Welfare Measurement under Threats of Environmental Catastrophes

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Abstract

Welfare measures under threats of environmental catastrophes are studied using the "parable" apparatus of Weitzman and Löfgren [31]. The occurrence probability of the catastrophic events is driven (at least partly) by anthropogenic activities such as natural resource exploitation. Ecological systems are particularly vulnerable to sudden collapse because their nonlinear dynamics often give rise to instabilities, sensitivity to various thresholds and hysteresis phenomena. Without external effects, the green NNP is a genuine welfare measure vis-à-vis a particular parable economy. Often, however, the occurrence hazard constitutes a public bad, treated as an externality by agents who ignore their own contribution to its accumulation. In such cases the green NNP, although accounting for the event hazard rate per se, fails to properly internalize future effects on the hazard rate of current economic activities and as a result overestimates welfare. The bias term associated with the green NNP is derived and expressed in a simple and interpretable form.

Keywords: biodiversity; green NNP; ecological events; hazard rate; uncertainty

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1 Introduction

In a widely cited paper, Weitzman [26] has shown how to relate national accounting to a welfare measure at any given time. The method has since been extended and applied in a wide range of environmental related issues, including sustainability [27, 5, 7], technical change and economic growth [3, 27, 31], environmental policy [13, 19] and intergenerational equity [6]. The recent books of Weitzman [30] and Aronsson et al. [1] provide detailed accounts on this line of research. In this work we apply the green accounting methodology to situations involving threats of environmental catastrophes which are (at least partly) induced by anthropogenic activities. Examples of such catastrophic events include nuclear accidents [10, 2], global warming related calamities [16, 23, 14], pollution-induced events [9, 24], and biodiversity loss and species extinction [22, 25, 18].

The biodiversity example is particularly relevant to our considerations. Biodiversity loss is often induced by a sudden collapse of the ecosystem that shifts the underlying ecology from an existing, species-rich regime to a new, species-poor regime. This is so because ecosystems are inherently complex and their nonlinear dynamics often give rise to instabilities, sensitivity to various thresholds and hysteresis phenomena (see [20, 11, 4] and references they cite). A well-known example is the development of resistance in pests [15] and bacteria [17, 12] due to the intensive use of pesticides and antibiotics. The loss of biodiversity, in these cases, corresponds to the complete take-over by the immune variants, which constitutes a severe threat to agricultural production or to human health. Moreover, ecosystems are vulnerable to such events as habitat destruction by forest fires, disease outbreaks, or invading populations, which are genuinely stochastic in nature (see [25] and references cited therein). These circumstances give rise to discrete ecological events with uncertain occurrence conditions.

An event is classified as "catastrophic" when its abrupt occurrence inflicts a significant damage that should not be ignored in the national accounts. Moreover, the conditions that trigger the events are not completely understood or controlled, and the exact occurrence time cannot be predicted in advance. The nature of the events and the scale of the damage inflicted often render complete insurance schemes infeasible.

The presence of environmental threats has undesirable welfare implications and a question arises regarding whether these implications are captured by measures such as the green NNP. The answer depends on whether the market prices that underlie the NNP properly account for the environmental hazard, which in turn depends on whether the hazard involves external effects. Without external effects, the event risk is fully reflected by the market prices and the green NNP can be related to a stationary-equivalent welfare measure by means of the "parable" apparatus of Weitzman and Löfgren [31]. We show that the green NNP represents the stationary consumption rate of a "nonchalant" parable economy that, up to the occurrence date, consumes all of its income regardless of the event threat.

Biodiversity considerations, however, are often fraught with externalities, as individual agents tend to treat the hazard as a public bad. Indeed, the green accounting methodology is very explicit in requiring that accounting is "comprehensive", implying that no effect is left out, including external hazard effects (see [3, 27, 31] and references they cite for studies of green accounting with external effects in an economic growth context). When agents ignore their own contribution to hazard accumulation and the stocks of the mitigating environmental resources decrease over time, the green NNP overestimates welfare. This is so because the market prices of the hazard-mitigating assets, while reflecting the current hazard per se, fail to properly account for future changes in its rate due to current exploitation activities.

