# Noah's non-concavity: On the existence of non-trivial interior solutions to the problem of cost-effective conservation planning

by

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Brief Title: **Noah's Non-concavity**

**Abstract**

The idea that species loss diminishes future information flows is a cornerstone of arguments for conservation planning. In his seminal work entitled "The Noah's Ark Problem", Weitzman [9] examines the problem of cost-effective conservation planning from a theoretical perspective accounting for the affect planning has on the expected size of the biosphere's informative potential. We extend Weitzman's analysis by examining how his conclusions are altered by the introduction of a conservation authority that considers the *value of information* contained in the biosphere. We find that introducing non-quasiconcave preferences for the information contained in each species substantially modifies the characterization of a cost-effective conservation plan. In particular, we find that a cost-effective plan generally includes partial funding for many species and funds no species completely. Our investigation is motivated by theoretical contributions to the information economics literature, *a la* Radner and Stiglitz [7], showing that the value function for information tends to exhibit increasing returns.

Discovery requires both effort and substance. Inquisitive persons most often purposely provide the effort and nature the substance. The fruit of discovery is information, prized because it improves decision making. The belief that the diverse genetic heritage of The Planet's extant species holds a vast reserve of as-yet-undiscovered information has been a cornerstone of arguments for species preservation. In this paper, we ask two closely related questions concerning the information theoretic aspects of conservation planning: (1) What are the essential characteristics of the biosphere's information reserve that should be considered in a cost-effective approach toward its management? (2) What are the notable features of an optimal conservation plan when the informational aspects of species loss predominate? Drawing upon results in the information economics literature (Radner & Stiglitz [7]; Chade & Schlee [3]), we find that the oft-mentioned non-concavity in the value of information is crucial to a well-formed optimal biodiversity management problem. Contrary to prior results in the literature (Weitzman [9]), we find that cost-efficient management of the biosphere's informational resource generally requires a conservation plan that gives partial preservation funding to *many* species and funds no species fully.

Using evolutionary biology's "descent-by-modification" theory, Weitzman [9] provides a clear theoretical structure for analyzing the impact of species loss on the size of society's information reserve in his paper titled *The Noah's Ark Problem (NAP).* Weitzman shows that by pursuing the objective of maximizing expected diversity,

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a conservation authority simultaneously maximizes the expected size of society's biological information reservoir.[2]  The rigid structure between species imposed by $NAP$ generates an optimal solution that is "extreme" in the sense that it suggests concentrating the entire conservation budget on those species contributing most to the biosphere's genetic diversity.  We extend Weitzman's analysis to allow the conservation authority to explicitly consider the *value of information* contained in the biosphere's gene reserve.  Our results tend to be antipathetic to the "extreme policy solution" of $NAP$ and suggest an optimal policy which concentrates funding on the most diverse member of a particular group of closely related species.  However, among the funded species the optimal distribution of funds corresponds to partial funding for many of the species in most cases. This rather surprising result is a direct consequence of the well-known non-concavity in the value function for information, a regular feature of many problems in informational decision theory.

*The Noah's Ark Problem* represents a milestone in the economic analysis of biodiversity.  The paper exposes the two-date problem of a cost-minimizing conservation authority, $\mathcal{CA}$ (i.e., Noah), deciding how best to allocate a fixed budget (i.e., Ark space) among species whose probability of surviving an extinction event (i.e., The Flood) will be affected by the level of funding received. [3]  The $NAP$ assumes an

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[2]Weitzman uses the term "information content" to capture the same idea.

[3]The two-date structure of Weitzman's [9] problem is implicit.  He considers the *ex ante* optimization problem of a conservation authority trying to allocate a budget prior to the realization of

additively separable objective for the $\mathcal{CA}$ with two components. The first component is intended to represent the impact conservation measures will have on the "direct use" value obtained from preserving a species. The second component is given the two-fold task of representing the "serendipity" value and the "pure" diversity value of preserving a species. Here, "pure" diversity value means the contribution to human welfare from having a genetically diverse set of species as co-inhabitants of The Planet. "Serendipity" value captures the idea that biodiversity provides the substance for discovery. [4] In his exposition, Weitzman develops a rationale for concluding that ex-

any extinctions. For example, Weitzman does not consider how conservation budget allocations may change over time in response to the realized extinction of certain species. Additionally, Weitzman is also concerned with the eventual resolution of uncertainty regarding the discovery of information embodied in preserved species. That is, Weitzman assumes that only the information contained in species surviving the extinction event will be revealed. This assumption allows all of the concerns raised by Weitzman (i.e., extinction and discovery) to be well represented by only considering two dates.

