T_S$: the $\tilde{L}$ collapses. Indeed simple substitution of S_0^c by S_{0S}^c, T by T_S and T^c by T_S shows that the system* (A.3a), (A.3b) *becomes after simplification*

$$r\beta S_0^f = (\hat{b} + nk^c - (n+1)k^f)(rT_S - 1 + e^{-rT_S}) + (\alpha - \tau - \hat{b})rT_S, \tag{A.15a}$$

$$r\beta S_{0S}^c = n(k^f - k^c)(rT_S - 1 + e^{-rT_S}), \tag{A.15b}$$

which holds by definition of T_S and S_{0S}^c.

Remark 2 *Similarly when $S_0^c = \hat{S}_0^c$ then $T = \hat{T}_S$ and $T^c = \hat{T}_S + \hat{T}_{LM}$ solve the system*

$$\begin{aligned} r\beta S_0^f = &\left(\alpha - \tau + nk^c - (n+1)k^f\right) rT - n(\hat{b} - k^c)e^{-rT^c}(1 - e^{rT}) \\ &- (n+1)(\hat{b} - k^f)(1 - e^{-rT}), \end{aligned} \tag{A.16a}$$

$$\begin{aligned} r\beta S_0^c = &\, n(k^f - k^c)rT + n(\hat{b} - k^f)(1 - e^{-rT}) + n(\hat{b} - k^c)e^{-rT^c}(1 - e^{rT}) \\ &+ (\alpha - \tau - \hat{b})r(T^c - T). \end{aligned} \tag{A.16b}$$

Indeed we know that they satisfy

$$r\beta S_0^f = (\alpha - \tau + nk^c - (n+1)k^f)(rT - 1 + e^{-rT}), \tag{A.17a}$$

$$\begin{aligned} r\beta S_0^c = &\, (\hat{b} - (\alpha - \tau) + n(k^f - k^c))(rT - 1 + e^{-rT}) \\ &+ (\alpha - \tau - \hat{b})rT^c, \end{aligned} \tag{A.17b}$$

$$(\hat{b} - k^c)e^{-rT^c} = \left[\left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha - \tau}{n} - k^c\right] e^{-rT}. \tag{A.17c}$$

substituting $(\hat{b} - k^c)e^{-rT^c}$ *by* $\left[\left(1 + \frac{1}{n}\right)\hat{b} - \frac{\alpha-\tau}{n} - k^c\right] e^{-rT}$ *into* (A.16a) *yields the result.*

Proof of Lemma 8. The proof consists of showing that for any S_0^f and for any $S_0^c \in \left[S_{0S}^c, \hat{S}_0^c\right]$ there exists T and $T^c \geq T$ such that (A.3a) and (A.3b) are satisfied. The sum of (A.3a) and (A.3b) reads

$$r\beta\left(S_0^f + S_0^c - \frac{\alpha - \tau - \hat{b}}{\beta}(T^c - T)\right) = (\alpha - \tau - k^f)rT - (\hat{b} - k^f)(1 - e^{-rT}). \tag{A.18}$$

Condition (A.18) defines a unique relationship between the duration of limit pricing $T^c - T$ and the time of transition (or duration of the S phase); we rewrite this condition as

$$T^c - T = \mathbf{H}\left(T, S_0^f, S_0^c\right) \equiv \frac{r\beta\left(S_0^f + S_0^c\right) - (\alpha - \tau - k^f)rT + (\hat{b} - k^f)(1 - e^{-rT})}{(\alpha - \tau - \hat{b})r}.$$

Manipulations allow to rewrite (A.3a) as $e^{r(T^c - T)} = \mathbf{Z}\left(T, S_0^f\right)$, with

$$\mathbf{Z}\left(T, S_0^f\right) \equiv \frac{n(\hat{b} - k^c)(1 - e^{-rT})}{r\beta S_0^f + (n+1)(\hat{b} - k^f)(1 - e^{-rT}) - (\alpha - \tau + nk^c - (n+1)k^f)rT}.$$

Substituting $T^c - T = \mathbf{H}\left(T, S_0^f, S_0^c\right)$ allows to characterize T as the solution to

$$\mathbf{W}\left(T, S_0^f, S_0^c\right) = 0, \text{ with } \mathbf{W}\left(T, S_0^f, S_0^c\right) \equiv e^{\mathbf{H}\left(T, S_0^f, S_0^c\right)} - \mathbf{Z}\left(T, S_0^f\right). \tag{A.19}$$

