A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources*

Aïchouche Oubraham
GERAD, HEC Montréal, Canada

Georges Zaccour
Chair in Game Theory and Management, GERAD, HEC Montréal, Canada

May 6, 2017

Abstract

In this paper, we survey the literature applying viability theory to the sustainable management of renewable resources. After a refresher on the main concepts of viability theory, we provide a general map of the contributions and next discuss them by area of application, including ecosystems and population biology, climate change, forestry and others. We conclude by pointing out issues that deserve more attention and should be part of a research agenda.

Key Words: Viability theory; Sustainability; Renewable resources; Fisheries; Climate change; Forests.

JEL Codes: Q01; Q22; Q57.

Introduction

It is not new that societies care about their environment and resources and take actions to protect them.[1] What is however of recent vintage is the awareness that (i) immoderate human activity, e.g., burning fossil fuels, over fishing or excessive deforestation, have direct undesirable consequences, such as loss of biodiversity and deterioration in environmental quality, and (ii) some concerted actions are urgently needed to preserve these resources. A pivotal date in first gaining this awareness was probably the publication of *Limits of Growth* in 1972 (Meadows et al. [81]), a study that triggered fervent debate and stroked the popular imagination, since some of the simulated growth scenarios predicted the collapse of the global system. Later in the same decade, it was argued that economic development could be sustained indefinitely, but only if it were to take into account its ultimate interaction with the natural environment. This marked the advent of the concept of *ecological management,* which paved the way for the notion of *sustainable development,* which was coined by the International Union for the Conservation

*Research supported by Social Sciences and Humanities Research Council, Canada, grant 435-2013-0532.

[1] The following website offers an environmental history timeline with a list of events and actions related to environmental protection: http://66.147.244.135/~enviror4/2012/07/16/about-environmental-history/#more-164

of Nature and Natural Resources (IUCN) in 1980; see (Allen et al. [2]). Although at that time a precise definition of sustainable development was lacking, the idea itself very quickly gained in popularity among scientists, decision makers and activists.[2] A second notable date is the publication in 1987 of the Brundtland Report, which provided a unifying definition of sustainable development:

> "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs."(Brundtland et al. [30])

This definition has since been adopted by all stakeholders, although refinements have occasionally been considered, implicitly or explicitly, in some studies.

This paper provides a comprehensive review of the literature on applications of *viability theory* to the sustainable management of renewable resources including ecosystems and populations such as fisheries and non-marine species, the environment (with a focus on climate change and GHG concentration), and other resources (e.g., forests and soil). In a nutshell, "Viability theory is an area of mathematics that studies the evolution of dynamical systems under constraints on the system's state (Aubin [7], Aubin et al. [12]). It was developed to formalize problems arising in the study of various natural and social phenomena, and has close ties with the theories of optimal control and set-valued analysis."[3] As in optimal control, the basic ingredients of viability theory (VT) are control and state variables, and a dynamical system whose evolution is governed by differential (or difference) equations, which are functions of the state and control variables and some parameters. The system evolution can be deterministic or not, and is subject to some (viability) constraints. A notable difference with optimal control is the absence of an objective functional to be optimized. As we will see, the main objects of viability theory are sets, hence the link made above to set-valued analysis. The theory was initiated by Jean-Pierre Aubin in the late 1970s and the fundamental results established in the 1980s (see Haddad [61]).

In Aubin [6], viability theory is described as a mathematical theory based on three main features, namely: (i) non-determinism of evolutions; (ii) viability constraints; and (iii) inertia principle. The two first features concern the state evolution of the studied system and reflect the fact that a system can evolve in many different and possibly unpredictable ways depending on its initial state, its past evolution, the environment in which it evolves or anything else (non determinism), and also the fact that, for many reasons, the evolution of a system is restrained by some constraints that must be satisfied at each instant of time. These are the two founding pillars of viability theory models.[4] The last feature (inertia principle) concerns the control variables and stipulates that these controls are changed only when required for maintaining viability. To find a viable solution (or a set of viable solutions), VT follows a backward (or inverse)

[2]For a list of some definitions of sustainable development used in between 1980 and 1988, see the Appendix in Pezzey [86].

[3]https://en.wikipedia.org/wiki/Viability_theory

[4]Besides, Aubin et al. [12] present this theory as a mathematical translation of Jacques Monod's *Chance and Necessity* ([82]) in which there appears a quotation from Democritus stating that "the whole universe is but the fruit of two qualities, chance and necessity." Chance refers to the non-determinism of evolutions, and necessity expresses the need to meet certain conditions or criteria, which results in viability constraints.

method, that is, starting from a set of given viability constraints, one looks for the set of initial states from which the system can be indefinitely viable.

Other approaches than VT are of course available to determine sustainable exploitation of a renewable resource, in particular the so-called *policy optimization* and *policy evaluation* (Weyant et al. [106]). In the former, as the name suggests, one defines an objective functional that typically measures the relevant costs and benefits of possible decisions, and the optimization is carried out subject to a series of constraints. In policy evaluation, some feasible scenarios are assessed and eventually the best one is selected. While these approaches have obvious merits, they often involve trade-offs between the different environmental, economic and social facets of sustainability, which may not be desirable. As mentioned above, there is no (intertemporal) objective to be optimized in a VT model, and sustainability is addressed through the viability constraints. Therefore, a VT model avoids the contentious issue of weighting different sustainability facets, or making trade-offs between short- and long-term considerations. Writing down an intertemporal objective requires an assessment of future options. In a VT model, such knowledge of the future is not mandatory because the choice of controls at any given initial time is not final, and can be adapted to eventual changes in the system's environment (Aubin [5]). However, viability theory does not allow us to obtain directly, that is, as an explicit mathematical rule, the viable controls or strategies. These can only be numerically approximated by constructing regulation maps acting as feedback control. This is somehow similar to what is done in the *policy guidance approach* (PGA), which was recently proposed and has been referred to by different names in different areas, e.g., *tolerable window approach* in climate change and GHG management, *population viability analysis* in conservation biology or *safe minimum standards* in fisheries. Indeed, the basic idea behind the PGA is to maintain the system as long as possible within some predefined bounds (De Lara and Doyen [41]).

The rest of the paper is organized as follows: In Section 2, we provide a short refresher on viability theory. In Section 3, we show how viability theory is used to address sustainable development problems, and give a general map of the contributions. In Section 4, we review the applications of viability theory to the management of renewable resources, which is the main block of interest. In Section 5, we briefly conclude. A table summarizing all reviewed papers is given in the Appendix.

1 A refresher on viability theory

In this section, we recall some concepts of viability theory that are useful for appreciating its applications in renewable resources. For a rigorous introduction to viability theory, the interested reader may consult the books by Aubin (1991), Aubin [6], Aubin et al. [12], Aubin [7] and De Lara and Doyen [41].

We shall distinguish in the sequel between deterministic viability and stochastic viability. Although in both settings the main questions are the same, e.g., how to remain viable, to reach a target or to restore viability if lost during the process, the concepts and techniques used to answer these questions will be different, at least to a certain extent.

Denote by $x(t)$ the state of a system of interest at time $t \in [0, +\infty)$, and let $X \subset \mathbb{R}^n$ be the state space. The evolution of the state is described by

$$\mathcal{S}\begin{cases} x'(t) = f(x(t), u(t)) \\ u(t) \in U(x(t)) \end{cases}, \tag{1}$$

where $u(\cdot)$ is the control variable and $U(x(t))$ is the set of admissible controls at time t, which depends on the state of the system at that time. We shall refer to $\mathcal{S}$ as the controlled-evolution system.

At each time t and starting from any state x, the system can follow different evolutions depending on the applied control u and other parameters. We denote by $\mathcal{S}(x)$ the set of all admissible evolutions starting from x and governed by $\mathcal{S}$ (1), that is,

$$\mathcal{S}(x) = \{x(\cdot) | x(0) = x \text{ and } (1) \text{ satisfied}\}.$$

Let $K \subset X$ be the set of (viability) constraints. In its simplest expression, this set would involve lower and upper bounds on the control variables, i.e.,

$$K = \left\{ x \in X \middle| \begin{array}{l} \underline{x}_1 \leq x_1 \leq \overline{x}_1 \\ \vdots \\ \underline{x}_n \leq x_n \leq \overline{x}_n \end{array} \right\},$$

but of course, in general, the constraints can be much more complex. Let $C \subset K$ be a target.

1.1 Viability kernel

The *viability kernel* is a cornerstone of viability theory. To define it, we first need to recall what is meant by a viable evolution. An evolution of the system is said to be viable on a time interval if it satisfies the viability constraints at each moment of this time interval. A mathematical definition follows.

Definition 1 (Viable evolution)**.** *An evolution $x(\cdot)$ is said to be viable in K on the time interval $[0, T)$ $(T \leq +\infty)$ if*

$$\forall\, t \in [0, T), x(t) \in K.$$

The set of all viable evolutions in K on $[0, T)$ $(T \leq +\infty)$ is

$$\mathcal{V}(K) = \{x(\cdot) | \forall\, t \in [0, T), x(t) \in K\}.$$

We shall later on give an overview of the viability constraints that have been considered in the context of the sustainable exploitation of renewable resources, and the list of these constraints in each contribution.

The viability kernel is the set of all initial states from which **at least one** viable evolution starts.

Definition 2 (Viability kernel)**.** *The viability kernel of K for system $\mathcal{S}$ is the set*

$$Viab_{\mathcal{S}}(K) = \{x_0 \in K | \exists\ x(\cdot) \in \mathcal{S}(x_0)\ such\ that\ \forall t \geq 0, x(t) \in K\}.$$

The viability kernel is a tool that allows us to check whether a system is viable, and in particular if the current state (as initial state) is viable. If the current state does not belong to the viability kernel, then a first conclusion is that the system is not sustainable. A natural follow-up question is then: can viability be restored? We will come back to this below.

A more restrictive notion is the *invariance kernel*, which corresponds to the set of all initial states such that **all** evolutions starting from these states are viable.

Definition 3 (Invariance kernel)**.** *The invariance kernel of K for system $\mathcal{S}$ is the set*

$$Inv_{\mathcal{S}}(K) = \{x_0 \in K | \forall\ x(\cdot) \in \mathcal{S}(x_0), \forall t \geq 0, x(t) \in K\}.$$

Clearly, the invariance kernel is a subset of the viability kernel.

1.2 Capture basin

In some problems, the aim is to reach a target in finite time rather than to maintain the state in a viable set at each instant of time. In this case, the relevant concept is the *capture basin*, and the following three definitions are the corresponding alternatives to the above three definitions.

In presence of a target, we will be interested by the so-called "capturing evolutions" rather than viable ones. An evolution of the system captures a target if it permanently satisfies the viability constraints before reaching the target in finite time.

Definition 4 (Capturing evolution)**.** *The evolution $x(\cdot)$ captures the target C if*

$$\exists\ T < +\infty | \forall t \in [0, T), x(t) \in K\ \&\ x(T) \in C.$$

The set of all capturing evolutions of C is

$$\mathcal{K}(K, C) = \{x(.) | \exists\ T < +\infty\ such\ that\ x(.) \in \mathcal{V}(K)\ on\ [0, T]\ and\ x(T) \in C\}.$$

The alternative notion to the viability kernel when a target is involved is the capture basin, which is the set of all initial states from which **at least one** capturing evolution starts.