[4] The challenge to natural resource economists of characterizing the impact of changes in the level of biodiversity on the welfare of current and future generations has been long-standing and formidable. In his seminal work detailing the evolving natural resource economics literature, John Krutilla [5] began enumerating the essential welfare impacts of policies affecting biodiversity: recreation value, medicinal value, stability value, existence value, option value, and "serendipty" value. While not as readily recognized as other members of the list, "serendipity" value has had an equally important influence on the way the literature currently frames discussions of biodiversity. "Serendipity" value attempts to create a label recognizing the contribution that biodiversity makes to humanity

pected diversity is a first-order approximation of a species's contribution to society's "information reserve". [5] In consolidating the "pure" diversity value of a species with its contribution to the social "information reserve", the $NAP$ presumes a $\mathcal{CA}$ that has (strictly) quasiconcave preferences for information. The results following from this presumption imply an optimal policy that allocates funding among species to achieve the highest expected diversity and, consequently, the largest expected biological "information reserve." Substantial evidence from the information economics literature suggests that preferences for information tend to be non-quasiconcave (Radner & Stiglitz [7]; Chade & Schlee [3]), thus providing the impetus for further study.[6]

Our contribution is unique because of its specific concern for generating a complete analysis of the information theoretic aspects of cost-effective biodiversity management within the context of Weitzman's Noah's Ark structure. Other noteworthy contribu-

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as a vast reserve of undiscovered ideas and new information.

[5]Weitzman [9] uses the term "information content", which we find somewhat less evocative of the underlying resource question that is essentially an optimal depletion problem. In Weitzman's Theorem 1 the following is established: $I(P,\varepsilon) \approx \varepsilon \cdot W(P)$, where $I(\cdot)$ is "information content", $W(P)$ is the expected number of Bernoulli trial associated with conservation plan $P$, and $\varepsilon$ is the probability of a "success." Thus, $\partial I/\partial P_i = \varepsilon \cdot \partial W/\partial P_i$.

[6]Hirshliefer [4] demonstrates that the analytical distinction between information gained by human effort, "discovery", and information gained without human effort, "foreknowledge", is a sterile one. Thus, in what follows we simply characterize information from the biological reserve as a gift of nature without considering the implications of costly research effort needed for its extraction.

tions to the literature have been critical of Weitzman's framework on the grounds of feasibility or suggested modification along other lines. Mainwaring [6], for example, criticizes Weitzman's policy implication on the basis of practicability due to data constraints and lack of consideration of species interdependence. Weikard [8] argues that Weitzman's analysis can be made both practical and applicable by considering ecosystems as the unit of analysis rather than species.

The remaining sections of the paper are organized as follows: We first introduce a two-species example similar to Weitzman's [9]. In Section 2, we develop a Blackwellian [1] index based on Weitzman's description of the information discovery process as independent Bernoulli trials. This exercise provides motivation for applying the Blackwell index to measure the size of the information reserve under different conservation plans. Establishing the use of the Blackwell index also provides a solid justification for employing the non-concavity in our analysis. Section 3 generalizes the two-species example to N species. Concluding observations follow.

## 1 A Two-Species Information Reserve

In this section, we focus on analyzing an extension of the two-species example developed in the *NAP*. The "two-species example" attempts to capture in its simplest form the problem a $\mathcal{CA}$ faces when allocating a limited budget among species whose probability of surviving an extinction event can be modified by conservation funding.

In developing his analytical structure, Weitzman uses the metaphor of a species as a library to aid his exposition. We will employ the same metaphor. For those readers who are unfamiliar, we briefly recount the rudimentary details. The basic idea is that each species can be viewed by the $\mathcal{CA}$ as a "library" whose value is derived from the library building and its collection of books. These metaphorical details are meant to provide mental cues that lead the reader to recall three components of value associated with each distinct species/library: direct use value, diversity value, and serendipity value. Direct use value stands for the dimension of species value associated with the immediate uses of the extant species population (i.e., the value of the building). The collection of "books" in the species/library stands for the species's genetic code. It is assumed that distinctiveness in the genetic code of a species is, in itself, of value to the conservation authority; call this dimension "diversity value." Weitzman also argues that the collection of books may also be valued for the pragmatic reason that they contain information; call this dimension "serendipity value." He then introduces two closely related functions, $V(\cdot)$ and $W(\cdot)$, that measure the deterministic and probabilistic informative potential of a species/library assemblage, $Q \subseteq S$, respectively. $S$ represents the set of all species to be considered for possible funding. The function $V(\cdot)$ measures the number of independent Bernoulli trials available if a species/library survives the extinction event and becomes the focus of research efforts, where the probability of a "success" for each trial (i.e., the probabil-

ity that any book chosen at random contains the piece of information being sought), $\varepsilon$, is arbitrarily close to zero. In keeping with the library metaphor Weitzman calls each trial a "different book." $W(P)$ measures the expected number of trials in a species/library assemblage prior to the extinction event, where $P$ is a vector whose components, $P_i$ for $i \in Q$, are the survival probabilities are each species in a particular collection. So, in the parlance of the species/library metaphor the "information content" of a species is $W(s_i) = P_i V(s_i)$. In the deterministic case, $V(s_i) = W(s_i)$. The "information content" of a species/library assemblage, $Q \subseteq S$, employed in the development of $NAP$ is as follows:

$$I(P; \varepsilon) \equiv \sum_{Q \subseteq S} \left( \prod_{j \in Q} P_j \right) \left( \prod_{k \in S \setminus Q} (1 - P_k) \right) [H(Q; \varepsilon)] \qquad (1)$$