To focus attention on environmental hazards, we consider in Section 2 a simple economy with a single composite consumption good and two capital goods, of which one is the environmental capital affecting the probability of the event occurrence. (Extensions to more general settings, such as in [28, 29, 8], will not change the nature of the results.) In Section 3 we present the main theme of this work, which relates the green accounting methodology to the literature on catastrophic event uncertainty. A stationary-equivalent welfare measure under event uncertainty is defined and compared to the green NNP. The competitive case (of agents that treat the event hazard as an externality) and the socially optimal solution are considered and contrasted. The bias term associated with the competitive NNP is derived and expressed in a simple and interpretable form. Section 4 concludes.

2 The economy

Except for the modifications needed to account for the environmental events, we follow the formulation of Weitzman [26, 27] and Weitzman and Löfgren [31]. We consider an economy with a single composite consumption good, denoted C, and several capital assets, of which we single out a natural resource capital stock Q. The other capital stocks, denoted K, represent the traditional stocks used for production as well as other environmental assets. For the present purpose no generality is lost by considering a two-dimensional capital vector (K, Q). The constant (consumption) discount rate is denoted r.

The special role of the environmental stock Q is manifest via its effect on the hazard rate $h(Q)$ of abrupt occurrence of some detrimental event such that $h(Q)dt$ measures the conditional probability that the event will occur during $[t, t + dt]$ given that it has not occurred by time t when the resource stock is Q. Let T represent the random event-occurrence time with the probability distribution and density functions $F(t)$ and $f(t)$, respectively. For a given $Q(t)$ process, the hazard $h(Q(t))$ is related to the distribution $F(t)$ according to $h(Q(t)) = f(t)/(1 - F(t)) = -d[\ln(1 - F(t))]/dt$, yielding

$$F(t) = 1 - e^{-\Omega(t)} \quad \text{and} \quad f(t) = h(Q(t))e^{-\Omega(t)}, \tag{2.1}$$

where

$$\Omega(t) = \int_0^t h(Q(\tau))d\tau. \tag{2.2}$$

Let I_K and I_Q represent the net investment rates in K and Q, respectively,

$$\dot{K}(t) = I_K(t) \quad \text{and} \quad \dot{Q}(t) = I_Q(t). \tag{2.3}$$

Given the capital stocks (K, Q), consumption-investment decisions are constrained to the convex production possibilities set $S(K, Q)$, i.e., the combination (C, I_K, I_Q) is feasible if

$$(C, I_K, I_Q) \in S(K, Q). \tag{2.4}$$

Let $\varphi(K,Q)$ denote the post-event value function, representing the maximal present value of all future consumption streams from the occurrence time T onward discounted to time T, given that $K = K(T)$ and $Q = Q(T)$. At time t prior to occurrence, a feasible consumption-investment policy $\{C(\tau),\ I_K(\tau),\ I_Q(\tau),\ \tau \geq t\}$ gives rise to the expected present value (discounted to time t)

$$E_T\{\int_t^T C(\tau)e^{-r(\tau-t)}d\tau + e^{-r(T-t)}\varphi(K(T),Q(T))|T>t\} =$$
$$\int_t^\infty \{C(\tau)\frac{1-F(\tau)}{1-F(t)} + \frac{f(\tau)}{1-F(t)}\varphi(K(\tau),Q(\tau))\}e^{-r(\tau-t)}d\tau =$$
$$\int_t^\infty \{C(\tau) + h(Q(\tau))\varphi(K(\tau),Q(\tau))\}e^{-R(\tau,\,t)}d\tau,$$

where

$$R(\tau,t) = r(\tau-t) + [\Omega(\tau) - \Omega(t)] = \int_t^\tau [r + h(Q(s))]ds \qquad (2.5)$$

and E_T denotes expectation with respect to the distribution of T. The maximal present value of all feasible consumption streams at time t is, therefore, given by

$$V(K,Q) = \max_{\{C(\tau),\,I_K(\tau),\,I_Q(\tau)\}} \int_t^\infty \{C(\tau) + h(Q(\tau))\varphi(K(\tau),Q(\tau))\}e^{-R(\tau,\,t)}d\tau \qquad (2.6)$$

subject to (2.3) and (2.4), given $K = K(t)$ and $Q = Q(t)$. Notice that $V(K,Q)$ depends on t only through the initial capital stocks $K(t)$ and $Q(t)$ (changing t and keeping the initial stocks fixed is equivalent to a mere shift of the origin of the time index). It is assumed that (2.6) admits a unique solution.