Definition 5 (Capture basin)**.** *The capture basin of C for system $\mathcal{S}$ is the set*

$$\begin{aligned} Capt_{\mathcal{S}}(K,C) &= \{x_0 \in K | \exists\ (x(\cdot),T) \in \mathcal{S}(x_0) \times \mathbb{R}_+ \\ &\quad such\ that\ \forall\, t \in [0,T]\,, x(t) \in K\ and\ x(T) \in C\}\,. \end{aligned}$$

Finally, equivalently to the notion of the invariance kernel, we define the *absorption basin* of a target, which corresponds to the set of all initial states such that **all** evolutions starting from these states capture the target.

Definition 6 (Absorption basin)**.** *The absorption basin of C for system $\mathcal{S}$ is the set*

$$\begin{aligned} Abs_{\mathcal{S}}(K,C) &= \{x_0 \in K | \forall x(\cdot) \in \mathcal{S}(x_0), \exists\ T \leq +\infty \\ &\quad such\ that\ \forall\, t \in [0,T]\,, x(t) \in K\ and\ x(T) \in C\}\,. \end{aligned}$$

We note that the absorption basin is a subset of the capture basin.

1.3 Restoring viability

As alluded to above, it may well be the case that viability is not at hand, which occurs when, e.g., the viability kernel is empty or the initial state of the system is not viable. In such cases, one may wonder how much time will elapse before the constraints are violated, whether the system's viability is compromised definitively and, if it is possible to restore it, how can it be restored it and how long will it take? The *exit function* and the *crisis function* [50] are the starting points for such an analysis. The exit function measures the maximum time during which the system evolution can satisfy the constraints. The crisis function measures the minimum time that an evolution starting from a given state spends outside the viability kernel.

Definition 7 (Exit function)**.** *The exit function associates to a state $x \in X$ its maximum exit time $\tau_K(x)$:*

$$\begin{aligned} \tau_K\ :\ X &\rightarrow \mathbb{R}_+ \cup \{+\infty\}, \\ x &\mapsto \tau_K(x) = \sup_{x(.) \in \mathcal{S}(x)} \inf\{t \geq 0 | x(t) \notin K\}. \end{aligned}$$

Definition 8 (Crisis function)**.** *The crisis function associates to a state $x \in X$ its minimum crisis time $\mathcal{C}_K(x)$:*

$$\begin{aligned} \mathcal{C}_K\ :\ X &\rightarrow \mathbb{R}_+ \cup \{+\infty\}, \\ x &\mapsto \mathcal{C}_K(x) = \inf_{x(.) \in \mathcal{S}(x)} \lambda_l(t \geq 0 | x(t) \notin K), \end{aligned}$$

where λ_l is the Lebesgue measure.

One can easily deduce that a viable state will have an infinite exit time and a crisis time equal to zero, while a non-viable one will have a finite exit time and positive (finite or not) crisis time.

To restore viability, we can for example apply the *viability multiplier* to change the initial dynamics, use *reset mapping* (impulse controls) to change the initial conditions of the system, and other methods. For more details, see Aubin et al. [12], chapter 12.

1.4 Non-deterministic viability

In many problems, the evolution of the system of interest may depend on some uncertain parameters. In such cases, the dynamics of the system will involve some random variables describing the uncertainty. System $\mathcal{S}$ (1) then becomes

$$\mathcal{S}\begin{cases} x'(t) = f(x(t), u(t)) + g(\zeta) \\ u(t) \in U(x(t)) \end{cases}, \tag{2}$$

where ζ is a random variable following a certain probability distribution $\mathcal{P}$, which can be known or unknown.

Stochastic viability or robust viability can be used to deal with such contexts. In the stochastic viability framework, the assumption is that the uncertain events obey a probability law, which is inferred from some historical observations, experiences, etc. Here, the satisfaction of the viability constraints is stated in terms of a given confidence level. (Of course, one can conduct a sensitivity analysis that varies this level.) Robust viability is a special case of stochastic viability in the sense that the confidence level is set at 100%, i.e., the constraints must be satisfied whatever the uncertainties. This approach is related to the concept of *ambiguity* and is preferred when the probability law of the uncertain event is unknown, or the decision maker is seeking a strategy against the worst-case scenario. Both approaches have been considered in many other areas and are by no means limited to viability theory. However what is particular here is the adaptation of the above definitions to a non-deterministic setting. To illustrate, the next two definitions give the viability kernel in the context of stochastic and robust approaches.

Definition 9 (Stochastic viability kernel)**.** *The stochastic viability kernel of K under system $\mathcal{S}$ (2) to the confidence level of m% is the set*

$$Viab_{\mathcal{S}}^{m}(K) = \left\{x_0 \in K | \exists\, x(.) \in \mathcal{S}(x_0) \text{ such that } \forall t \geq 0, \mathbb{P}(x(t) \in K) \geq \frac{m}{100}\right\},$$

where $\mathbb{P}(x(t) \in K)$ is the probability of realization of the event $x(t) \in K$.

Definition 10 (Robust viability kernel)**.** *The robust viability kernel of environment K under system $\mathcal{S}$ (2) is the set*

$$Viab_{\mathcal{S}}^{R}(K) = \{x_0 \in K | \exists\, x(.) \in \mathcal{S}(x_0) \text{ such that } \forall t \geq 0, \mathbb{P}(x(t) \in K) = 1\}.$$

2 Applications of viability theory

Devising a VT model to study the sustainability of a system essentially involves the following inputs:

A description of the dynamical system. The ingredients here are state variables (e.g., stock of fish, size of a forest, pollution stock, population), control variables (e.g., fishing effort; deforestation and reforestation efforts; emissions; birth, death and migration rates), some uncontrollable factors (weather, epidemics, state of the economy, etc.), and their interrelationships.

An operationalization of sustainability. In the context of renewable resources, and as implied by the definition in the Brundtland Report, environmental, economic and social variables are needed to construct the validity of sustainable management (or exploitation of a resource). Practically speaking, the sustainable domain is described by a series of (viability) constraints that are imposed on the state variables (and possibly on their velocities), on the control variables, and on some joint constraints involving both types of variables. The satisfaction of the constraints is one way of handling the multi-criteria feature of sustainability, without, however, having to aggregate these facets into one index.

Depending on the context, the output is the viability kernel or the capture basin, or their more restrictive versions, that is, the invariance kernel or the absorption basin. Also, we obtain the controls that must be exerted to remain in one of these sets. These controls are interpreted as policy guidance.

We make the following remarks:

1. Sustainability must in some way refer to intergenerational equity to account for the principle stated in the Brundtland Report, namely, of meeting "the needs of the present without compromising the ability of future generations to meet their own needs." This intergenerational equity is inherently preserved in VT because the constraints must be satisfied at *each* instant of time, independently of which generation is living at that instant, which means that all generations are treated equally.

2. As VT proceeds numerically, the functions describing the dynamical system and the constraints can be of any form. This huge flexibility comes at the cost that the controls needing to be exerted to remain viable can only rarely be described in closed form.

3. A VT model can have as many control variables as the situation dictates. The number of state variables is non-restricted in theory, but in practice, it is very hard to go beyond a four-dimensional state. In fact, in applications of VT to renewable resources, the dimension of the state space is generally less than three. Note that all other alternative approaches involving dynamic optimization also suffer from this curse of dimensionality.

Table 1 reports the number of papers applying VT to renewable resources by area. The main takeaway is that ecosystems and population biology are by far the most studied areas, with fisheries accounting for half of all applications of VT to renewable resources (50%).

Table 1: Viability theory applications by area

	Number of articles	%
Ecosystems and population biology:		
Fisheries	40	50
Other non-marine species	14	18
Farming and agro-ecology	9	11
Climate change	6	7
Forests	4	5
Renewable resources (general)	3	4
Other	4	5

From Table 2, we learn that most models have infinite time horizons, that discrete-time models are slightly more popular then continuous-time models and that two-thirds of publications assumed a deterministic world. Stochastic viability is used slightly more often than robust viability when uncertainty is considered.

Table 2: Type of model

	Percentage
Discrete time model	52
Continuous time model	48
Infinite time horizon	61
Finite time horizon	39
Deterministic viability	67
Stochastic viability	19
Robust viability	14

From Table 3, we notice that most articles involve a practical numerical application, with 47% using empirically estimated values from real situations. The other studies either give a numerical illustration using some suitable values or do not provide any numerical examples.

Table 3: Type of numerical application or illustration

	Percentage
Practical application	47
Arbitrary values	29
No application	24

Finally, we note that of the 80 papers selected for this survey, 45 (or 56%) were published during the period from 2010 to 2015.

2.1 Viability studies in ecosystems and population biology

Early contributions of viability theory in renewable resources are related to ecosystems and population biology; see Křivan ([69], [70], [71]) and Bonneuil [23]. Křivan was mainly interested in the following question: "How can we modify a dynamical system to make it viable, (i.e., having solutions that do satisfy the constraints), knowing the dynamical behavior of the system without the state constraint?" ([69]). Bonneuil's contribution, in [23], was to revisit the Malthus-Boserup explanatory framework of population biology using the point of view of viability theory.

Within this group of studies, fishery is by far the most popular topic. One possible explanation for this is that optimal-control models, which share a number of commonalities with VT models, were already widely used in fisheries, and therefore, the transition from one methodological framework to the other was somewhat easy. Whatever the precise objective being pursued, e.g., protection of an endangered species or preservation of biodiversity, this literature will typically have a population state space $X \subset \mathbb{R}^n$ where $n \geq 1$ is the number of different species considered, or age classes in the case of age-structured populations, and $x(t) = (x_i(t))_{i=\overline{1,n}}$ is the biomass or stock level of each species $i \in \{1, ..., n\}$ at time t. Of course, other state variables may be considered, such as biodiversity or economic indicators. In the continuous-time case, the evolution of the (population) state variables is described by the following system of differential equations:

$$\begin{aligned} x'(t) &= f(x(t), u(t)), \\ u(t) &\in U(x(t)), \end{aligned}$$

where function f captures the evolutionary characteristics of each species (e.g., reproduction and fertility, natural mortality) as well as the interactions with other species (e.g., predation, cohabitation).

The literature can be divided along different lines. One is multi-species studies (e.g., Béné and Doyen [18], De Lara and Martinet [42], Martinet et al. [77], Gourguet et al. [59], Lercari and Arreguín-Sánchez [72], Krawczyk et al. [68], Mullon et al. [84], Doyen et al. [52], Maynou [80], Martinet and Blanchard [76]) versus single-species studies (e.g., Chavas [36], Doyen and Béné [48], Eisenack et al. [56], Péreau et al. [85], De Lara et al. [45], Ferchichi et al. [58], Sanogo et al. [98], Curtin and Martinet [39], Eisenack and Kropp [55], Alais et al. [1]) or age structured population studies (e.g., De Lara et al. [43], De Lara and Martinet [42], Doyen et al. [51], Gourguet et al. [59], Curtin and Martinet [39], Maynou [80], De Lara et al. [45], Alais et al. [1], De Lara et al. [44], Chavas [36]) or even sex-structured population studies (e.g., Gourguet et al. [60], Ferchichi et al. [58]). In a multi-species context, the focus is on marine (and sometimes non-marine) ecosystems and food webs. The resulting models are, generally speaking, more complex than in single-species models, as all relevant interactions between the different species must be taken into account. Each control variable may concern one or many of these species, and all of them may be involved in the economic or environmental viability constraints. In the single-species category, only one resource stock is considered, and the (often implicit) assumption is that the effect of the other species on this stock is captured by the mortality and fertility parameters, while the effect of variations in the considered species on the others can be captured through some biodiversity indicators.