$H(Q; \varepsilon) \equiv 1 - (1 - \varepsilon)^{V(Q)}$ is a function giving the probability that a collection, $Q$, contains useful information. Notice in the case of two species, $s_i$ and $s_j$, that $H(s_j; \varepsilon) > H(s_i; \varepsilon)$ iff $V(s_j) > V(s_i)$. In this case, the contribution of species $j$ to the "information content" of the entire collection is greater than that of species $i$, *ceteris paribus*. Recognizing the implicit ranking of species with respect to their contribution to the "information content" of the universal collection, $S$, we propose two modifications to the $NAP$ characterization of "information content" and the relative "informativeness" of different species. First, we propose to index the *ex post* "informativeness" of a species collection, $H(Q; \varepsilon)$, so the Blackwell [1] ranking may be applied. Specifically, the Blackwell ranking indexes Bayesian information

structures such that an increase in the value of the index corresponds to a more informative structure. The set of possible rankings is a closed and bounded set on $\mathbb{R}_+$, $\Theta \equiv [0, \bar{\theta}]$. An arbitrary ranking, $\theta \in \Theta$, is completely "uninformative" if $\theta = 0$. An "uninformative" structure adds nothing to an agent's *a priori* information set. Formally, we assume the existence of a rank order preserving mapping $\zeta : H(Q; \varepsilon) \to \Theta$. (This assumption will be given a formal justification based on the Bernoulli trial description of the discovery process in Section 2.) The second modification deals directly with the issue of modeling the decision problem underlying the *NAP*'s concern for the information content of species. The *NAP* captures the conservation authority's concern for the consequences that preservation efforts will have on the future flow of information by choosing an objective that will maximize information content, *ceteris paribus*. In contrast, we consider an objective for the $\mathcal{CA}$ that maximizes the expected *value of information* rather than the expected *size* of the information reserve. In the next section, we will show that our justification for modifying the $\mathcal{CA'}s$ objective follows from a precise interpretation of Weitzman's description of the discovery process, which leads to a 'hand-in-glove' application of the Radner-Stiglitz non-concavity theorem.

For example, consider a benevolent $\mathcal{CA}$ has been given a fixed budget, $B$, that may be allocated to increase the survival probabilities of two species, $i$ & $j$. The authority has preferences defined over three species attributes: usefulness, distinctiveness, and

information. The coefficient $U_h$, $h = i, j$, measures the estimate of each species's "direct use" value used by the conservation planner. "Distinctiveness" is a measure of a species's contribution to biodiversity which is represented by the number of different genes contained in the species. Each species is assumed to have $M_h$, $h = i, j$, different genes of which $E_h$, $h = i, j$, are unique and $J$ are common to both species. Thus, $M_h = E_h + J$, $h = i, j$. Based on the developments in the previous paragraph, we simplify notation and suppress explicit mention of $\zeta(\cdot)$. Thus, measure the information contained in each species $(i, j)$ and their union $(ij)$ using the Blackwell index $\theta_h$, $h = i, j, ij$. In what follows, the function, $\xi(\theta_Q)$, for $Q \subseteq S$, measures the value of the information contained in each surviving species subset. Accounting for the survival probability of each species subset and consolidating, the $\mathcal{C}\mathcal{A}'s$ objective reduces to the following:

$$\max_{p_i p_j} \Pi = [U_i + M_i + \xi(\theta_i)]p_i + [U_j + M_j + \xi(\theta_j)]p_j \tag{2a}$$

$$+[\xi(\theta ij) - \xi(\theta_i) - \xi(\theta_j) - J]p_i p_j$$

subject to

$$B = c_i \left( \frac{p_i - \underline{p_i}}{\overline{p_i} - \underline{p_i}} \right) + c_j \left( \frac{p_i - \underline{p_j}}{\overline{p_j} - \underline{p_j}} \right) \tag{2b}$$

$$\overline{p_i} \geq p_i \geq \underline{p_i} \tag{2c}$$

$$\overline{p_j} \geq p_j \geq \underline{p_j} \tag{2d}$$

The first constraint represents the relationship between the *ex post* conservation sur-

vival probability of each species and the authority's budget. The coefficients in the budget constraint, $c_i$ & $c_j$, measure the cost of raising each of the respective species's survival probabilities from an *ex ante* conservation level, $\underline{p}$, to the highest possible level, $\overline{p}$. The term captured in parentheses introduces consideration of some level of conservation between the two bounds. The budget constraint can be written more succinctly by collecting terms:

$$\widetilde{B} = \left(\frac{c_i}{\widehat{p}_i}\right) p_i + \left(\frac{c_j}{\widehat{p}_j}\right) p_j \tag{3}$$

where

$$\widehat{p}_i = \overline{p}_i - \underline{p}_i, \ \ \widehat{p}_j = \overline{p}_j - \underline{p}_j$$

$$\widetilde{B} = B + \left(\frac{c_i}{\widehat{p}_i}\right) \underline{p}_i + \left(\frac{c_j}{\widehat{p}_j}\right) \underline{p}_j$$

