GRANDMA OR THE WOLF?
A REAL OPTIONS FRAMEWORK FOR MANAGING
HUMAN-WILDLIFE CONFLICTS

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Abstract

As wildlife habitats shrink, some species are pushed into extinction, while conflicts with others increase and cause significant economic damages. This paper proposes a simple real options framework to analyze wildlife management policies that account for ecological uncertainty and the risk of extinction. Our application to wolves provides an economic justification for their reintroduction and highlights the importance of existence value. Our sensitivity analysis shows that the optimal management policy depends on the growth rate, the volatility, and the minimum viable density of the wolf population, but little on damages, existence value, and the discount rate for the parameters considered.

Keywords: endangered species; reintroduction; uncertainty; irreversibility; real options.

JEL Codes: D81, Q20, Q28

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“In those days we had never heard of passing up a chance to kill a wolf…Since I have lived to see state after state extirpate its wolves… I have seen every edible bush and seedling browned, first to anemic desuetude, and then to death. … I now suspect that just as a deer herd lives in mortal fear of its wolves, so does a mountain live in mortal fear of its deer…. Perhaps this is the hidden meaning in the howl of the wolf, long known among mountains, but seldom perceived among men.” Aldo Leopold [23].

1. Introduction

Conflicts between wildlife and humans, fueled by population growth and the conversion of natural areas, are creating increasingly tough challenges for policy makers. All over the U.S., pet owners are concerned by coyotes visiting urban or suburban areas [36] [39] [32]; ranchers and hunters are complaining about wolves in Western [16] and Midwestern states [4]; and Californians are worried by mountain lion sightings [7] and the aggressive behavior of black bears with a taste for human cuisine [3]. At the same time, the eradication of predators is to blame for a surge in the deer population that dramatically increased the number of road collisions [13], and for an explosion in the goose population that led to significant pollution from goose droppings [42]. The situation is worse in many developing countries. In India, home to half of the world’s remaining wild tigers, poor villagers are killed every year as they battle tigers for the use of protected areas [22]. In Zimbabwe, the government is erecting electric fences around villages to reduce the threat to human life and crops from elephants and other wild animals [51].

The task of wildlife managers is particularly difficult for the long-term management of potentially dangerous, endangered predators such as wolves, grizzly bears, or tigers, at a time where the preservation of these species appears ever more essential for the health of entire
ecosystems [41]. Yet the environmental economics literature does not offer simple management rules that account for biological uncertainty, the risk of extinction and the possibility of reintroduction (when available). This paper proposes a continuous time real options framework to derive such rules. Although our approach may be applied to any species that enters in conflict with humans, we focus here on large carnivores. Our application to wolves highlights the importance of existence value for the preservation of endangered species (a point also made in [1]), and it provides an economic justification for the reintroduction of wolves. Our sensitivity analysis shows that the optimal management policy depends on the growth rate, the volatility, and the minimum viable density of the wolf population, but not much on damages (for the parameters considered), existence value (provided it remains large), and the discount rate (≥2%).

As emphasized by a number of conservation biologists, large carnivores are especially important among endangered predators for several reasons [2] [43]. First, they are good indicators of ecosystem health because the areas where they still live typically retain most native species and have a high potential for ecological integrity [30]. Second, they often provide a “protective umbrella” to many other wild species.¹ Indeed, recent evidence supports the view that many carnivorous predators have a strong positive influence on biodiversity by limiting the overexploitation of vegetation by herbivores, the so-called “top-down” theory [27] [41]. Third, as large carnivores often require large expanses of land to maintain viable populations, focusing on their preservation is a useful strategy to conserve large-scale ecosystems for the long term.² Finally, large predators have become a symbol for the ongoing public debate on conservation [19] because big animals tend to inspire people in ways that microorganisms probably never will.

Although the Endangered Species Act explicitly bans the use of benefit cost analysis, the tools of environmental economics are useful for informing decision makers about the tradeoffs
entailed in the management of recovering endangered species. Two features of this problem are particularly noteworthy: uncertainty and irreversibility. First, there is uncertainty in the benefits and costs associated with different wildlife population levels, and they tend to vary randomly over time: unfavorable conditions may cause a predator to prey more on livestock, or public interest for a species may increase following positive press coverage. Second, the ecology of a number of predators is still insufficiently understood [8]. Irreversibility stems from the risk of extinction, although it may be overcome with reintroduction. The importance of uncertainty and irreversibility is now well-known in the economic literature on investment [12] so these features should be incorporated in wildlife management models. This motivates our approach, which extends and corrects the harvesting framework proposed by Saphores [38] to include a flow of amenities between controls and the loss of existence value from extinction [21].