The damage function is defined as

$$\psi(K,Q) = V(K,Q) - \varphi(K,Q), \tag{2.7}$$

such that the expected loss associated with occurrence during $[t, t+dt]$ is $\psi(K(t), Q(t))h(Q(t))dt$. For example, a "doomsday" event that ceases all further economic activities entails $\varphi(K,Q) = 0$ and $\psi(K,Q) = V(K,Q)$. Recurrent events may destroy some appreciable amounts D_K and D_Q of the existing capital stocks, in which case $\varphi(K,Q) = V(K-D_K, Q-D_Q)$, or affect the production feasibility set S, changing the possibilities (and welfare) for post-event performance. Regardless of the exact specification, we assume that the damage function is known and sufficiently large to rule out the possibility of complete insurance coverage. When the extent of damage is also subject to uncertainty, we take ψ to represent its expected value.

Let P_K and P_Q be the current-value costate variables of K and Q, and define the functions

$$Y(K,Q,P_K,P_Q) = \max_{(C,\, I_K,\, I_Q)\in S(K,Q)} \{C + P_K I_K + P_Q I_Q\} \tag{2.8}$$

and

$$G(K,Q,P_K,P_Q) = Y(K,Q,P_K,P_Q) - h(Q)\psi(K,Q). \tag{2.9}$$

We now show that the following conditions hold along the optimal trajectory $\{K^*(\tau), Q^*(\tau), C^*(\tau), I_K^*(\tau), I_Q^*(\tau), P_K^*(\tau), P_Q^*(\tau)\}$ corresponding to (2.6):

$$Y(K^*,Q^*,P_K^*,P_Q^*) = C^* + P_K^* I_K^* + P_Q^* I_Q^*, \tag{2.10}$$

$$\dot{P}_K^*(\tau) - rP_K^*(\tau) = -\partial G/\partial K \tag{2.11}$$

and

$$\dot{P}_Q^*(\tau) - rP_Q^*(\tau) = -\partial G/\partial Q, \tag{2.12}$$

where the dot indicates derivative with respect to τ and the derivatives on the right-hand side of (2.11) and (2.12) are evaluated at the optimal arguments.

Condition (2.11) can be rewritten, using the property $\partial V/\partial K = P_K^*$, as

$$\dot{P}_K^* - [r + h(Q^*)]P_K^* = -[\partial Y/\partial K + h(Q^*)\partial\varphi/\partial K], \tag{2.11*}$$

while condition (2.12) assumes different forms depending on whether the effect of Q on the hazard $h(Q)$ is internalized. In the competitive case, agents consider the environmental hazard as a "public bad". Thus they ignore their own effect on Q and treat the hazard as an exogenous function of time evolving along the equilibrium path $h^*(\tau) = h(Q^*(\tau))$. In this case, $\partial G/\partial Q = \partial Y/\partial Q + h^*(\tau)\partial\varphi/\partial Q - h^*(\tau)P_Q^*$, yielding

$$\dot{P}_Q^* - [r + h^*(\tau)]P_Q^* = -[\partial Y/\partial Q + h^*(\tau)\partial\varphi/\partial Q], \tag{2.12*}$$

where we use the property $\partial V/\partial Q = P_Q^*$. In contrast, a socially optimal behavior is obtained when agents are induced to account for the hazard effects of their own resource exploitation by standard regulatory instruments (such as Pigouvian taxes and subsidies or pollution permits). In this case, the effect of Q on h is internalized, $\partial G/\partial Q = \partial Y/\partial Q + h(Q)\partial\varphi/\partial Q - h(Q)P_Q - h'(Q)\psi$ and (2.12) becomes

$$\dot{P}_Q^* - [r + h(Q^*)]P_Q^* = -[\partial Y/\partial Q + h(Q^*)\partial\varphi/\partial Q - h'(Q^*)\psi]. \tag{2.12**}$$

The last term on the right-hand-side of (2.12**) implies a different evolution of the price P_Q^* relative to the competitive evolution corresponding to (2.12*). The significance of this distinction to the welfare interpretation of the NNP is considered in the following section.