The resulting Kuhn-Tucker Necessary Conditions for the appropriately formulated Lagrangian, $\mathcal{L}$ :

$$\left[\frac{\partial \mathcal{L}}{\partial p_i}\right] p_i = \left[\begin{array}{c} [U_i + M_i + \xi(\theta_i)] \\ +[\xi(\theta ij) - \xi(\theta_i) - \xi(\theta_j) - J]p_j - \lambda\left(\frac{c_i}{\widehat{p}_i}\right) \end{array}\right] p_i = 0 \tag{4a}$$

$$\left[\frac{\partial \mathcal{L}}{\partial p_j}\right] p_j = \left[\begin{array}{c} [(U_j + M_j + \xi(\theta_j)] \\ +[\xi(\theta ij) - \xi(\theta_i) - \xi(\theta_j) - J]p_i - \lambda\left(\frac{c_i}{\widehat{p}_i}\right) \end{array}\right] p_j = 0 \tag{4b}$$

$$\left[\frac{\partial \mathcal{L}}{\partial \lambda}\right] = \left[\widetilde{B} - \left(\frac{c_i}{\widehat{p}_i}\right) p_i - \left(\frac{c_j}{\widehat{p}_j}\right) p_j\right] = 0 \tag{4c}$$

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## 1.1 Sufficiency

Characterization of the solution shows that the shape of the level sets depends critically on the non-concavity in the value of information function, $\xi(\cdot)$. For the sake of notational convenience, let $\widetilde{U} = [U_h + M_h + \xi(\theta_h)]$ for $h = i, j$ and $\Delta\xi(\theta_{ij}) = [\xi(\theta ij) - \xi(\theta_i) - \xi(\theta_j)]$. If the level sets are concave, then Weitzman's "extreme policy solution" is a good general characterization of the optimal conservation plan. However, if the level sets are convex a policy that spends at least some of the conservation budget on each species and funds no species entirely is best. The level sets of the $\mathcal{CA}$'s objective are:[7]

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\begin{aligned}
d\Pi &= \left(\frac{\partial\Pi}{\partial p_i}\right)dp_i + \left(\frac{\partial\Pi}{\partial p_j}\right)dp_j = 0 \qquad (5)\\
&= \left(\widetilde{U}_i + (\Delta\xi(\theta_{ij}) - J)p_j\right)dp_i + \left(\widetilde{U}_j + (\Delta\xi(\theta_{ij}) - J)p_i\right)dp_j = 0\\
&\Rightarrow \frac{dp_i}{dp_j} = \frac{-\left(\widetilde{U}_j + (\Delta\xi(\theta_{ij}) - J)p_i\right)}{\left(\widetilde{U}_i + (\Delta\xi(\theta_{ij}) - J)p_j\right)} < 0\\
&\Rightarrow \frac{d^2p_i}{dp_j^2} = \frac{2(\Delta\xi(\theta_{ij}) - J)\left(\widetilde{U}_j + (\Delta\xi(\theta_{ij}) - J)p_i\right)}{\left(\widetilde{U}_i + (\Delta\xi(\theta_{ij}) - J)p_j\right)^2} \overset{?}{\gtrless} 0
\end{aligned}
$$

The shape of the authority's budget constraint depends on the magnitudes of $B, c_i$, and $c_j$. In any event, the budget produces a piecewise linear graph in the $p_i \times p_j-$ plane. Examination of the second derivative, $\frac{d^2 p_i}{dp_j^2}$, shows that the parameter restrictions imposed by the $NAP$ description of the relationship between species imply

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[7]Note that the second partial requires treating $p_i$ as a function of $p_j$.

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that $sign(\frac{d^2 p_i}{dp_j^2}) = sign(\Delta\xi(\theta_{ij}) - J)$. Interpretation of this finding characterizes one of our main results:

**Proposition 1** *Holding constant the increase in information from preserving both species, $\Delta\xi(\theta_{ij})$, the greater the number of genes in common between the two species, J, the more likely will be the case that concentrating funding on the preservation of one of the species at the exclusion of the other is optimal. Conversely, if the species do not share many genes in common then an optimal distribution of conservation funding between the two species will more likely include some level of funding for both species.*

We will show in the subsequent section that this characterization holds in the more general setting where conservation planning includes N species, but the intuition of the result is most easily seen in the two-species example. Equivalently, we could have examined the properties of the resulting bordered Hessian matrix to derive the same conclusion. In the next section, we provide the analytical details needed to bridge the gap between Weitzman's characterization of "information content" and ours.

## 2   A Blackwellian Interpretation

Question: Is information inherently valuable? The common-sense answer based on a foundation of rigorous thought in the information economics literature: Only if

the information makes a behavioral improvement, net of all costs, possible. Radner and Stiglitz [7] demonstrate that the value of information function is "flat" in a neighborhood around the origin when graphed against an index of information structures ranging from the non-informative (i.e., null information $= 0$) to the perfectly informed, where increases in the index represent greater "informativeness." The non-concavity has important consequences for economic problems that introduce the possibility of information acquisition by agents because it implies that the value of information exhibits increasing marginal returns over some range. The wide applicability of the Radner and Stiglitz result was not fully appreciated until recently. Chade and Schlee [3] have shown that the non-concavity is a robust feature of many information decision problems and cannot be avoided without significantly restricting the scope of analysis. Since the current mode of thinking about conservation planning recognizes species information content as a central motive, understanding the potential impact of the non-concavity on an optimal distribution of funding among species seems particularly important to advancing the literature.