We analyze the problem of a wildlife manager who intends to maximize the stream of net benefits from a species by controlling at what density culling is necessary, and the size of culling. For a fixed territory, the wildlife density follows a diffusion process (see [18]) due to fluctuations in natural conditions that affect reproduction, migration, food availability, and losses from predators. In a first class of models, if the wildlife population thrives, its density increases to the point where the flow of net benefits is negative so culling becomes necessary. If, on the other hand the wildlife population becomes extinct, there is a loss of existence value. In a second class of models, we allow for reintroduction as soon as the wildlife density reaches a lower threshold $K_0$. Comparing both classes of models is useful for assessing the value of the option to reintroduce. We then apply our framework to the management of wolves, probably the most controversial and charismatic predators around [33].

Two fairly recent papers study the inter-temporal trade-offs associated with the
management of valuable wildlife populations that cause damages at high densities. Using optimal control, Rondeau [34] analyzes the transitory dynamics following the reintroduction of a wildlife species. He investigates the timing of costly control measures and conditions under which reintroduction is not desirable. An application to deer management illustrates the losses resulting from delays in implementing the optimal control policy, and provides a very informative discussion of deer management issues. However, Rondeau’s analysis deals neither with uncertainty nor with the risk of population extinction. In a follow-up paper, Conrad and Rondeau [35] build a discrete time dynamic model to study deer management. They show that pulsing controls are more efficient than steady state policies over a wide range of conditions, including biological uncertainty, and discuss the acceptability of various deer control measures. They do not, however, discuss the risk of extinction and the resulting loss in existence value.

In modeling wildlife population dynamics, it would clearly be useful to account for mutualistic and predator-prey relationships; a logical and promising approach would be to build a general equilibrium ecosystem as in [14]. However, our biological understanding of many species is still too poor at this point for this type of models. This is the case for many large carnivore species [48] because their secretive behavior, low densities, and the long intervals between successive generations make their study expensive [8]. While the approach we propose in this paper is not ideal, it provides resource managers with simple decision rules until biological complexities are better understood.

The paper is organized as follows. In the next section, we provide a brief overview of the management policies applied to wolves in the United States. Section 3 presents a general framework for managing species that enter in conflict with humans. Section 4 applies this framework to the management of wolves. Section 5 summarizes our conclusions.
2. The case of wolves

Wolves typically generate strong emotions; they are emblematic of our difficult relationship with nature [19]. Their qualities make them tough competitors: they are highly intelligent with an acute hearing and an exceptional sense of smell [9]. Like humans, they adapt remarkably well to very diverse climatic conditions: they are distributed between 20° N. latitude (mid-Mexico and India) and the North Pole [28]. North America has two main species of wolves: gray wolves (Canis lupus, or timber wolves), used to range from Canada to Mexico, while red wolves (Canis rufus) occupied mostly the Southeastern United States. Apart from the color of their pelage, red wolves differ from gray wolves by their slightly smaller size and their more slender head.

Wolves have been the object of both fascination and age-old fears.³ According to the legend, a she-wolf fed Romulus, the founder of ancient Rome, and his brother Remus. In addition, most Native Americans revered gray wolves and tried to emulate their remarkable hunting skills [49]. On the other hand, most American settlers inherited the traditional European antipathy towards wolves (Remember the big bad wolf in Little Red Riding Hood or in Peter and the Wolf).⁴ This attitude is partly based on religious beliefs that give people dominion over the Earth’s creatures, and hold that the wilderness is godless and should be conquered [19]. It is therefore not surprising that the American colonies’ first wildlife act targeted wolves with the creation in 1630 by the Plymouth Colony pilgrims of a one-cent bounty [26]. As settlers moved west, they decimated animal populations traditionally preyed upon by wolves (such as deer, elk, moose, or bison [9]), so wolves turned increasingly to sheep and cattle. Wolves became “an object of pathological hatred” [25] and ranchers created bounty programs to eliminate them, along with other predators. Eradication efforts were boosted by technological improvements in
traps, firearms, and poisons; they became systematic when the government created predator control programs at the beginning of the 20th century [8]. Some of these programs continued until 1965 [44]. By the middle of the 20th century, wolves had been wiped out of the continental United States, except for several hundred gray wolves in Minnesota, an isolated population on Michigan’s Isle Royale, and a few red wolves in Louisiana and Texas. Wolves, however, continued to thrive in Canada and were not on the brink of extinction in Alaska.