To verify that (2.10), (2.11*) and (2.12*) are necessary conditions for a solution of (2.6) when $h(\tau) = h(Q(\tau))$ and $\Omega(\tau)$ are treated as exogenous functions of time, notice that, with the effective discount rate $r + h(\tau)$, the current-value Hamiltonian is $C + h(\tau)\varphi + P_K I_K + P_Q I_Q$ and the conditions follow from the Maximum Principle.

When the effect of Q on h is internalized, we treat Ω as a third state variable satisfying $d\Omega/d\tau = h(Q)$ (c.f. (2.2)) with the initial value $\Omega(t)$, and define the present-value (discounted to time t) Hamiltonian

$$H = [C + h(Q)\varphi(K, Q)]e^{-R(\tau,\, t)} + \lambda_K I_K + \lambda_Q I_q + \mu h(Q), \tag{2.13}$$

where $\lambda_K,\ \lambda_Q$ and μ are the present-value costates of $K,\ Q$ and Ω, respectively. Noting (2.5), $d\mu/ds = -\partial H/\partial\Omega = [C(s)+h(Q(s))\varphi(K(s), Q(s))]e^{-R(s,\, t)}$. Integrating from τ to ∞ along the optimal path and using the transversality condition $\lim_{s\to\infty} \mu(s) = 0$, gives $\mu^*(\tau) = -V(K^*(\tau)), Q^*(\tau))e^{-R^*(\tau,\, t)}$. With the current-value costates $P_K(\tau) = \lambda_K(\tau)e^{R^*(\tau,\, t)}$ and $P_Q(\tau) = \lambda_Q(\tau)e^{R^*(\tau,\, t)}$, conditions (2.10), (2.11*) and (2.12**) follow from the Maximum Principle.

It is instructive to consider the equivalent formulations (2.11) and (2.11*) (or (2.12) and (2.12*)-(2.12**)) under two extreme specifications of the damage function. For "doomsday" events φ vanishes and (2.11*) shows the main effect of the hazard in increasing the effective rate of discount. When the damage is vanishingly small, G reduces to Y (c.f. (2.9)) and (2.11)-(2.12) reduce to the conditions corresponding to an event-free economy.

In fact, equations (2.11) and (2.12) describe the arbitrage conditions in an economy operating under the occurrence hazard $h(Q)$. To see this, suppose that at time t prior to occurrence, the economy owns the capital stocks

(K, Q). With these stocks, the consumption equivalent of $Y(K, Q, P_K(t), P_Q(t))\delta$ can be produced during an infinitesimal time interval $[t, t+\delta]$. With probability $1 - h(Q)\delta$ the event will not occur during the interval, leaving the value $V(K, Q)e^{-r\delta}$. With probability $h(Q)\delta$ the event occurs, leaving the value $\varphi(K, Q)e^{-r\delta}$. The expected value at time $t + \delta$ (discounted to time t) is thus

$$Y(K, Q, P_K(t), P_Q(t))\delta + V(K, Q)e^{-r\delta} - h(Q)\delta e^{-r\delta}\psi(K, Q).$$