This section of the paper examines whether Weitzman's [9] stylized version of the discovery process may be described using Blackwell's [1] ranking of information structures. Formally, we wish to establish that the description of discovery as a series of independent Bernoulli trials used in the $NAP$ implies a set of information structures that may be ranked by Blackwell's method. Establishing a well-founded

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connection between the $NAP$ discovery process and Blackwell's ranking provides a formal justification for replacing the number of different genes in a species collection, $V(Q \subseteq S)$, with the Blackwell ranking, $\theta_Q$, when examining the affect the shape of the value of information has on the optimal conservation plan.

Begin by recalling the $NAP$'s description of discovery. Weitzman defines $\varepsilon$ as the probability of finding a particular trait (i.e., piece of sought after information) in a single gene. $\varepsilon$ is assumed to be the same for each different gene in the grand collection of all different genes, $V(S)$. The probability of discovering the trait is also assumed to be independent among the different genes. Notice that for any arbitrary collection of different genes, $V(Q \subseteq S)$ , the process of discovery conducted with genes in the collection corresponds to a series of $V(Q \subseteq S)$ independent Bernoulli trials with parameter $\varepsilon$. Given this observation, is there some sense in which it is meaningful to say that having a larger collection (i.e., number of Bernoulli trials) is more informative? The answer to this question turns out to be affirmative and demonstrating the affirmation merely requires an interpretation of Weitzman's description of discovery assuming that the $\mathcal{CA}$ is a Bayesian.

As a Bayesian, the $\mathcal{CA}$ views the information contained in each species as a way of sharpening his probability estimates of events that will ultimately affect his well-being. Note that this view is consistent with Weitzman's description of the $\mathcal{CA}$ having a "utilitarian" interest in species conservation motivated by "information content."

However, it also implies the existence of an underlying resource allocation problem that the $\mathcal{CA}$ is trying to solve and that he believes the species information reserve might contain information relevant to his decision. From the perspective of the species conservation choice being considered presently, the importance of the underlying decision problem is captured entirely by a very general description of the problem's information theoretic aspects. Thus, we will describe $\mathcal{CA}$'s underlying decision problem in terms of the "likelihood" of states of nature, $\omega_h, h = 1, 2$, given different messages that may be received from the species information discovery process previously described, $m_h, h = 1, 2$, and a distribution of prior probabilities of the states, $\gamma_h, h = 1, 2$. Our indices on the states, messages, and priors indicate that for the $NAP$ we will only be considering a dyad of each. The dyads give a sufficient basis for describing the $NAP$'s family of information structures because of the Bernoulli trial discovery process assumed by Weitzman, though more general structures could be accommodated without affecting our results. Before moving on to describe the $NAP$ family information structures, we would like to acknowledge our cognizance of some of the conceptual subtleties in moving from Weitzman's comparatively straight forward description of discovery to ours. Shortly, the payoff of this added complexity will be obvious. Rather than delving into these subtleties, we defer to another classic contribution to the information economics literature that gives them the attention they deserve: Hirshleifer [4] AER 1971.

Returning to our description of the $NAP$'s family of information structures, we describe each of the elements in turn. We begin with the set of states, $\omega_h, h = 1, 2$. In the $NAP$, there are two relevant states of the world. The first of these, $\omega_1$, we label the "good" state. The description of this state captures all the economically relevant details for the $\mathcal{CA}$ having realized the state that the "discovery" was supposed to make more (subjectively) likely. Thus, $\omega_2$ is the "bad" state. The messages can be similarly interpreted. We label $m_1$ as the "discovery" message, which indicates that at least one of the Bernoulli trials preserved as a result of conservation was a "success." The message $m_2$ is the "no discovery" message. The distribution of prior probabilities, $\gamma_h, h = 1, 2$ is similarly defined. Consider the following illustration, which parallels one used by Weitzman [9]. Suppose the $\mathcal{CA}$ is interested in finding a cure to some disease and that he believes that the cure might be hidden in the vast species information reservoir. In this case, $\omega_1$ would be the "no disease" state. Clearly, there is some probability that the cure is found without any species conservation measures. For instance, the cure might be found through other means or be in the set of species that survives the extinction event without conservation funding. It is also possible that no cure is found, but that the "no disease" state is realized anyway. For example, it is possible that there is an effective quarantine of infected individuals, that no new cases occur, and that those with the disease heal on their own or expire. In any case, it is obvious that the $\mathcal{CA}$ has some prior probability

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for the "no disease" state occurring that is distinct from the influence that species conservation funding may have on his estimate of the probability. The "discovery" message in this example would mean that the $\mathcal{CA}$ found a compound (i.e., a successful Bernoulli trial) in the funded collection and that this discovery made the "no-disease" state more (subjectively) probable. Surely, the discovery of a compound that has a curative effect on a disease does not ensure that everyone will be able to get the compound and that the disease will be eliminated.