We argue that for any $S_0^c \in \left[S_{0S}^c, \hat{S}_{0S}^c + S_{LM}\right]$ there exists a solution T to (A.19). From Remark 1 and Remark 2 above we get

$$\mathbf{W}\left(T_S, S_0^f, S_{0S}^c\right) = 0 = \mathbf{W}\left(\hat{T}_S, S_0^f, S_{0S}^c + S_{LM}\right).$$

As $\mathbf{W}\left(T, S_0^f, S_0^c\right)$ is an increasing function of S_0^c, we have for any $S_0^c \in (S_{0S}^c, S_{0S}^c + S_{LM})$

$$\begin{aligned}
&\mathbf{W}\left(T_S, S_0^f, S_0^c\right) > \mathbf{W}\left(T_S, S_0^f, S_{0S}^c\right) = 0, \\
&\mathbf{W}\left(\hat{T}_S, S_0^f, S_0^c\right) < \mathbf{W}\left(\hat{T}_S, S_0^f, S_{0S}^c + S_{LM}\right) = 0.
\end{aligned}$$

Since $\mathbf{W}\left(T, S_0^f, S_0^c\right)$ is a continuous function of T (CHECK) we can therefore state that for any $S_0^c \in [S_{0S}^c, S_{0S}^c + S_{LM}]$ there exists a solution $T \in \left[T_S, \hat{T}_S\right]$ to $\mathbf{W}\left(T, S_0^f, S_0^c\right) = 0$.

We still need to check that for any $S_0^c \in (S_{0S}^c, S_{0S}^c + S_{LM})$ we have $\mathbf{H}\left(T, S_0^f, S_0^c\right) \geq 0$. Note that $\mathbf{H}\left(T, S_0^f, S_0^c\right) > \mathbf{H}\left(T, S_0^f, S_{0S}^c\right)$. Furthermore, $\mathbf{H}$ is a decreasing function of T since $k^f < \hat{b} < \alpha - \tau$:

$$\mathbf{H}_T = \frac{-\left(\alpha - \tau - k^f\right) + (\hat{b} - k^f)e^{-rT}}{(\alpha - \tau - \hat{b})} < 0 \text{ for } T \geq 0.$$

Therefore, we have

$$\mathbf{H}\left(T, S_0^f, S_0^c\right) > \mathbf{H}\left(T, S_0^f, S_{0S}^c\right) > \mathbf{H}\left(T_S, S_0^f, S_{0S}^c\right) = 0. \square$$

Proof of Lemma 8. The proof consists of showing that there exist T_1 and T_2 such that the system (A.4a)-(A.4c) holds, where $T^c = T_2 + \hat{T}_{LM}$. Note that given S_0^f the solution to $r\beta S_0^f = (\alpha - \tau + nk^c - (n+1)k^f)(rT - 1 + e^{-rT})$ is unique and therefore T_1 is the same as the duration of the S phase when $S_0^c = S_{0S}^c + S_{LM}$ that is when the regime reads $S \to \hat{L}$.

The proof now consists of showing that there exists T_2 that solves

$$\begin{aligned}\mathbf{Y}(T_2) \equiv & -r\beta S_0^c + \frac{n}{n+1}\Big((n+1)k^f - nk^c - (\alpha-\tau)\Big)\left(rT_1 - 1 + e^{-rT_1}\right) \\ & + \frac{n}{n+1}\left(\frac{n+1}{n}\hat{b} - k^c - \frac{\alpha-\tau}{n}\right)\left(rT_2 - 1 + e^{-rT_2}\right) \\ & + (\alpha-\tau-\hat{b})rT^c = 0.\end{aligned}$$

We know that $\mathbf{Y}(T_1) < 0$ for $S_0^c > \hat{S}_0^c$ since $\mathbf{Y}(T_1) = 0$ when $S_0^c = \hat{S}_0^c$. Moreover, we have $\lim_{T_2\to\infty} \mathbf{Y}(T_2) = \infty$ and $\mathbf{Y}' > 0$, which implies the existence and unicity of T_2 that solves $\mathbf{Y}(T_2) = 0$. $\square$

Proof of Proposition 4.

Proof of Proposition 2.

A.3 First-best and perfectly competitive equilibrium