A second distinction can be made between studies that consider human intervention (Béné and Doyen [17], Sinclair [100], Cissé et al. [37] and Eisenack et al. [56]), and those that do not (see, e.g., Bonneuil [25], Křivan [69], Křivan [70], Bonneuil and Müllers [27], Křivan and Colombo [71], Bonneuil [23], Rougé et al. [92], Bonneuil and Saint-Pierre [28], Aubin and Saint-Pierre [10]). When human action is absent, the long-term evolution of the system will depend only on inter-species interactions and possibly some unforeseen events, and can then be considered a benchmark for assessing the impact of human intervention.

A third distinction is between deterministic models and those where some form of uncertainty is considered. In population-biology and fisheries models, this uncertainty can be of a biological nature, for instance uncertainties in the population's rate of reproduction, inter-species relationships and rate of predation, or the initial biomass stock size; see, e.g., Regnier and De Lara [90], Chapel et al. [35] and Křivan and Colombo [71]. It can also be related to the environment, e.g., the uncertainty related to climate change or the effect of pollution on the species; see, e.g., Doyen et al. [52], Křivan [70] and Martinet et al. [78]. Finally, the uncertainty can be related to market conditions (demand and price) or to the evolution of technology; see, e.g., Gourguet et al. [59].

Although this literature is dense, it is interesting to note that the different contributions share a lot of common features when it comes to selecting the control and state variables and defining the viability constraints. With the following list of variables and constraints, we account for a large extent of what has been considered in this literature:

State variables: The most common variables are (i) the biomass stock of the species; and (ii) some biodiversity indicators. See, e.g., Doyen and Béné [48], Hardy et al. [62], Cisse et al. [38] and De Lara et al. [46].

Control variables: The most frequently considered variables are (i) the harvest level (e.g., De Lara et al. [43], Béné and Doyen [17], Doyen and Béné [48], De Lara et al. [44] and Curtin and Martinet [39]); and (ii) the catching effort (e.g., Doyen et al. [52], De Lara and Martinet [42] and De Lara et al. [45]).

Viability constraints: Ecological viability constraints typically refer to the non-extinction of species (e.g.,. Bonneuil [25]), minimum biomass stock of the resources (used by a large majority of studies), or minimum levels for some biodiversity indicators (Doyen et al. [52], Hardy et al. [62], Cisse et al. [38], Béné and Doyen [18] and Cissé et al. [37]). Economic viability constraints include the satisfaction of demand or guaranteeing food security (Eisenack et al. [56], Cissé et al. [37], Hardy et al. [62], Regnier and De Lara [90], Hardy et al. [63], Thébaud et al. [101], Cisse et al. [38] and De Lara et al. [44]), or minimum revenue or productivity level (e.g., Doyen et al. [52], Meadows et al. [81], Béné and Doyen [17] and Doyen et al. [51]). Social constraints are rarely addressed, but still, a few examples are available, e.g., limiting the number of layoffs per period, which in a fishery context requires to lower-bound the fleet size (Meadows et al. [81]) or maintaining a minimum level of activity for fishermen (Lercari and Arreguín-Sánchez [72], Martinet et al. [77], Sanogo et al. [97], Péreau et al. [85], Sanogo et al. [98], Krawczyk et al. [68], Ferchichi et al. [58], Sinclair [100] and Alais et al. [1]).

2.2 Viability studies in climate change and GHG management

Schematically, the main question when it comes to climate change and GHG management is how to limit the rise in temperature to below a given threshold (two degrees is the most cited number) by a certain date (the end of the century). The assumption is that surpassing two degrees will lead to a long series of problems such as loss of biodiversity, rise in the sea level, and droughts, with considerable negative impacts on all living species and their ecosystems. Any attempt to answer this question requires that a dynamical system be defined that adequately describes the evolution of the environment as a function of some control variables and uncontrollable factors. It then suffices to introduce relevant constraints to have a viability model. Actually, this viability theory philosophy is embedded in the Tolerable Window Approach proposed in the nineties by the German advisory council on global change (Scientific Advisory Council on Global Change [99]), even though the viability study per se only began to appear ten years later.

In the few published papers that use the tools of viability theory, the state variables are the same as those used in other methodological frameworks, namely, GHG concentration (Bernardo and Saint-Pierre [21], Aubin et al. [11], Von Bloh et al. [104], Andrés-Domenech et al. [3] and Aubin [8]), mean global temperature (Bernardo and Saint-Pierre [21]) and, a novelty, emission flows (Aubin [8] and Andrés-Domenech et al. [3]). The rationale behind seeing emissions flows as a state variable rather than a control variable lies in the fact that emissions are a by-product of the production of goods and services, and thus, modifying emissions cannot be easily feasible for technological or economic reasons. However, their rate of change can be controllable.

Commonly considered control variables include GHG-emissions (or -abatement) rates (Bernardo and Saint-Pierre [21] and Aubin et al. [11]), investments in green technologies, or intensity of industrial activities (Aubin [8]) and emission rights allocations (Aubin et al. [13]). In models where forests are included as carbon sinks that reduce GHG concentrations in the atmosphere, deforestation and reforestation rates are also retained as decision variables (Andrés-Domenech et al. [3]). Control variables are often lower- and/or upper-bounded to account for some hard technological and economic constraints. For instance, one should not decrease emissions beyond a certain level to avoid massive short-term economic losses or because it is impossible to take too many cars off the road overnight.

It is not surprising that environmental viability constraints take the form of an upper bound on GHG concentrations in the atmosphere (Bernardo and Saint-Pierre [21], Aubin et al. [11] and Andrés-Domenech et al. [3]) or an upper bound on the global mean temperature (Bernardo and Saint-Pierre [21]). Popular economic viability constraints are either limits imposed on the cost that can be borne when changing emissions levels, which can be operationalized by an upper bound on the velocity of the cumulative emissions, or they can constrain the minimum revenues from industrial activities responsible for GHG emissions to be no lower than a given vital threshold (see, e.g., Bernardo and Saint-Pierre [21], Andrés-Domenech et al. [3]).

Finally, we observe that, with few exceptions (see, e.g., Aubin et al. [13]), not much has been done to incorporate uncertainty in VT climate models.

2.3 Other renewable resources

In this miscellaneous category of applications of VT, we have contributions dealing with forest management (4 papers), farming and agro-ecological systems (9 papers), water resources (Martin [74], Rougé et al. [91] and Alais et al. [1]) and ecotourism-based systems (Wei et al. [105]).

For the Food and Agriculture Organization, "[Forests] are to provide renewable raw materials and energy, maintain biological diversity, mitigate climate change, protect land and water resources, provide recreation facilities, improve air quality and help alleviate poverty" (see *Global Forest Resources Assessment* 2005 [88]). The world's forests cover nearly one-third of the Earth's surface, but are shrinking at an alarming rate, with an area equivalent to the size of Costa Rica being deforested every year (FAO 2010 [57]). The main reason for deforestation is agriculture, which brings revenues but by the same token eliminates some of the benefits listed above. A viability model essentially aims at preserving the forest while balancing its competing uses. In the few available studies, the state variable is typically the forest's size (Bernard and Martin [20] and Andrés-Domenech et al. [4]) or the number of trees (Mathias et al. [79]), although other variables have also been considered, such as forest biodiversity indicators or the size of the population whose life quality depends on the forest and their wealth or the stock of timber (Mathias et al. [79], Bernard and Martin [20] and Andrés-Domenech et al. [4]). Examples of control variables are forestation and deforestation rates, frequency of these activities, monetary transfers to forest owners to incentivize them to protect their forests, or measures to control the size of a population living around the forest (as suggested in Andrés-Domenech et al. [4], Bernard and Martin [20], Mathias et al. [79]). Environmental viability constraints include imposing a minimum forest size (Andrés-Domenech et al. [3], Mathias et al. [79] and Andrés-Domenech et al. [4]), minimum level of biodiversity, maximum level of deforestation or constraints related to the composition of the forest in terms of species or age of the trees (Mathias et al. [79]). Typical economic constraints are the satisfaction of the demand for timber (Andrés-Domenech et al. [3] and Andrés-Domenech et al. [4]) and a minimum revenue from forest exploitation (Andrés-Domenech et al. [3], Mathias et al. [79] and Andrés-Domenech et al. [4]).

The applications in agro-ecological and farming problems mostly relate to herd- and grazing-management systems (Tichit et al. [102], Baumgärtner and Quaas [15], Sabatier et al. [93], Sabatier et al. [94], Tichit et al. [103], Martin et al. [75] and Sabatier et al. [95]). The state variables are the grass biomass or height (as in Baumgärtner and Quaas [15], Sabatier et al. [93], Sabatier et al. [94], Tichit et al. [103], Martin et al. [75] and Sabatier et al. [95]), the herd composition or size(Tichit et al. [102] and Baumgärtner and Quaas [15]) or the abundance of some protected wildlife leaving in the grassland (Mouysset et al. [83], Sabatier et al. [93], Tichit et al. [103] and Sabatier et al. [94]). The control variables are grazing frequency and intensity (Tichit et al. [103], Baumgärtner and Quaas [15], Sabatier et al. [93], Martin et al. [75], Sabatier et al. [94] and Sabatier et al. [95]) or breed composition within the herds (Tichit et al. [102]). Examples of viability constraints include the preservation of the grassland (as in Baumgärtner and Quaas [15] and Martin et al. [75]), the satisfaction of cattle feeding requirements (like in Sabatier et al. [93], Sabatier et al. [94], Tichit et al. [103] and Sabatier et al. [95]), the guarantee of a minimum income to the farmers (Tichit et al. [102], Sabatier et al. [94], Mouysset et al. [83], Baumgärtner and Quaas [15], Martin et al. [75] and Sabatier et al. [95]), maintain acceptable level of biodiversity (Mouysset et al. [83]) and protect wildlife leaving or breeding in the grassland (Tichit et al. [103], Mouysset et al. [83], Sabatier et al. [93] and Sabatier et al. [94]). Climatic and environmental risk are the most often considered type of uncertainty (Tichit et al. [102], Baumgärtner and Quaas [15], Mouysset et al. [83] and Sabatier et al. [95]).

Soil preservation problems are considered in (Durand et al. [53]). The soil quality (state variable) is measured by a composite index involving physical, chemical and biological characteristics. The control variables refer to the choice of activities (agriculture, cattle breading, etc.), the sequence of plantation, the type of agricultural practices (traditional or intensive), the investment in green technologies, etc. In the same reference ([53]), the main ecological viability constraint is a lower bound on soil quality, and the economic constraints are related to the cash balance, the total revenue from agricultural activity, or investments. In this farming and agro-ecology category, the principal types of uncertainty that have been considered are climatic (Tichit et al. [102]) or in the parameter values of the population dynamics (Sabatier et al. [93]).

3 Concluding remarks

We surveyed in this paper the applications of viability theory to the sustainable exploitation of renewable resources, i.e., population biology and ecosystems (fisheries and other species), climate change, farming, forests and other resources. We wish to conclude by pointing out some issues and topics that deserve some attention from researchers.