Weitzman's [9] description of the discovery process indicates that there is a chance, $\varepsilon$, of finding the information being sought in each different gene in the set of all species, $S$. Thus, we can deduce that the $\mathcal{CA}$ has a prior belief, $\gamma_1$, that the information would be discovered in the absence of conservation efforts. Likewise, $\mathcal{CA}$'s prior belief that the information will not be discovered is $\gamma_2$. Weitzman's description also provides us with the probability of finding the information given a search of some subset of the total extant population, $Q \subseteq S$, *conditional* on the information actually existing, $q_{m=1|\omega=1} = 1 - (1-\varepsilon)^{V(Q \subseteq S)}$, where the subscript on $q$ indicates the conditional probability of receiving a message, $m = 1, 2$, about the discovery given the true state of the world, $\omega$. Staying as close to the $NAP$ setting as possible, we assume that a message of discovery precludes the possibility of the uninformed state, $q_{m=1|\omega=2} = 0$. The other two conditional probabilities can be similarly deduced: $q_{m=2|\omega=1} = (1-\varepsilon)^{V(Q \subseteq S)}$, $q_{m=2|\omega=2} = 1$. The first member of this pair simply states

the probability of not making the discovery even though the information may indeed exist elsewhere. The second member means that if the information does not exist, then there is no chance of receiving a message that it does. We use the following definition of the Blackwell ranking to demonstrate that as the number of different genes in the *ex post* collection increases so does the informativeness of the information structure it represents [2]:

**Definition 2** $Y$ *is a set of messages/signals with* $M$ *members,* $L$ *is a* $\times M$ *matrix of conditional probabilities,* $\gamma$ *is a vector of unconditional probabilities for receiving each of the messages in* $Y$, *and* $P$ *is a* $\Sigma \times M$ *matrix of conditional (Bayesian) posterior probabilities for each state. Information structure* $(Y', L')$ *is more informative than information structure* $(Y, L)$ *in the Blackwell sense if and only if for every vector of prior beliefs,* $\overrightarrow{\gamma}$, *there exists a* $M \times M'$ *Markov matrix* $T$ *such that* $P = TP'$ *and* $\gamma T = \gamma'$.

**Theorem 3** *Let* $Y$ *be the set of signals for a sequence of* $n \in \mathbb{N}$ *independent Bernoulli trials that indicates whether any of the trials was a "success." (1) If* $\Sigma \times M$ *matrix* $L(n)$ *is a Markov matrix giving a distribution of signals conditional on the state* $\omega \in$ , *then* $n$ *provides an ordinal ranking of the family of information structures* $\{Y, L(n)\}$ *that is consistent with the Blackwell ranking. (2) Moreover, no information structure* $\{Y, L(\widehat{n})\}$ *from this family can be more informative in the Blackwell sense than another member of the family with* $n > \widehat{n}$.

**Proof.** *By induction, using Definition 2 above. A matrix $\widetilde{T}$ with the following properties $P(1) = \widetilde{T}P(2)$ and $\gamma(1)\widetilde{T} = \gamma(2)$ can be easily constructed, where 1 & 2 can be used to indicate the respective member of the family $\{Y, L(n)\}$. A similar construction follows for $\widehat{T}$ that demonstrates that the same relationship holds for $n$ and $n + 1$.* ∎

**Corollary 4** *Weitzman's "Information Content" provides an ordinal ranking of species and species coalitions that is a strictly monotone transformation of Blackwell's ranking. Remark: Weitzman's "Information Content" is an increasing function of $V(Q \subseteq S)$ as is the Blackwell ranking.*

Theorem 3 and Corollary 4 are meant to serve as a foundation upon which a bridge between the *NAP* and non-concavity literature can be built. The idea is to ensure that there is no equivocation taking place in our analysis caused by introducing the Blackwell ranking of species information content. This is a particularly important semantic detail. In the absence of a clear connection between Weitzman's concept of "information content" and the Blackwell ranking, one might reasonably doubt the relevance of the non-concavity and whether our concern for valuing the information content of the species informational reservoir might have been addressed in the original *NAP*. We hope the preemptive efforts on our part have alleviated any such concerns. There is an additional technical detail to be considered when using the results of the non-concavity literature. The detail centers on the requirement

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that the value of information $\xi(\cdot)$ be a *smooth* function of the Blackwell index in a neighborhood of the ranking for null information, $\theta = 0$. Because of the Bernoulli process of experimentation underlying the description of the family of information structures $\{Y, L(n)\}$ considered by the $\mathcal{CA}$, the ranking of the structures is not continuous in $\theta$. However, in the remainder of the paper we assume continuity of the ranking of the family structures as a convenience. Continuity is not necessary to obtain the result, but it improves the flow of the exposition. Henceforth, we choose a Blackwell index of the information structures $\theta_Q \equiv \ln(V(Q) + 1)$ for $V(Q) \in \mathbb{R}_+$.