Alternatively, $P_Q(t)\varepsilon$ units of consumption can be traded for ε units of the environmental stock, producing $Y(K, Q + \varepsilon, P_K(t), P_Q(t))\delta$ during the same short interval, and possessing a stock worth $V(K, Q+\varepsilon)]e^{-r\delta}$ with probability $1-h(Q+\varepsilon)\delta$ that the event will not occur during $[t, t+\delta]$, or $\varphi(K, Q+\varepsilon)e^{-r\delta}$ with probability $h(Q + \varepsilon)\delta$ of occurrence. The discounted expected value in this case is

$$\begin{aligned} -P_Q(t)\varepsilon + Y(K, Q + \varepsilon, P_K(t), P_Q(t))\delta \quad &+ \quad V(K, Q + \varepsilon)e^{-r\delta} \\ &- \quad h(Q + \varepsilon)\delta e^{-r\delta}\psi(K, Q + \varepsilon). \end{aligned}$$

Equilibrium conditions correspond to indifference between these two options. At time $t + \delta$, $\partial V/\partial Q = P_Q(t + \delta)$, and the arbitrage condition becomes

$$-P_Q(t)\varepsilon + \frac{\partial Y}{\partial Q}\delta\varepsilon + P_Q(t + \delta)\varepsilon e^{-r\delta} - \frac{\partial(h\psi)}{\partial Q}\varepsilon\delta e^{-r\delta} = 0.$$

With $e^{-r\delta} = 1 - r\delta + o(\delta)$, we obtain, after neglecting terms of order $o(\varepsilon\delta)$ and dividing by $\varepsilon\delta$, $\dot{P}_Q - rP_Q = -\partial G/\partial Q$, as stated in (2.12). Condition (2.11) can be verified in the same way.

We note again that the distinction between the competitive and socially optimal prices depends on whether or not account is taken of the dependence of h on Q in the arbitrage condition above. Having verified that the prices P_K^* and P_Q^* satisfy this condition, we identify $Y^*(t) \equiv Y(K^*(t), Q^*(t), P_K^*(t), P_Q^*(t))$ of (2.10) with the green NNP. We turn now to investigate the relation between the green NNP and a stationary-equivalent welfare index for the economy.

3 Welfare measurement

To define a stationary-equivalent welfare measure corresponding to the value $V(K(t), Q(t))$, we consider a "parable model" *à la* Weitzman and Löfgren [31] of a simplified stationary economy operating under event uncertainty and relate its stationary rate of consumption (prior to occurrence) to the welfare of the real economy. At some initial pre-event time t the (hypothetical) parable economy possesses the capital stocks $(K(t), Q(t))$ which allow to produce the net output Y^P and is under the risk of event occurrence at some future random time T. From time t to occurrence, all net output is consumed and the capital stocks are kept fixed at $(K(t), Q(t))$. Thus, the output and hazard rate also remain fixed at Y^P and $h = h(Q(t))$, respectively. Upon occurrence, the value $\varphi = \varphi(K(t), Q(t))$ is obtained. The expected present value of the nonchalant parable policy $C^P = Y^P$ is

$$\begin{aligned} V^P(K(t), Q(t)) &= E_T\{\int_t^T C^P e^{-r(\tau-t)} d\tau + e^{-r(T-t)}\varphi | T > t\} \\ &= \int_t^\infty [C^P + h\varphi] e^{-(r+h)(\tau-t)} d\tau = \frac{C^P + h\varphi}{r+h} \end{aligned} \quad (3.1)$$

(the adjective "nonchalant" is attached to the parable policy to reflect the property that all output is consumed regardless of the hovering event threat).

The parable consumption rate $C^P = Y^P$ constitutes a welfare index for the parable economy because it describes the stationary rate of consumption that can be supported prior to occurrence. To establish a link with the real economy, we look for the parable output (and consumption) C^P that gives rise to equal values for the two economies, i.e. $V^P(K(t), Q(t)) = V(K(t), Q(t))$ of (2.6). Solving for C^P, we denote the solution by $W(t)$ and interpret this quantity as the stationary-equivalent pre-event welfare measure appropriate for the real economy. Observing (3.1) and (2.7), the welfare index we seek is

$$\begin{aligned} W(t) &= [r + h(Q(t)]V(K(t), Q(t)) - h(Q(t))\varphi(K(t), Q(t)) \\ &= rV(K(t), Q(t)) + h(Q(t))\psi(K(t), Q(t)), \end{aligned} \tag{3.2}$$

which exceeds the standard index rV by the expected immediate damage at time t. It is recognized at this point that other specifications of the welfare index can also be considered. The reasons for preferring the index (3.2) over the standard index are discussed at the end of this section.