Computation (algorithms): A series of algorithms for solving a viability problem, e.g., determining its viability kernel, are available; see, e.g., Saint-Pierre [96], Bonneuil [26], Deffuant et al. [47], Aubin et al. [12], Krawczyk and Pharo [67] and Maidens et al. [73]. Except in some special cases, all these algorithms are subject to the curse of dimensionality. (The size of the state space matters here, rarely the number of control variables is an issue.) Facing this problem, the strategy in the applied literature was to limit the number of state variables to one or two (of course some applications included more than that). Developing efficient algorithms using machine learning techniques, see, e.g., Deffuant et al. [47], Chapel et al. [35], and approximate dynamic programming, see, e.g., Bertsekas [22], Powell [87], will allow dealing with practical problems in higher dimensions and improving computational time.

Computation (users): One reason why some methods (think of statistical methods and linear programming) are more used than others is clearly the availability of (friendly users) software. An economist or an ecologist looking for a viable solution to his/her system will be very much reluctant to jump in the area unless he/she could easily have access, if not to a fully canned software, to at least some programs written in, e.g., Matlab, Python or in other widely available computational environment. This is to say that popularity of viability theory is bounded to increase with the availability of computational tools. In some sense, we launch a call to the community to share its resources in order to converge in the long term to developing a platform for solving viability problems that could be accessible to researchers (and eventually) practitioners in this area.

Viability and strategic interactions: We mentioned before that viability theory offers a framework that accommodates for the presence of more than one stakeholders and more than one objectives. This works quite well as long as these stakeholders can "coordinate" in some sense when drawing the list of viability constraints and they are not playing strategically. In many situations, the resource considered is of open access, that is, more than one agent can exploit the stock of resources, e.g., exploitation of high-sea fisheries and the environment. Here, coordination is much harder to achieve because the players are seeking their individual interests and competing. Can viability theory help in addressing this class of problems? If a regulator can impose that the dynamical system describing the evolution of the resource remains viable, then the answer is yes. The resulting noncooperative game is then played à la Rosen (1965), that is, the players are subject to coupling constraints and a generalized (or normalized) Nash equilibrium is sought. For an introduction to coupling constraint noncooperative games, see, e.g., Haurie et al. [64]. Further, it is worth mentioning that some mathematical analysis and numerical methods links have been established between zero-sum two-player differential games and viability theory, see, e.g., Cardaliaguet et al. [33, 34], Cardaliaguet and Plaskacz [32].

Missing applications: Our survey showed a variety of applications and also some cases where the potential of using viability theory is far from having been reached. This is surely the case in, e.g., soil management, forest preservation and sustainable tourism. Further, some clearly relevant interactions between systems, e.g., forests and climate change, or soil preservation and protection of some animal species, have not yet attracted the attention they deserve. Finally, little research effort has been devoted till now to develop viability models to better account for uncertainties in climate change, forestry, soil exploitation and possibly other areas.

Appendix A: Summary table

In the following table we present a brief summary of the papers reviewed in this survey. The papers in the table are first sorted according to the domain of application and next ordered by publication year.

The first column gives the reference, and the second column provides information about the following elements:

1. The studied model or problem and its characteristics.
2. The viability theory tools used, with some details on their use.
3. Information about the numerical applications, if any.

The third and fourth columns give the state and control variables, while the fifth column displays the list of viability constraints. Each one of these constraints is tagged with a specific sign according to its type: "🍃" for environmental constraints, "$" for economic constraints and "#" for social ones. Combinations of these symbols are used to mark constraints of more than one type (for example "$#" designates socio-economic constraints). It is important to mention that there are some physical constraints (like non negativity of a physical stock, carrying capacities, etc.) that are used in some studies but are not listed in this table because they are obvious constraints and are explicitly or implicitly considered in all concerned studies.

The information about the model's horizon (finite, infinite) and time (discrete, continuous) appears in the sixth column. The last column indicates if uncertainty has been considered in the model, and eventually the approach used (robust or stochastic).

To simplify the table, the abbreviations listed below are used:

VT: Viability theory.

VK: Viability kernel.

SVK: Stochastic viability kernel.

RVK: Robust viability kernel.

VD: Viability domain.

VP: Viability probability.

CB: Capture basin.

IK: Invariance kernel.

TCF: Time of crisis function.

IF: Inertia function.

RA: Robust approach.

SA: Stochastic approach.

RS: Regulated system.

SS: Stabilized system.

NA: Numerical application.

UB: Upper bound.

LB: Lower bound.

ULB: Upper and Lower bounds.

C/D: Continuous / Discrete (time).

F/ ∞: Finite / Infinite (time).

Ref	Details	State variables	Control variables	Viability constraints	T	U
Climate change						
[11] 2005	1)GHG accumulation model. 2)Uses VK and associated feedbacks to study the sustainability of the system and choose the sustainable management strategies with minimum transition cost. 3) NA with chosen parameters' values.	-GHG concentration.	-Short-term pollutant emissions.	🍃 UB on GHG concentration level in the atmosphere.	C ∞	No
[104] 2008	1)Atmospheric CO2 concentration model parametrization problem. 2)Uses VK and associated feedbacks to estimate the values of the unknown parameters of the model (biogenic enhancement factor of weathering). 3)NA on empirical data.	-Atmospheric CO2 concentration.	-None (RS). The regulon is the biogenic enhancement factor of weathering.	-UB on error degree (Coherence of the state variable with observed data) -UB on the velocities of the regulon (No big changes in the parameters' value in a small time interval).	D ∞	No
[8] 2010	1)Economic-Climatic coupled system. 2)Uses the IF to measures the transition cost of the industrial activity necessary to maintain the GHG concentration at acceptable level. 3)No NA.	-GHG Concentration. -Short-term pollution rate.	-None (RS).The regulon is the intercity of the industrial activities.	🍃 UB on GHG concentration level.	C ∞	No
[3] 2011	1)GHG Accumulation-Deforestation coupled system. 2)Uses VK to study the sustainability conditions of the system in different situations and assess the sustainability of world's forests to limit CO2 concentration. 3)NA on estimated parameters' values.	-CO2 emissions. -CO2 concentration in the atmosphere. -Forest surface.	-Deforestation rate. -Reforestation rate. -Speed of CO2 emission adjustment. -Monetary transfers.	🍃 UB on CO2 stock. $ ULB on emission rate. $ LB on revenue. $ LB on wood quantity production.	C ∞	No
[13] 2012	1)Emission rates allocation problem. 2) Uses VK and the associated feed-backs to determine the sustainable initial emission rates and their corresponding dynamical allocation knowing the maximum emission growth rates of polluters. 3) NA on chosen parameters' values.	-Global emissions. -Individual emissions (by polluter).	-Emission rights allocations.	🍃 UB on global emissions level. 🍃 UB on each polluter's allowed emission level. $ LB on each polluter emission right. $ UB on each polluter's emission rate growth.	C F	RA
[21] 2015	1) Climate changes problem. 2) Proposes VT based measurement tools and climate indicators and uses VK and associated feedbacks to study the sustainability of the system and the viable management strategies. 3) NA on estimated parameters' values.	-CO2 concentration. -Global mean temperature. -Cumulative CO2 emission.	-Anthropogenic CO2 emission.	🍃 UB on CO2 concentration. 🍃 UB on global temperature. 🍃 UB on cumulative emissions. $ ULB on state variables' velocities. $ ULB on the emission level.	C ∞	No
Forest protection						
[31] 2011	1) Single agent savanna management problem. 2) Uses VK to study the viability and resilience of the model. 3) NA on estimates parameters' values.	-Density of trees in the savanna.	-Grazing pressure (More grazing = less grass = more trees).	🍃 ULB on the density of trees.. $ ULB on grazing pressure. $ UB on grazing pressure variation.	C ∞	No

[20] 2013	1) Forest urbanisation management problem. 2) Uses VK and associated feedbacks to study the system's sustainability conditions and best management strategies. Also shows the importance of monetary transfers to achieve viability. 3) NA on the rain forest in the corridor of Fianarantsoa (Madagascar).	-Size of built area. -Population size. -Total wealth of the population.	-Urbanizing effort. -External workers proportion. -Monetary transfers. -Demographic growth rate.	🍃 LB on forest size (UB on built area). $ LB on capital/capita value. $ Increasing individual wealth over time. $ ULB on the control variables. # LB on population size.	C ∞	No
[4] 2014	1) Single agent single species forest management system. 2) Uses VK to show the unsustainability of the current state and practices in the Androy forest and to derive some possible ways to recover sustainability. 3) NA on the Androy forest in Madagascar.	-Size of forest area. -Population size. -Physical capital of zebu.	-Forestation. -Births rate. -Per capital consumption. -Deforestation rate for agriculture. -Deforestation rate for wood. -Deforestation rate for cattle breeding. -Monetary transfers.	$ Non decreasing per capita level of consumption. $ Non decreasing absolute and relative levels of capital. $# Covering population's basic need of wood at any time.	D ∞	No
[79] 2015	1) Single agent single species forest management system. 2) Discusses the efficiency of different forest management strategies (bounds' values in the constraints) using VK and the corresponding values of the flexibility indicator. 3) NA on the univen-aged silver fir forest in "Quatre montagnes" (France).	-Number of trees in both strata of the forest. -Volume of deadwood. -Timber stock.	-Intensity and frequency of harvesting wood in both strata. -Deadwood retention volume.	🍃 ULB on trees quantity in each stratum. 🍃 ULB on per hectare deadwood quantity. $ ULB on timber stock level.	C ∞	No
Fisheries						
[17] 2000	1) Single agent single species fishery subject to resource and market seasonal oscillation. 2) Uses VK to study the role of storage regulation in maintaining the system's viability. 3) NA on the French Guyana shrimp fishery.	-Storage volume of the harvested resource.	-Export flow (Fishing flow).	$ Positive profit. $ Catches bounded by demand. $ LB on storage level. $ Limited storage capacity.	C ∞	No
[19] 2001	1) Single species single agent bio-economic marine system. 2) Uses VK and associated feedbacks to identify overexploitation situations preventing regulation controls. Uses also TCF to study the reversibility of overexploitation situations. 3) No NA.	-Biomass stock level. -Fishing effort.	-Time variation of fishing efforts (Velocity of the fishing effort).	🍃 LB on biomass stock level. $ ULB on fishing effort level. $ Positive global and net benefit at any time.	C ∞	No
[48] 2003	1) Single agent single renewable resource protected area. 2) Uses IK to study the efficiency of marine reserves in protecting resources and its sensitivity to uncertainty. 3) NA on chosen parameter values.	-Biomass stock level.	-Harvest rate.	🍃 LB on biomass stock level.	D ∞	RA
[54] 2003	1) Co-managed single species fishery. 2) Uses VT modelling approach combined to a qualitative approach to study the sustainability of the system. 3) No NA.	-Biomass stock. -Capital accumulated in the fishery.	-Catch recommendation.	🍃 LB on biomass stock level. $# LB on total harvest (for acceptable employment level, food safety and economic profitability).	C ∞	No