## 3  The Noah's Ark Problem II

In what follows, we consider a N-species version of the $NAP$ that explicitly accounts for the value of information contained in a collection of species. The end goal is to demonstrate that Weitzman's [9] "extreme policy solution" is tempered by consideration of the non-concavity. Thus, we begin with an objective that closely resembles the original $NAP$. The objective incorporates the original preferences of the $\mathcal{CA}$ with respect to diversity and direct utility of species. The problem ($NAP\ II$) is to select values of $\{P_i\}$ to achieve

$$\max_{\{P_i\}} \left[ W(\{P_i\}) + U(\{P_i\}) + X(\{P_i\}) \right] \tag{6}$$

where $X(\{P_i\})$ is the expected value of information associated with the chosen conservation plan. Specifically,

$$X(\{P_i\}) = \sum_{Q \subseteq S} \left( \prod_{j \in Q} P_j \right) \left( \prod_{k \in S \setminus Q} (1 - P_k) \right) \xi(\theta_Q)$$

and

$$W(\{P_i\}) = \sum_{Q \subseteq S} \left( \prod_{j \in Q} P_j \right) \left( \prod_{k \in S \setminus Q} (1 - P_k) \right) (V(Q))$$

The maximization in (6) occurs subject to the $n$ probability constraints

$$\underline{P_i} \leq P_i \leq \overline{P_i}, \quad \forall i$$

and the budget constraint

$$\sum_i C_i \left( \frac{P_i - \underline{P_i}}{\overline{P_i} - \underline{P_i}} \right) = B.$$

We now seek to contrast our results with those given in Weitzman's Theorem 2.

**Theorem 5**  *There exists a $\delta^* \in \mathbb{R}_+$, such that if $\xi(\theta_Q) \geq \delta^*$ for some $Q \subseteq S$, then the $\arg\max NAP\ II$ contains at least two interior elements.*

**Proof.**  The thrust of our argument is to show that the non-concavity in $\xi(\cdot)$ prevents the required deduction in Weitzman's proof of the "extreme policy solution." Following Weitzman, we write the portion of the objective dealing with diversity and information content to facilitate taking the second mixed partial derivatives with

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respect to an arbitrary species pair, $s = 1, 2$.

$$W(\{P_i\}) + X(\{P_i\}) =$$

$$\sum_{Q \subseteq S \backslash 1 \backslash 2} [K(Q)] \cdot [P_1 P_2 V(Q \cup 1 \cup 2) + P_1(1 - P_2)V(Q \cup 1)$$

$$+ P_2(1 - P_1)V(Q \cup 2) + (1 - P_1)(1 - P_2)V(Q)]$$

$$+ \sum_{Q \subseteq S \backslash 1 \backslash 2} [K(Q)] \cdot [P_1 P_2 \xi(\theta(V(Q \cup 1 \cup 2))) + P_1(1 - P_2)\xi(\theta(V(Q \cup 1)))$$

$$+ P_2(1 - P_1)\xi(\theta(V(Q \cup 2))) + (1 - P_1)(1 - P_2)\xi(\theta(V(Q)))]$$

where $K(Q)$ is defined as:

$$K(Q) \equiv \left( \prod_{j \in Q \backslash 1 \backslash 2} P_j \right) \cdot \left( \prod_{k \in S \backslash Q \backslash 1 \backslash 2} (1 - P_k) \right) \geq 0$$

and set notation is written in shorthand without the braces. Taking the second mixed partial reveals

$$\frac{\partial^2 W}{\partial P_1 \partial P_2} + \frac{\partial^2 X}{\partial P_1 \partial P_2} = \sum_{Q \subseteq S \backslash 1 \backslash 2} [K(Q)] \cdot [\Delta \xi(Q) - J(Q)]$$

where

$$J(Q) \equiv [V(Q \cup 1 \cup 2) - V(Q \cup 1) - V(Q \cup 2) + V(Q)]$$

and

$$\Delta \xi(Q) \equiv [\xi(\theta(V(Q \cup 1 \cup 2))) - \xi(\theta(V(Q \cup 1))) - \xi(\theta(V(Q \cup 2))) + \xi(\theta(V(Q)))]$$

Weitzman [9] argues that $-J(Q)$ is necessarily negative for all $Q \subseteq S$, so that in absence of other considerations the mixed partial $\frac{\partial^2 W}{\partial P_i \partial P_j}$ is negative. This implies that

the objective is convex in any two of its variables. However with the introduction of $X(\cdot)$ into the calculus of optimal conservation this result is no longer guaranteed. The condition that leads to an interior solution for at least two species is $\Delta\xi(\widetilde{Q}) - J(\widetilde{Q}) > 0$, which requires that $\xi(\theta(V(\widetilde{Q} \cup 1 \cup 2) > \delta^* = \xi(\theta(V(\widetilde{Q} \cup 1))) + \xi(\theta(V(\widetilde{Q} \cup 2))) - \xi(\theta(V(\widetilde{Q}))) + J(\widetilde{Q}) + \sum_{Q \subseteq S\setminus\widetilde{Q}\setminus1\setminus2} [K(Q)] \cdot [\Delta\xi(Q) - J(Q)]$ for some $\widetilde{Q} \subseteq S\setminus1\setminus2$. The implication is that at least two species will receive partial funding given a sufficiently steep value of information, $\xi(\cdot)$, and sufficiently strong preferences for information relative to diversity. In fact, generally it is to be expected that if the non-concavity is relevant for any $Q \subseteq S$, then the sign of the second mixed partial is ambiguous. Moreover if $\xi(\cdot)$ exhibits increasing returns globally, then it may be the case that the second mixed partial is *positive for all species pairs*, which implies the possibility of an optimal policy that gives partial funding to all species. ∎

Theorem 5 shows that when the information contained in species become a predominate motive for conservation planning, then a funding strategy that includes partial funding for multiple species may be optimal. In these situations, predominace of information content in the planner's funding calculus can be traced to two underlying causes. The first of these is mentioned casually in the proof of Theorem 5. It is the relative unimportance of other conservation motives. More specifically, the general non-concavity in the value of information is sufficient in the absence of other considerations to guarantee a partial funding solution to *NAP II*. The second

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reason that information content may dominate the influence of other conservation motives is the convexity of $\xi(\cdot)$. As $\xi(\cdot)$ becomes increasingly convex, the magnitude of $\Delta\xi(Q)$ grows. Thus, other things equal greater convexity in $\xi(\cdot)$ makes it more likely that the optimal conservation plan will partially fund several species.