It turns out that when the effect of Q on the hazard rate h is internalized, the NNP equals the welfare measure defined by (3.2). However, when the hazard is treated by agents as an externality, the NNP introduces a bias, as established by the following Proposition:

Proposition 1. (a) *When the effect of Q on the hazard rate h is internalized,*

$$W^*(t) = Y^*(t) \tag{3.3}$$

and the NNP agrees with the welfare measure (3.2) *evaluated at* $(K^*(t), Q^*(t))$.

(*b*) *When the hazard is treated by agents as an externality, i.e., when* $h(\tau) = h(Q(\tau))$ *and* $\Omega(\tau)$ *are taken as exogenous functions of time,*

$$W^*(t) = Y^*(t) + Z^*(t) \tag{3.4}$$

where

$$Z^*(t) = -\int_t^\infty \dot{h}^*(\tau)\psi(K^*(\tau), Q^*(\tau))e^{-R^*(\tau,t)}d\tau. \tag{3.5}$$

Proof: (*a*) When the effect of Q on h is accounted for, the property $dH/d\tau = \partial H/\partial\tau$ of the present value Hamiltonian (2.13) implies

$$dH(\tau,t)/d\tau = -r[C^*(\tau) + h(Q^*(\tau))\varphi(K^*(\tau), Q^*(\tau))]e^{-R^*(\tau,t)}.$$

Integrating from t to ∞ along the optimal path and using the transversality condition $\lim_{\tau\to\infty} H(\tau,t) = 0$ (see [21]), gives

$$H(t,t) = rV(K^*(t), Q^*(t)). \tag{3.6}$$

Recalling that $P_K^*(\tau) = \lambda_K^*(\tau)e^{R^*(\tau,\,t)}$, $P_Q^*(\tau) = \lambda_Q^*(\tau)e^{R^*(\tau,\,t)}$ and $\mu^*(\tau) = -V(K^*(\tau), Q^*(\tau))e^{-R^*(\tau,\,t)}$ (see Section 2), the Hamiltonian (2.13) at $\tau = t$ reduces to

$$H(t,t) = Y^*(t) - h(Q^*(t))\psi(K^*(t), Q^*(t)). \tag{3.7}$$

Comparing with (3.6) and (3.2) verifies (3.3).

(*b*) When $h(\tau)$ and $\Omega(\tau)$ are taken as exogenous functions of time, the present-value Hamiltonian is

$$H(\tau,t) = [C + h(\tau)\varphi(K,Q)]e^{-R(\tau,t)} + \lambda_K I_k + \lambda_Q I_Q \tag{3.8}$$

and the property $dH/d\tau = \partial H/\partial\tau$ gives

$$\begin{aligned} dH(\tau,t)/d\tau &= \{\dot{h}^*(\tau)\varphi(K^*(\tau),Q^*(\tau)) - \\ &\quad [r+h^*(\tau)][C^*(\tau)+h^*(\tau)\varphi(K^*(\tau),Q^*(\tau))]\}e^{-R^*(\tau,t)}. \end{aligned}$$

Integrating from t to ∞ along the optimal path and using the transversality condition $\lim_{\tau\to\infty} H(\tau,t) = 0$ implies

$$H(t,t) = [r+h^*(t)]V(K^*(t),Q^*(t)) - Z^*(t). \tag{3.9}$$

where

$$\begin{aligned} Z^*(t) &= \int_t^\infty \{[h^*(t)-h^*(\tau)][C^*(\tau)+h^*(\tau)\varphi(K^*(\tau),Q^*(\tau))] \\ &\quad + \dot{h}^*(\tau)\varphi(K^*(\tau),Q^*(\tau))\}e^{-R^*(\tau,t)}d\tau \end{aligned} \tag{3.10}$$

Taking the derivative of (3.10) with respect to t gives $\dot{Z}^*(t)-[r+h^*(t)]Z^*(t) = \dot{h}^*(t)\psi(K^*(t),Q^*(t))$, which is integrated along the optimal path to yield (3.5).