[84] 2004	1) Single agent multi-species marine ecosystem. 2) Uses VK calculated different scenarios (with or without exploitation) to study the sustainability of the system. 3) NA on the Benguela ecosystem.	-Biomass of each species.	-global mortality and interspecies consumption as regulons. -Catches as control variable (scenarios with exploitation).	🍃 ULB on each species biomass level.	D ∞	RA
[40] 2005	-Explains how viability theory can be applied to study the sustainability of ecosystem based fisheries.	X	X	X	X	X
[56] 2006	1) Co-managed single species fishery (one decision maker). 2) Uses VD to study the sustainability of the system and the efficiency of three control strategies. 3) No NA.	-Biomass stock level	-Catches recommendation.	🍃 LB on biomass stock level. $# LB on total harvest (Food safety).	C ∞	No
[55] 2006	1) Single species participatory fishery management model. 2) Explains how to use VT modelling and VK and viable feedbacks to study the sustainability of the system. 3) No formal mathematical model or NA.	-Biomass stock of the resource.	-Total annual allowable catches. -Catches allocation among the fishing firms.	🍃 LB on biomass stock level.	C ∞	No
[43] 2007	1) Single agent single species age structured fishery. 2) Uses VD to study the efficiency of the spawning-stock biomass and fishing mortality as indicators of sustainability in the precautionary approach. 3) NA on the northern hake and Bay of Biscay anchovy.	-Vector of abundance of the stock at each age.	-Fishing mortality multiplier.	🍃 LB on spawning-stock biomass. 🍃 UB on fishing mortality over predetermined age range.	D ∞	No
[52] 2007	1) Single agent exploited food-web with marine reserves. 2) Uses VK to study the influence of protected areas upon environmental and economic sustainability of the system. 3) NA on the Aboré coral reef reserve in New Caledonia.	-Biomass stock for each species. -State of the habitat.	-Harvesting effort for each species.	🍃 Preservation of all the spaces. 🍃 LB on a biodiversity indicator value. $ LB on utility from catches.	D F	SA
[81] 2007	1) Single agent single species fishery. 2) Uses VK to identify the viable states of the system and the TCF to study the recovery possibilities of the non-viable ones. 3) NA on the bay of Biscay Nephrops fishery.	-Biomass stock level.	-Fleet size and fishing effort.	🍃 LB on biomass stock. $ LB on per vessel benefice. # LB on the fleet size. # UB on size changing speed (velocity of fleet size)	D ∞	No
[35] 2008	1) Single agent multi-species ecosystem fishery. 2) Uses VK to study the effect of fishing some species of fish and determines sustainable yield policies. 3) NA on the southern Benguela ecosystem.	-Biomass stock level of each species.	-Yields on harvested fish (Pelagic fish and Demersal fish).	🍃 ULB on biomass stock levels. $ LB on the yield.	C ∞	RA
[42] 2009	1) Single agent multi-species age structured fishery with one exploited species and one non exploited one. 2) Uses VK and associated feedbacks to determine the fishing strategies maximising the viability probability. 3) NA on the nephrops-hake fisheries on the Bay of Biscay.	-Biomass stock of each species (Nephrps and hakes) at different ages.	-Harvesting effort for Nephrops fish.	🍃 LB on abundance level of mature hakes fish. $ Profitability of the fishery.	D F	SA
[76] 2009	1) Single agent multi-species exploited ecosystem with one exploited species (Shrimp) and one non exploited species (Frigate bird feeding on fishery discards). 2) Uses VK to study the sustainability of the system. 3) NA on the French Guiana shrimp fishery.	-Biomass of the shrimp stock.	-Fishing effort.	🍃 LB on fishing discards level (To feed and conserve the Frigate bird population). $LB on catches level per unit of effort.	D ∞	No

[16] 2009	1)Multi-agent multi-species fishery. 2) Uses VT modelling combined with simulation to compare different fishing scenarios. 3) NA on chosen parameter values.	-Biomass stock of the different species.	-Fishing effort.	🍃 Preservation of all species. $ Profitability of the fishing activity for all the fishers.	D ∞	No
[72] 2009	1) Single agent multi-species fisheries. 2) Uses VT modelling combined with simulation to study the sustainability of the system and determine viable harvesting strategies. 3) NA on the Northern Gulf of California ecosystem.	-Biomass levels of the different species.	-Fishing effort.	🍃 UB on ecosystem deterioration level. 🍃 LB on biomass recovery level for endangered species. $ Profitability of the fisheries. # Maintain fishermen jobs. # Respect the government regulation plans of fisheries.	C ∞	No
[77] 2010	1) Single agent single species fishery. (Fleet composed of multiple vessels: single decision maker). 2) Uses VK to study the sustainability of the system and TCF to determine acceptable recovery paths from non-viable states. 3) NA on the Bay of Biscay nephrops fishery.	-Biomass stock of the exploited resource. -Fleet size.	-Fishing effort. -Changes in the fleet size.	🍃 LB on biomass level. $ LB on profit per vessel. LB on the fleet size.	D ∞	No
[65] 2010	1) Two models for single species single agent fishery one with and the other without protected area. 2) Uses the VK to study the sustainability of the systems in order to investigate the benefits of protected areas. 3) No NA.	-Stock level of the resource in each area considered in the model.	-Harvesting effort in each area considered in the model.	🍃 LB on stock level. $ LB on fishermen income.	C ∞	No
[45] 2011	1) Single agent single species age structured monotone harvest fishery. 2) Uses VK to study the sustainability of the system. 3) NA on two Chilean fisheries (Sea bass and Alfonsino).	-Abundance of population at different ages.	-Fishing effort.	🍃 LB on spawning stock biomass of the resource. $ LB on yield from fishing.	D ∞	No
[66] 2012	1) Two single agent single species commercial fishing models (with and the other without a price state variable). 2) Uses VK to study the sustainability of the systems and determine the best exploitation strategies (combinations of resource stock, price, capital and investment). 3) No NA.	-Density of fish population. -Capital investment in fishing activity. -Price	-Investment rate.	🍃 LB on resource stock level. $ LB on catches level. $ LB on capital investment	C ∞	No
[51] 2012	1) Multi-agent multi-species age structured fishery. 2) Uses the SVK to determine the exploitation strategies maximizing the viability probability of the fishery. 3) NA on the nephrops and hake fisheries in the bay of Biscay (France).	-Abundance of the species at different ages.	-Fishing mortality associated with the fleets (Target species for each fleet).	🍃 LB on abundance level of each species at each age. $ LB on each fleet income.	D F	SA
[97] 2012	1) Two-agents single species fishery. 2) Uses VK to study the sustainability of the system. 3) No NA.	-Biomass of the exploited resource.	-variation rate of fishing effort of the two fleets. (investments)	🍃 LB on biomass stock level. $ LB on each fleet income. $# ULB on the fishing efforts.	C ∞	No
[85] 2012	1) Multi-agent single species transferable quota based management fishery. 2) Uses VK to study the sustainability of the system with asymmetric agents. 3) NA on the Bay of Biscay nephrops fishery.	-Biomass level of the resource.	-Total allowable catches (the sum of all the quotas attributed to the agents).	🍃 LB on biomass level. $ Profitability of the fishery. # LB on the number of active fishers.	D ∞	No

[46] 2012	1) Single agent multi-species ecosystem based fishery. 2) Uses VK to study the sustainability of the system. 3) NA on the Hake-Anchovy couple in the Peruvian Upwelling ecosystem.	-Biomass level of each species.	-Harvesting effort for each species.	🍃 LB on each species biomass. $ LB on catch levels (yield).	D ∞	No
[37] 2013	1) Multi-fleet multispecies ecosystem based management fishery. 2) Uses VT modelling approach combined with simulation to evaluate the sustainability of a set of management strategies. 3) NA on the costal fishery of French Guiana.	-Biomass of each species.	-Fishing effort of each fleet.	🍃 LB on the Species richness indicator (SR) (biodiversity level). 🍃 LB on the trophic marine index indicator (MTI) (total biomass level). 🍃 LB on the Simpson diversity index(SI). $ LB on harvest (food security). $ Positive profit for each fleet.	D F	No
[59] 2013	1) Multi-agent multi-species age structured fishery. 2) Uses VK to compare the efficiency of different management strategies in sustainability. 3) NA on the demersal fishery in the bay of Biscay.	-Abundance of the species at different ages.	-Fishing efforts multipliers (allocation of the vessels).	🍃 LB on species abundance levels. $ Positive profit for each vessel.	D F	SA
[39] 2013	1) Single species age structured regulated transboundary fishery (2 countries with different technology). 2) Uses VK to study the sustainability of the system and assess the viable management strategies. 3) NA on the France-Spain Bay of Biscay anchovy fishery.	-Biomass stock of fish at each age.	-Annual total catches for each age class of fish and the fishing quota allocation.	$ LB on each country's profit. # Fairness between the countries in the quota allocation.	D ∞	No
[98] 2013	1) Single agent single species fishery. 2) Uses VK to study the viability of the system and assess the sustainable management options. 3) No NA.	-Biomass stock. -Available catching effort.	-Investments rate in catching efforts.	🍃 LB on biomass stock level. # ULB on catching effort levels.	C ∞	No
[68] 2013	1) Multi-agents multispecies by-catch fishery. 2) Uses VK to study the sustainability of the system. 3) NA on chosen data.	-Biomass stock. -Catching effort	-The catching effort variation.	🍃 LB on biomass level. $ Positive profits for the fishery's fleets. # ULB on catching efforts.	C ∞	No
[62] 2013	1) Multi-agent multispecies small scale fishery. 2) Uses VT modelling combined with simulation to identify the system's sustainability conditions. 3) NA on the Solomon islands' small scale fisheries.	-The biomass stock of each species.	-Vector of fishing efforts allocated to each fleet.	🍃 LB on "Species richness" and "Simpson index" ecological indicators (biodiversity). $# Food and cash security.	D ∞	No
[63] 2013	1) Multi-agent single species artisanal fishery. 2) Uses TCF to study the resilience of the system in case of cooperation or non-cooperation between the agents. 3) NA on the Solomon Islands' small scale fisheries.	-Biomass stock of the resource. -The number of fishermen.	-Fishing effort allocation among the agents.	$# LB on each agent catches (food security and acceptable cash income for each agent).	D ∞	No
[58] 2014	1) Single species hermaphrodite maturity stage structured population fishery (3 stages: Juvenile, male, female). 2) Uses VK to study the viability domain and sustainable management strategies of the system. 3) No NA.	-Resource density at each maturity stage. -The number of fishermen.	-Fishing effort for each class of the resource.	🍃 LB on female density. $ LB on fishermen revenue. # LB on fishing activity at any time.	C ∞	No
[80] 2014	1) Two fleets multispecies and age structures fishery. 2) Uses VP to study, compare and rank some management scenarios. 3) NA on the main western Mediterranean Spanish fisheries.	-Abundance of each species at each age.	-Strategies for fishing mortality reduction.	🍃 LB on spawning stock biomass level for all species. $ Positive economic profit for each fleet.	D F	SA