Theorem 5 reaffirms our prior result in the two species example that for species pairs containing jointly many distinct genes Weitzman's [9] extreme solution is likely to hold. However, the basis of this conclusion begs the question of whether species diversity is inherently valuable. In the original $NAP$, the case for including $W(P)$ in the $\mathcal{CA}'s$ objective was bolstered by its connection with the idea of "information content." In our analysis, the relationship between $W(P)$ and the $\mathcal{CA}'s$ concern for preserving valuable information captured in the species gene reserve are no longer directly linked. In the absence of $W(P)$, whether more than one species receives partial funding depends entirely on the shape of the value function for information. If diversity is dropped from the objective, then the presence of the non-concavity can only strengthen the possibility that partial funding for more than one species is optimal.

Rather than formally develop a ranking criterion based on the characterization of $NAP\ II$, we will merely point out the notable features of a priority ranking of species based on our solution. Our solution implies that funding will tend to be concentrated on the most diverse members of what we will a call "sister" species.

By "sister" species, we mean two species that are closely related in the sense that they share many genes in common that are not shared by any other species. Given the tendency of the $J(Q)$ function to cause a concentration of funding when two species are closely related, a good general characterization of the optimal plan is that one member of each of the pairs of sister species receives funding. Among those individuals that are funded, the optimal policy should not, *a priori,* strictly prefer to concentrate funding on any species at the exclusion of others. Thus, our optimal policy requires the same fundamental data as the Weitzman "myopic ranking criterion", but also sophisticated knowledge of the informational aspects of Noah's decision problem.

## 4  Conclusion

In the foregoing analysis we have extended Weitzman's Noah's Ark Problem to include the possibility of a non-concavity in the value of information contained in species. The results of this extension for the characterization of a cost-effective conservation plan have, in some respects, been dramatic and antipathetic to those obtained by Weitzman. However to emphasize this perspective would be to gloss over many of the most important features of our analysis. Most importantly our findings are that: (1) Weitzman's "extreme policy solution" does hold when the non-concavity is small relative to the number of genes species have in common. And, (2) when Weitzman's

result does not hold the optimal distribution of funding concentrates more resources on those species that are "distinctive" but includes partial funding for many species.

One obvious yet important question remains that cannot be answered through the analysis of theory alone, "Should a non-concavity in the value of information factor significantly in the funding decisions of real-world conservation planners?" While we will refrain from arguing that the consequence of ignoring the non-concavity point to a planning failure in every case, our hope is that illustrating the charateristics of a cost-effective plan will help planners by making them aware that something may be overlooked in a plan looks very different what we have derived. If the "information content" of species is a significant motive for conservation planning, as we and many others believe it is, then correctly placing a value on the information preserved as a result of different conservation strategies should help us to avoid the most dramatic planning failures. In many other economic contexts, the non-concavity has played a significant role in describing sometimes counter-intuitive, but none-the-less optimal, policy advice. In the extreme, if returns to small improvements in the amount of information are high enough, then the optimal policy includes partial funding for *every* species. Drawing comfort from realizing this case is highly unlikely, we might also find bemusement from noting that when the *real* Noah's Ark Problem was solved - *every* animal was given a place on the Ark.

# 5 References

**References**

D. Blackwell, Equivalent Comparisons of Experiments, The Annals of Mathematical Statistics 24 (1953) 265-272.

C. Campbell, Blackwell's Ordering and Public Information, Journal of Economic Theory *forthcoming.*

H. Chade, E.E. Schlee, Another Look at the Radner-Stiglitz Nonconcavity in the Value of Information, Journal of Economic Theory 107 (2002) 421-52.

J. Hirshleifer, The Private and Social Value of Information and the Reward to Inventive Activity, The American Economic Review 61 (1971) 561-574.

J.V. Krutilla, Conservation Reconsidered, The American Economic Review 57 (1967) 777-786.

L. Mainwaring, Biodiversity, Biocomplexity, and the Economics of Genetic Dissimilarity, Land Economics 77 (2001) 79-93.

R. Radner, J.E. Stiglitz, A Nonconcavity in the Value of Information, in M. Boyer, R.E. Kihlstrom (eds.), Bayesian Models in Economic Theory, North-Holland, New York, NY, 1984, pp. 33-52

H-P. Weikard, Diversity Functions and the Value of Biodiversity, Land Economics 78 (2002) 20-27.

M.L. Weitzman, The Noah's Ark Problem, Econometrica 66 (1998) 1279-1298.