Again, $P_K^*(\tau) = \lambda_K^*(\tau)e^{R^*(\tau,\,t)}$, $P_Q^*(\tau) = \lambda_Q^*(\tau)e^{R^*(\tau,\,t)}$ and the Hamiltonian (3.8) evaluated at the optimal policy at $\tau = t$ becomes

$$H(t,t) = Y^*(t) + h^*(\tau)\varphi(K^*(t),Q^*(t)). \tag{3.11}$$

Comparing (3.9) and (3.11), noting (3.2), verifies (3.4). $\square$

Part (a) provides a precise meaning to the welfare significance of the NNP without externalities in terms of the nonchalant parable. The bias term Z^* introduced in part (b) stems from the failure of the market prices to fully account for the effect of Q on the hazard rate. The bias vanishes for non-influential events (i.e., $\varphi = V$ and $\psi = 0$) or when h is independent of Q (see (3.5)). Often the environmental stock mitigates the hazard (i.e., $h'(Q) < 0$), in which case resource extraction above replenishment increases the occurrence threat over time and gives rise to $Z^*(t) < 0$. The presence of the externality enhances the tendency to overexploit the environmental resource. Although the shrinking stock Q is included in the green NNP, the ensuing increase in the hazard rate is not properly represented by its price and the NNP provides an over-optimistic measure of welfare.

Evidently, Proposition 1 is based on the "nonchalant" parable that identifies $W(t)$ of (3.2) as the appropriate welfare measure. An alternative index that might be considered is the standard index rV associated with the parable of a stationary economy that consumes all its output indefinitely, free of any risk of occurrence (see [31]). To see the implications of this choice we use (2.10), (3.6) and (3.7) and obtain (for a fully internalized hazard)

$$rV(K^*, Q^*) = C^* - h(Q^*)\psi(K^*, Q^*) + P_K^* I_K^* + P_Q^* I_Q^*, \tag{3.12}$$

which differs from the green NNP by the term $h\psi$. There are several ways to interpret this difference. One possible way is to complement the NNP by an hazard stock Ω that accumulates at the rate h and bears the price $-\psi$. Alternatively, one can say that the appropriate consumption rate for the economy is less than C^* because the amount $h\psi$ must be set aside as an insurance against future damage. However, when the environmental damage is too large to allow for complete insurance coverage (the case considered here), the standard parable (of an indefinitely stationary and risk-free economy) is inappropriate. In contrast, the "nonchalant" parable and its corresponding index $W(t)$ address this concern. This index reproduces Weitzman's result for the comprehensive (internalized hazard) case and is consistent with Weitzman and Löfgren's [31] approach.

4 Concluding comments

The notion that current market prices contain all the information relevant to determine long-term welfare is appealing, but its validity is subject to the assumption of comprehensibility. The green accounting literature has long recognized that this assumption is an idealization because of the pervasiveness of market failures. Here we find that, without externalities, the green NNP can still be interpreted as a welfare measure also in the presence of event uncertainty. This can be done because markets respond properly to the environmental threats by adjusting the time preferences from the "bare" discount rate r to the effective rate $r + h$ and modifying the prices associated with the relevant capital stocks.

Biodiversity, however, bears the characteristics of a public good. Agents, therefore, tend to treat the extinction hazard as an externality, hence the comprehensibility requirement is violated and the green NNP overestimates welfare. If the diversity-conserving environmental asset is at or near a steady state, the hazard rate is approximately constant over time and the NNP bias disappears. Otherwise, neglecting to account for their own contribution to increasing the hazard, agents overexploit the environmental resource, increasing the risk to the surrounding ecosystem and reducing the (expected) welfare for the whole economy. Standard regulation techniques, such as a Pigouvian tax on the exploitation of the hazard-mitigating resource based on its marginal hazard effect, can induce agents to internalize the effect and increase welfare to the value indicated by the green NNP.

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