[101] 2014	1)Single agent single species fishery. 2) Proposes a VT based approach to the evaluation of fisheries management strategies. 3) NA on the Ningaloo marine park of western Australia.	-Biomass level of the exploited resource.	-Exploitation strategies.	🍃 LB on regional and global spawning biomass level. $# LB on catches level.	D F	SA
[100] 2014	1) Three models: -a) Single agent single species fishery. -b) Single agent two independent species by-catch fishery. -c) Two agents two independent species fishery with one agent targeting only one species and the other fishes both. 2) Uses VK and associated feedbacks to study the sustainability of each system. 3) NA with chosen data values.	-Biomass of each resource species considered. -Fishing effort.	-Fishing effort adjustment.	🍃 LB on resources biomass. $ Economic profitability for all agents. $# LB on fishing effort level.	C ∞	No
[78] 2014	1) Single agent single species age structured fishery. 2) Uses VP to study and compare the effort based and quota based fishing strategies. 3) NA on the Chilean Jack-marckerel fishery.	-Biomass stock of the resource at each age class and spawning stock biomass level.	-Fishing effort.	🍃 LB on the spawning stock biomass indicator (SSB). $# LB on fishery yield. $# LB on fishing activity level.	D F	SA
[90] 2015	1) Single agent two species exploited ecosystem. 2) Uses RVK to study the effect of different types of uncertainty on the sustainability of the system. 3) NA on the anchovy-hake couple in the Peruvian upwelling ecosystem.	-Biomass of species.	-Harvesting effort for each species.	🍃 LB on each species biomass. $ LB on catch level for both species.	D F	RA
[60] 2015	1) Single agent multispecies sex-structured fishery in an ecosystem composed of 4 species (3 targeted and one non fished species). 2) Uses SVK to compare different management strategies and harvesting efforts allocations. 3) NA on the Australian northern prawn fishery.	-Biomass stock of each species.	-Harvesting effort for each targeted specie. -Fishing management strategy.	🍃 LB on spawning stock for all species (targeted or not). $ LB on annual net benefit from fishing.	D F	SA
[38] 2015	1) Multi-agent multispecies small scale fishery. 2) Uses VT modelling framework combined with simulation to study the sustainability of the system under three fishing scenarios. 3) NA on the French Guiana's coastal fishery.	-Biomass level of each species.	-Fishing effort of the fleets.	🍃 LB on the Species richness indicator (SR) (biodiversity). 🍃 LB on the trophic marine index indicator (MTI). $ LB on harvest level (food security). $ Positive profit for each fleet.	D F	SA
[29] 2016	1) a-Multispecies population growth model. b-Single agent multi-species by catch fishery model. 2) Proposes a VK algorithme and applies it the models. 3)NA on chosen parameters' values.	a-Size of each species populations and their evolution rate. b-Biomass of each species.	a-None (RS) The regulon is the evolution rate velocity. b-Fishing effort of the targeted species.	model a: 🍃 ULB on the populations sizes. model b: 🍃 LB on species biomass. $ ULB on fishing effort. $ LB on fishery profit. $# UB on fishing effort variation.	D F	No

Ecosystems and population biology

[69] 1991	1) Food web in an ecosystem composed of n species. 2) Proposes a VT model of population biology studies its sustainability using the G-projection method. 3) No NA.	-Biomass of each species.	-None (RS). The regulons are the choice of resource used by each species to feed.	🍃 Preservation of all species.	C F	No
[23] 1993	1) Boserupian system for population growth. 2) Uses VK to study the properties of the system subject to the possible technological changes. (For different bounds on the technological changes). 3) No NA.	-Population size. -Level of technological advance.	- None (RS). The regulon is the level of technological change (Velocity of technological evolution).	-ULB on technological changes.	C ∞	No
[70] 1995	1) Prey predator ecosystem composed of two areas where live one predator and two pray species. 2) shows how the viability theory is useful to model the interactions between populations competing for space in presence of uncertainties in an ecosystem. 3) No NA.	-Abundance of each type of population.	- None (RS) The regulons are the fractions of predator population in each area of the system and strategies of the populations.	🍃 Space limitation.	C F	RA
[27] 1997	1) Prey predator system with one predator and one prey. 2) Uses VK and associated feedbacks to study the sustainability of the system according to the preservation objectives (Preservation of one or both species). 3) NA on chosen data values.	-The density of the prey and predator species.	-None (RS) The regulons are the species' survival strategies.	🍃 LB on density of one or other species (accordingly to the considered situation).	C ∞	No
[71] 1998	1) Single species extinction problem. 2) Uses VT modelling to study the extinction possibilities of the population and estimate its extinction time. 3) NA on the grizzly-bear female population in the Yellowstone National Park.	-Abundance of the population.	None (RS). The regulon is the growth rate of the population (The uncertainty).	🍃 Reaching the extinction threshold at the final time. (endangered species: those which will reach their extinction threshold in finite time.)	C F	RA
[24] 1999	1) Explains how some game theory models of population growth and fishery can be reinterpreted trough application of viability theory. Model a) Bassori population-cattle interaction. Model b) Norwegian fishery: Multiagent fishery. 2)Uses VT modelling and VK to study the sustainability of the systems. 3) NA on predefined parameter values.	Model a: -Households' sizes in the Bassouri nomad organisation. -Households' herd sizes. Model b: -Agents' capital. -Agents' possible catches. -Probability of bad catching level.	Model a: -Sedentarization rate. -Predation rate. Model b: -Level of risk taking (probability of acting individually and not following the group).	Model a: 🍃 Food security of the population. 🍃 ULB on the predation levels. # ULB on the sedentarisation levels. Model b: $ UB on each agent's ruin probability.	C ∞	No
[25] 2003	1) Prey predator ecosystem with one predator and one prey species. 2) Studies the effect of additive and multiplicative viability multipliers on the viability of the system. 3) No NA.	-Density of the predator and prey species.	-None (SS). Looks for states ensuring sustainability trough natural equilibrium of the system.	🍃 Non-extinction of both predator and prey species.	C ∞	No

[28] 2005	1) Multispecies ecosystem composed of a 3-level food chain (prey, predator and super predator species). 2) Uses VK to study the sustainability conditions of the system. 3) NA with chosen parameters' values.	-Density of each species.	-None (RS) the regulons are the predation and competition strategies of the different species.	LB on each species density (Non extinction of the species).	C ∞	No
[9] 2007	1) Malthus population growth model. 2) Explains how to use VT and its tools to study renewable resources management problems in general with illustrations on population growth model. 3) No NA.	-Population size. Population's growth rate.	-None (RS) the regulon is the growth rate variation.	ULB on population size. -ULB on growth rate variation.	C ∞	No
[44] 2007	1) Single agent single species age structured population growth system. 2) Exploits the monotonicity properties to estimate the VK and study the sustainability of the system. 3) No NA.	-Abundance of the population at each age.	-Harvesting level.	LB on the population's abundance level at each age. $ LB on harvest level.	D ∞	No
[10] 2007	1) Verhulst model for population dynamics. 2) Illustrates the main concepts of VT by revisiting the Verhulst type models for population dynamics. 3) No NA.	-Stock level of the resource.	-None(RS). The regulon is the growth rate.	ULB on the resource stock.	C ∞	No
[18] 2008	1) Ecosystem with multiple species competing for one resource. 2) Uses VP to study the sustainability of the system in case of non-exploitation (No harvesting and without the economic constraint) and in the case of exploitation. 3) NA on chosen parameter's values.	-Abundance of each species. -Resource level.	-Harvesting intensity for each species.	LB on the Shannon biodiversity index. $ LB on the utility derived from the exploitation activity.	D F	SA
[92] 2014	1) Single species population growth model. 2) Uses SVK to study the sustainability and resilience of the system. 3) NA on chosen parameters' values.	-Population density. -Growth coefficient.	-None(RS). The regulon is the changes in growth coefficient.	ULB on population density level.	D F	SA
[36] 2015	1) Single species age structured population. 2) Explains how VK and CB can be useful to study the sustainability of the system and management strategies. 3) No NA.	-Abundance of the population at each age.	-Harvesting strategies (harvest for each age class).	LB on the total population abundance level.	D F	No
Renewable resources in general						
[89] 2006	1) Single agent single age-structured renewable resource with one mature harvestable age. 2) Uses VK and associated feedbacks to study the sustainability of the system and the harvesting strategies. 3) NA on chosen parameters' values.	-Available resource quantities at each age.	-Harvesting quantity.	$# LB on harvest at each time.	D ∞	No
[49] 2012	1) Multi-agent single renewable resource harvest system in presence of cooperation between the agent. 2) Uses VK to analyse the conditions under which cooperation promotes the sustainability of the system. 3) No NA.	-Resource stock level.	-Harvesting effort for agents inside and outside the coalition.	LB on the resource stock. $ Positive rent for each agent within the coalition.	D F	No

[14] 2013	1) Single agent multispecies renewable harvest system. 2) Uses RVK to study the sustainability of the system. 3) NA with chosen data values.	-Stock resource of the species. -The global harvest.	-None (RS) The regulon is the share of each species in the global harvest.	🍃 LB on the stock resource of the species. $ Satisfy the total and by species harvest demand.	C F	RA
Farming and agro-ecological systems						
[102] 2004	1) Single agent mixed herd composed of two species (Ilama and Sheep). 2) Uses RVK to study the sustainability of the system. 3) NA on the Bolivian highlands Ilama-sheep mixed herd.	-Wealth of the owners of the herds.	-Breeds management decisions: (rate of female's offtake and herd composition)	$ LB on income and wealth level at any time.	C ∞	RA
[103] 2007	1) Single agent grassland ecosystem which is the breeding habitat of 3 wader species and feeding resource 2 species suckling cattle (cow/calves). 2) Uses VK to study the sustainability of the system and the efficiency of different grazing strategies. 3) NA on measured and estimated data from European grasslands.	-Grass mass (live grass and standing dead grass).	-Grazing intencity.	🍃 Suitable sward state for the reproduction of the birds during their breeding period. $# Satisfy the cattle feeding requirement.	D F	No
[15] 2009	1) Single agent livestock grazing management system in semi-arid rangelands. 2) Uses VP to study the sustainability of the system and the management strategies. 3) NA on chosen parameters' values.	-Grass biomass (reserve and green biomass). -Herd size.	-grazing management strategy.	🍃 LB on grass biomass level. $ LB on income.	C F	SA
[93] 2010	1) Single agent grassland ecosystem with 2 wader species and cattle. 2)Uses VK to study the sustainability of the system and grazing practices as well as the effect of grazing on the conservation of wader species. 3) NA on the Ouest-du-Lay march (France).	-Biomass of grass (Alive and dead grass).	-Grazing intensity (cattle density and grazing rhythm).	🍃 Suitable sward state for the reproduction of the birds during their breeding period. 🍃 Eggs survival (UB on cattle density to limit the trampling impact on eggs). $# Satisfaction of cattle feeding requirement.	D F	RA
[75] 2011	1) Rangeland management model. 2) Uses VK, CB and associated feedbacks to study the sustainability and resilience of the system. 3) NA using parameters' values from the literature.	-Grass biomass	-grazing pressure.	🍃 LB on grass biomass. $ LB on grazing pressure.	D ∞	No
[94] 2012	1) Single agent grassland ecosystem. 2) Uses VK and associated feedbacks to study the sustainability of the system under different ecological constraints and determine the sustainable management strategies. 3) Application to the conservation of lapwing birds in the wet grasslands in France.	-Grass biomass (Live and standing grass). -Bird population size.	-Timing and intensity of grazing.	🍃 Conservation of the bird population (several constraints studied). $ Satisfaction of cattle feeding requirements. $ LB on productivity level (LB on grazing time)	D F	No
[83] 2013	1) Multispecies agro-ecological ecosystem. 2) Uses VP to identify sustainable management scenarios. 3) NA on bird population in small agricultural regions in metropolitan France.	-Abundance of each bird species at each region.	-Incentives (Subsides and taxes) to encourage specific crop or grass activities in the different agricultural regions.	🍃 LB on 3 biodiversity indicators. $ LB on income from the farming activities. $ LB on the budget allocated for farming activities.	D F	SA

[95] 2015	1) Single agent grassland agro system. 2) Uses RVK and associated feedbacks to study the sustainability of the system and of management strategies 3) NA on the cool-season grassland of south-central Wisconsin (USA).	-Grass biomass. -Production of the grassland system.	-Stocking rate and grazing sequences.	Satisfy the cattle daily needs of grass. $ LB on the system's production level.	D F	RA
[53] 2015	1) Single agent single parcel agro ecological system. 2) Uses CB to study the possibility of restoring the soil quality within the time horizon while maintaining acceptable economic performance. 3) NA on French West Endies.	-Soil quality indicator. -Cash balance.	-Agricultural strategy (planting sequences, agricultural activity and techniques).	Bring back the soil quality indicator to an acceptable level at the end of the exploitation period. (Target) $ Positive cash balance at any time.	D F	No
Others						
[74] 2004	1) Single agent lake eutrophication model. 2) Uses VK and TCF to study the sustainability of the system and find the best management strategies. 3) NA with parameters' values from the literature.	-Phosphorus quantity in water. -Annual phosphorus input from human activity.	-Variation of the annual phosphorus input.	UB on phosphorus quantity in water. UB on the phosphorus total input from human activity. $ LB on the total input level (LB on activity level).	C F	No
[105] 2013	1) Single agent socio-ecological tourism based system. 2) Uses VK to identify the sustainable situations, then uses CB calculated for different time horizons to estimate the required time to reach a sustainable state. 3) NA on chosen parameters' values.	-Tourist activity. -Quality of nature. -Capital (infrastructure)	-Investments in tourism. -Advertisement campaigns (to control the effect of competition).	ULB on the nature quality level. $ ULB on the tourism activity level. $ ULB on the capital value.	C ∞	No
[91] 2013	1) Single agent lake eutrophication model. 2) Uses SVK to study the sustainability and resilience of the system. 3) NA with parameters' values from the literature.	-Quantity of phosphorus in water. -Annual phosphorus input from human activity.	-Variation of the annual phosphorus input.	UB on phosphorus quantity in water. UB on the phosphorus total input. $ LB on the total input level (LB on activity level).	C F	SA
[1] 2015	1) Single hydroelectric dam under uncertainty and tourism constraints. 2) Uses VP to study the system's management strategies. 3) NA on data provided by the French electricity provider Electricité France.	-Water storage in the dam. -Dam inflow. -Electricity price	-Dam turbined flow.	LB on the guaranteed gain from the electricity production. $# LB on water storage level during the tourism season.	D F	SA

Features of the retained papers in the survey

References

[1] J-C Alais, P Carpentier, and M De Lara. Multi-usage hydropower single dam management: chance-constrained optimization and stochastic viability. *Energy Systems*, pages 1–24, 2015.

[2] R Allen et al. *World conservation strategy. Living resource conservation for sustainable development.* International Union for Conservation of Nature and Natural Resources., 1980.

[3] P Andrés-Domenech, P Saint-Pierre, and G Zaccour. Forest conservation and co2 emissions: A viable approach. *Environmental Modeling & Assessment*, 16(6):519–539, 2011.

[4] P Andrés-Domenech, P Saint-Pierre, P.S Fanokoa, and G Zaccour. Sustainability of the dry forest in androy: A viability analysis. *Ecological Economics*, 104:33–49, 2014.

[5] J-P Aubin. A survey of viability theory. *SIAM Journal on Control and Optimization*, 28(4): 749–788, 1990.

[6] J-P Aubin. *Viability theory with 14 illustrations.* Springer Science & Business Media, 1991.

[7] J-P Aubin. Viability theory. systems & control: Foundations & applications. *Birkhäuser, Boston. doi*, 10(1007):978–0, 1991.

[8] J-P Aubin. Une approche viabiliste du couplage des systèmes climatique et économique. *Natures Sciences Sociétés*, 18(3):277–286, 2010.

[9] J-P Aubin and P Saint-Pierre. An introduction to viability theory and management of renewable resources. *Decision Making and Risk Management in Sustainability Science*, pages 43–80, 2007.

[10] J-P Aubin and P Saint-Pierre. An introduction to viability theory and management of renewable resources. *Decision Making and Risk Management in Sustainability Science*, pages 43–80, 2007.

[11] J-P Aubin, T Bernardo, and P Saint-Pierre. A viability approach to global climate change issues. In Alain Haurie and Viguier Laurent, editors, *The Coupling of Climate and Economic Dynamics: Essays on integrated assessment*, chapter 5, pages 113–143. Springer, Notherland, 2005.

[12] J-P Aubin, A.M Bayen, and P Saint-Pierre. *Viability theory: new directions.* Springer Science & Business Media, 2011.

[13] J-P Aubin, L Chen, and M-H Durand. Dynamical allocation method of emission rights of pollutants by viability constraints under tychastic uncertainty. *Environmental Modeling & Assessment*, 17(1-2):7–18, 2012.

[14] J-P Aubin, L Chen, and M-H Durand. Dynamic decentralization of harvesting constraints in the management of tychastic evolution of renewable resources. *Computational Management Science*, 10(4):281–298, 2013.

[15] S Baumgärtner and M.F Quaas. Ecological-economic viability as a criterion of strong sustainability under uncertainty. *Ecological Economics*, 68(7):2008–2020, 2009.

[16] T BenDor, J Scheffran, and B Hannon. Ecological and economic sustainability in fishery management: a multi-agent model for understanding competition and cooperation. *Ecological Economics*, 68(4):1061–1073, 2009.

[17] C Béné and L Doyen. Storage and viability of a fishery with resource and market dephased seasonalities. *Environmental and Resource Economics*, 15(1):1–26, 2000.

[18] C Béné and L Doyen. Contribution values of biodiversity to ecosystem performances: A viability perspective. *Ecological Economics*, 68(1):14–23, 2008.

[19] C Béné, L Doyen, and D Gabay. A viability analysis for a bio-economic model. *Ecological Economics*, 36(3):385–396, 2001.

[20] C Bernard and S Martin. Comparing the sustainability of different action policy possibilities: Application to the issue of both household survival and forest preservation in the corridor of fianarantsoa. *Mathematical Biosciences*, 245(2):322–330, 2013.

[21] T Bernardo and P Saint-Pierre. Evaluation climate change impact and efficient threshold guardrails using adapted viability tools. To apear.

[22] D.P Bertsekas. *Dynamic programming and optimal control, Vol II: Approximate dynamic programming*. Athena Scientific., 2012.

[23] N Bonneuil. Malthus, boserup and population viability. *Mathematical population studies*, 5(1):107–119, 1994.

[24] N Bonneuil. Games, equilibria and population regulation under viability constraints: An interpretation of the work of the anthropologist fredrik barth. *Population: An English Selection*, pages 151–179, 1998.

[25] N Bonneuil. Making ecosystem models viable. *Bulletin of Mathematical Biology*, 65(6):1081–1094, 2003.

[26] N Bonneuil. Computing the viability kernel in large state dimension. *Journal of Mathematical Analysis and Applications*, 323(2):1444–1454, 2006.

[27] N Bonneuil and K Müllers. Viable populations in a prey–predator system. *Journal of Mathematical Biology*, 35(3):261–293, 1997.

[28] N Bonneuil and P Saint-Pierre. Population viability in three trophic-level food chains. *Applied mathematics and computation*, 169(2):1086–1105, 2005.

[29] A Brias, J-D Mathias, and G Deffuant. Accelerating viability kernel computation with cuda architecture: application to bycatch fishery management. *Computational Management Science*, 13(3):371–391, 2016.

[30] G Brundtland, M Khalid, S Agnelli, S Al-Athel, B Chidzero, L Fadika, V Hauff, I Lang, M Shijun, M.M de Botero, et al. Our common future (\'brundtland report\'). 1987.

[31] J.M Calabrese, G Deffuant, and V Grimm. Bridging the gap between computational models and viability based resilience in savanna ecosystems. In *Viability and Resilience of Complex Systems*, pages 107–130. Springer, 2011.

[32] P Cardaliaguet and S Plaskacz. Invariant solutions of differential games and hamilton-jacobi-isaacs equations for time-measurable hamiltonians. *SIAM Journal on Control and Optimization*, 38(5):1501–1520, 2000.

[33] P Cardaliaguet, M Quincampoix, and P Saint-Pierre. *Set-valued numerical analysis for optimal control and differential games*, pages 177–247. Birkhäuser Boston, 1999. ISBN 978-1-4612-1592-9. URL http://dx.doi.org/10.1007/978-1-4612-1592-9_4.

[34] P Cardaliaguet, M Quincampoix, and P Saint-Pierre. Pursuit differential games with state constraints. *SIAM Journal on Control and Optimization*, 39(5):1615–1632, 2000.

[35] L Chapel, G Deffuant, S Martin, and C Mullon. Defining yield policies in a viability approach. *Ecological Modelling*, 212(1):10–15, 2008.

[36] J-P Chavas. Dynamics, viability, and resilience in bioeconomics. *Annu. Rev. Resour. Econ.*, 7(1):209–231, 2015.

[37] AA Cissé, S Gourguet, L Doyen, F Blanchard, and J-C Péreau. A bio-economic model for the ecosystem-based management of the coastal fishery in french guiana. *Environment and Development Economics*, 18(03):245–269, 2013.

[38] AA Cisse, L Doyen, F Blanchard, C Béné, and J-C Péreau. Ecoviability for small-scale fisheries in the context of food security constraints. *Ecological Economics*, 119:39–52, 2015.

[39] R Curtin and V Martinet. Viability of transboundary fisheries and international quota allocation: The case of the bay of biscay anchovy. *Canadian Journal of Agricultural Economics/Revue canadienne d'agroeconomie*, 61(2):259–282, 2013.

[40] PH.M Cury, C Mullon, Garcia. S.M, and L.J Shannon. Viability theory for an ecosystem approach to fisheries. *ICES Journal of Marine Science: Journal du Conseil*, 62(3):577–584, 2005.

[41] M De Lara and L Doyen. *Sustainable management of natural resources: mathematical models and methods*. Springer Science & Business Media, 2008.

[42] M De Lara and V Martinet. Multi-criteria dynamic decision under uncertainty: A stochastic viability analysis and an application to sustainable fishery management. *Mathematical Biosciences*, 217(2):118–124, 2009.

[43] M De Lara, L Doyen, T Guilbaud, and M-J Rochet. Is a management framework based on spawning-stock biomass indicators sustainable? a viability approach. *ICES Journal of Marine Science: Journal du Conseil*, 64(4):761–767, 2007.

[44] M De Lara, L Doyen, T Guilbaud, and M-J Rochet. Monotonicity properties for the viable control of discrete-time systems. *Systems & control letters*, 56(4):296–302, 2007.

[45] M De Lara, P Gajardo, and H Ramírez. Viable states for monotone harvest models. *Systems & Control Letters*, 60(3):192–197, 2011.

[46] M De Lara, E Ocaña, R Oliveros-Ramos, and J Tam. Ecosystem viable yields. *Environmental Modeling & Assessment*, 17(6):565–575, 2012.

[47] G Deffuant, L Chapel, and S Martin. Approximating viability kernels with support vector machines. *IEEE transactions on automatic control*, 52(5):933–937, 2007.

[48] L Doyen and C Béné. Sustainability of fisheries through marine reserves: a robust modeling analysis. *Journal of Environmental Management*, 69(1):1–13, 2003.

[49] L Doyen and J-C Péreau. Sustainable coalitions in the commons. *Mathematical Social Sciences*, 63(1):57–64, 2012.

[50] L Doyen and P Saint-Pierre. Scale of viability and minimal time of crisis. *Set-Valued Analysis*, 5(3):227–246, 1997.

[51] L Doyen, O Thébaud, C Béné, V Martinet, S Gourguet, M Bertignac, S Fifas, and F Blanchard. A stochastic viability approach to ecosystem-based fisheries management. *Ecological Economics*, 75:32–42, 2012.

[52] L Doyen, A Cisse, S Gourguet, L Mouysset, P-Y Hardy, C Béné, F Blanchard, F Jiguet, J-C Pereau, and O Thébaud. Ecological-economic modelling for the sustainable management of biodiversity. *Computational Management Science*, 10(4):353–364, 2013.

[53] M-H Durand, A Desilles, P Saint-Pierre, V Angeon, and H Ozier-Lafontaine. Viability modeling of agro-ecological transition: enhancing the provision of soil ecosystem services to support farm sustainability in small island developing states. To apear.

[54] K Eisenack. Qualitative viability analysis of a bio-socio-economic system. In *Working Papers of 17th Workshop on Qualitative Reasoning. Brasilia*, pages 63–70. Citeseer, 2003.

[55] K Eisenack and J Kropp. Formal viability analysis of participatory fisheries management. 2006.

[56] K Eisenack, J Scheffran, and J.P Kropp. Viability analysis of management frameworks for fisheries. *Environmental Modeling & Assessment*, 11(1):69–79, 2006.

[57] FAO. Food and agriculture organization of the united nations, 2010.

[58] A Ferchichi, M Jerry, and S Ben Miled. Viability analysis of fisheries management on hermaphrodite population. *Acta biotheoretica*, 62(3):355–369, 2014.

[59] S Gourguet, C Macher, L Doyen, O Thébaud, M Bertignac, and O Guyader. Managing mixed fisheries for bio-economic viability. *Fisheries Research*, 140:46–62, 2013.

[60] S Gourguet, O Thébaud, S Jennings, L.R Little, C.M Dichmont, S Pascoe, R.A Deng, and L Doyen. The cost of co-viability in the australian northern prawn fishery. *Environmental Modeling & Assessment*, pages 1–19, 2015.

[61] G Haddad. Monotone viable trajectories for functional differential inclusions. *Journal of Differential Equations*, 42(1):1–24, 1981.

[62] P-Y Hardy, C Béné, L Doyen, and A-M Schwarz. Food security versus environment conservation: A case study of solomon islands' small-scale fisheries. *Environmental Development*, 8:38–56, 2013.

[63] P.Y Hardy, C Béné, L Doyen, J-C Pereau, D Miles, et al. Viability and resilience of small-scale fisheries through cooperative arrangements. Technical report, Groupe de Recherche en Economie Théorique et Appliquée, 2013.

[64] A Haurie, M Tavoni, and B.CC Van der Zwaan. Modeling uncertainty and the economics of climate change: recommendations for robust energy policy. *Environmental Modeling and Assessment*, 17(1):1–5, 2012.

[65] A Jerry, Mand Rapaport and P Cartigny. Can protected areas potentially enlarge viability domains for harvesting management? *Nonlinear Analysis: Real World Applications*, 11(2): 720–734, 2010.

[66] C Jerry and N Raissi. Optimal exploitation for a commercial fishing model. *Acta biotheoretica*, 60(1-2):209–223, 2012.

[67] J.B Krawczyk and A.S Pharo. Viability kernel approximation, analysis ans simulation application - vikaasa manual. 2011.

[68] J.B Krawczyk, A Pharo, O.S Serea, and S Sinclair. Computation of viability kernels: A case study of by-catch fisheries. *Computational Management Science*, 10(4):365–396, 2013.

[69] V Křivan. Construction of population growth equations in the presence of viability constraints. *Journal of Mathematical Biology*, 29(4):379–387, 1991.

[70] V Křivan. Differential inclusions as a methodology tool in population biology. In *Proceedings of the 9th European Simulation Multiconference"(M. Snorek, M. Sujansky, and A. Verbraeck, Eds.)*, page 544, 1995.

[71] V Křivan and G Colombo. A non-stochastic approach for modeling uncertainty in population dynamics. *Bulletin of Mathematical Biology*, 60(4):721–751, 1998.

[72] D Lercari and F Arreguín-Sánchez. An ecosystem modelling approach to deriving viable harvest strategies for multispecies management of the northern gulf of california. *Aquatic Conservation: Marine and Freshwater Ecosystems*, 19(4):384–397, 2009.

[73] J.N Maidens, S Kaynama, I.M Mitchell, M.K Oishi, and G.A Dumont. Lagrangian methods for approximating the viability kernel in high-dimensional systems. *Automatica*, 49(7): 2017–2029, 2013.

[74] S Martin. The cost of restoration as a way of defining resilience: a viability approach applied to a model of lake eutrophication. *Ecology and Society*, 9(2):8, 2004.

[75] S Martin, G Deffuant, and J.M Calabrese. Defining resilience mathematically: from attractors to viability. In *Viability and Resilience of Complex Systems*, pages 15–36. Springer, 2011.

[76] V Martinet and F Blanchard. Fishery externalities and biodiversity: Trade-offs between the viability of shrimp trawling and the conservation of frigatebirds in french guiana. *Ecological Economics*, 68(12):2960–2968, 2009.

[77] V Martinet, O Thébaud, and A Rapaport. Hare or tortoise? trade-offs in recovering sustainable bioeconomic systems. *Environmental modeling & assessment*, 15(6):503–517, 2010.

[78] V Martinet, J Peña-Torres, M De Lara, and H Ramírez. Risk and sustainability: Assessing fishery management strategies. *Environmental and Resource Economics*, pages 1–25, 2014.

[79] J-D Mathias, B Bonté, T Cordonnier, and F de Morogues. Using the viability theory to assess the flexibility of forest managers under ecological intensification. *Environmental Management*, 56(5):1170–1183, 2015.

[80] F Maynou. Coviability analysis of western mediterranean fisheries under msy scenarios for 2020. *ICES Journal of Marine Science: Journal du Conseil*, 71(7):1563–1571, 2014.

[81] D.H Meadows, D.L Meadows, J Randers, and W.W Behrens. The limits to growth. *New York*, 102, 1972.

[82] J Monod. *Chance and Necessity: Essay on the Natural Philosophy of Modern Biology*. Vintage books, 1971.

[83] L Mouysset, L Doyen, and F Jiguet. From population viability analysis to coviability of farmland biodiversity and agriculture. *Conservation biology*, 28(1):187–201, 2013.

[84] C Mullon, PH Cury, and L Shannon. Viability model of trophic interactions in marine ecosystems. *Natural Resource Modeling*, 17(1):71–102, 2004.

[85] J-C Péreau, L Doyen, L.R Little, and O Thébaud. The triple bottom line: Meeting ecological, economic and social goals with individual transferable quotas. *Journal of Environmental Economics and Management*, 63(3):419–434, 2012.

[86] J Pezzey. Sustainable development concepts. *World*, 1:45, 1992.

[87] W.B Powell. *Approximate dynamic programming: Solving the curses of dimensionality, 2nd edition*. Wiley Series in Probability and Statistics, 2011.

[88] C Profile. Global forest resources assessment 2005. 2005.

[89] A Rapaport, J-PH Terreaux, and L Doyen. Viability analysis for the sustainable management of renewable resources. *Mathematical and computer modelling*, 43(5):466–484, 2006.

[90] E Regnier and M De Lara. Robust viable analysis of a harvested ecosystem model. *Environmental Modeling & Assessment*, 20(6):687–698, 2015.

[91] C Rougé, J-D Mathias, and G Deffuant. Extending the viability theory framework of resilience to uncertain dynamics, and application to lake eutrophication. *Ecological indicators*, 29:420–433, 2013.

[92] C Rougé, J-D Mathias, and G Deffuant. Relevance of control theory to design and maintenance problems in time-variant reliability: The case of stochastic viability. *Reliability Engineering & System Safety*, 132:250–260, 2014.

[93] R Sabatier, L Doyen, and M Tichit. Modelling trade-offs between livestock grazing and wader conservation in a grassland agroecosystem. *Ecological Modelling*, 221(9):1292–1300, 2010.

[94] R Sabatier, L Doyen, and M Tichit. Action versus result-oriented schemes in a grassland agroecosystem: A dynamic modelling approach. *PLoS ONE*, 7(4):1–12, 04 2012. doi: 10.1371/journal.pone.0033257. URL http://dx.doi.org/10.1371%2Fjournal.pone.0033257.

[95] R Sabatier, L.G Oates, and R.D Jackson. Management flexibility of a grassland agroecosystem: A modeling approach based on viability theory. *Agricultural Systems*, 139:76–81, 2015.

[96] P Saint-Pierre. Approximation of the viability kernel. *Applied Mathematics and Optimization*, 29(2):187–209, 1994.

[97] C Sanogo, S Ben Miled, and N Raissi. Viability analysis of multi-fishery. *Acta biotheoretica*, 60(1-2):189–207, 2012.

[98] C Sanogo, N Raïssi, S Ben Miled, and C Jerry. A viability analysis of fishery controlled by investment rate. *Acta biotheoretica*, 61(3):341–352, 2013.

[99] A Scientific Advisory Council on Global Change. Scenario for the derivation of global co2 reduction targets and implementation strategies: Statement on the occasion of the first conference of the parties to the framework convention on climate change in berlin; adopted at the 26th session of the council, 17th february 1995, dortmund, 1995.

[100] S.D.N Sinclair. A viability analysis of sustainable by-catch fisheries under direct conservation measures. 2014.

[101] O Thébaud, N Ellis, L.R Little, L Doyen, and R.J Marriott. Viability trade-offs in the evaluation of strategies to manage recreational fishing in a marine park. *Ecological Indicators*, 46:59–69, 2014.

[102] M Tichit, B Hubert, L Doyen, and D Genin. A viability model to assess the sustainability of mixed herds under climatic uncertainty. *Animal Research*, 53(5):405–417, 2004.

[103] M Tichit, L Doyen, J.Y Lemel, O Renault, and D Durant. A co-viability model of grazing and bird community management in farmland. *Ecological Modelling*, 206(3):277–293, 2007.

[104] W Von Bloh, C Bounama, K Eisenack, B Knopf, and O Walkenhorst. Estimating the biogenic enhancement factor of weathering using an inverse viability method. *Ecological Modelling*, 216(2):245–251, 2008.

[105] W Wei, I Alvarez, and S Martin. Sustainability analysis: Viability concepts to consider transient and asymptotical dynamics in socio-ecological tourism-based systems. *Ecological Modelling*, 251:103–113, 2013.

[106] J Weyant, O Davidson, H Dowlabathi, J Edmonds, M Grubb, E.A Parson, R Richels, J Rotmans, P.R Shukla, R.S.J Tol, et al. *Integrated assessment of climate change: an overview and comparison of approaches and results*. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 1996.