In March 1967, following the Endangered Species Act (ESA) of 1966, wolves were listed as endangered in parts of the Southwest, and as threatened in the rest of the Southwest and in most of the Eastern United States. This first ESA did not protect species, however; it just called on federal agencies to pay attention to them and to foster their recovery. Wolves became effectively protected in August 1974, after the passage of the 1973 ESA [28].

In spite of illegal takings early on, the wolf population in the lower 48 states began to grow after 1974. Gray wolves from Minnesota migrated to Wisconsin, Michigan [29], and the Dakotas [24]. Canadian wolves spread into Montana, Washington State and Idaho [29]. In 1995 and 1996, 31 gray wolves from Canada were reintroduced in Yellowstone and 35 in Idaho [44]. By the end of 2002, there were approximately 664 wolves in Northern Montana, Central Idaho, and in the Yellowstone area, 295 in Michigan (including 17 on Isle Royale), 2,445 in Minnesota, and 323 in Wisconsin [44]. An additional 21 “Mexican” gray wolves lived in western Texas, central Arizona, and northern Mexico.

Following this success, gray wolves were reclassified as “threatened” in the Rocky Mountain region; wolves in Yellowstone and Central Idaho were designated as “non-essential, experimental” populations [45]. Following a court challenge by environmental organizations, this measure was overturned by a U.S. District court judge in Oregon who ruled that USFWS'
2003 decision was based only on healthy wolf populations in a small part of what used to be the gray wolf's range in the continental U.S. This delisting was also declared illegal because it combined regions in the North East with adequate habitat but no wolves, with the Great Lakes region, and declared recovery in both regions. Following this ruling, gray wolves were re-listed as endangered in February 2005, except for the 'non-essential experimental populations in Yellowstone and Idaho and the 'threatened' population in Minnesota, who retained their status.

The reintroduction of the red wolf was also successful: approximately 100 red wolves now live in the wild in North Carolina and another 156 are kept for breeding to insure genetic diversity [45]. They are still endangered, except in parts of North Carolina and Tennessee [46].

The persecution of wolves was mostly based on misconceptions. Little scientific information on wolves was available until the late 1960s [28]; it is now clear that they play a very important biological role. First, they regulate the populations of large hoofed animals (such as bison or elk), removing the weakest individuals from their populations [45]. This enhances plant diversity [1], and it makes biological room for smaller herbivores (e.g., beavers and small rodents). Wolves also displace some coyotes, allowing smaller predators such as foxes to thrive [10]. In addition, they indirectly support a number of other species such as grizzly bears, foxes, wolverines, vultures and ravens that feed on the carcasses of preys killed by wolves [44]. Many insect species also benefit: nearly 450 beetle species use these carcasses in Yellowstone [15].

In spite of their much-improved reputation [20], the reintroduction of wolves has generated a lot of opposition. Contrary to what the title of this paper suggests, wolf attacks on humans are rare in North America, and they took place in areas where wolves were attracted to garbage or were hand-fed [44]. Western ranchers and pet owners were the most affected: between 1987 and 2002, wolves killed approximately 600 sheep, 200 heads of cattle, and 50 pet
dogs [33]. Some elk hunters also blame wolves for a drop in the elk herd north of Yellowstone, although biologists point instead to severe droughts and other predators [33].

To calm public anger and limit further conflicts, lethal control has been exercised. Although this angers some conservationists [40], it still appears the most effective way of curbing wolf damage [28], so culling is the control we consider in our models. Indeed, traditional husbandry techniques relying on shepherds with guard dogs or electric fences are uneconomical; translocation does not work because wolves tend to return to their territories; and aversive conditioning is ineffective for wolves [28].

3. Models of Wildlife Management under Uncertainty

We consider an area jointly occupied by humans and a wildlife population, whose density is denoted by $X$. We suppose that there is a minimum viable wildlife density $K_0>0$, so as soon as $X$ reaches $K_0$, the wildlife population becomes extinct with certainty, which results in the loss of existence value $L$. $K_0$ therefore acts as an absorbing barrier for $X$ [38]. We assume that $X$ follows the autonomous diffusion process:

$$dX = m(X)dt + v(X)dz,$$  \hspace{1cm} (1)

where $m(.)$ and $v(.)>0$ are continuous functions defined on $[K_0, +\infty)$; $dt$ is an infinitesimal time duration; and $dz$ is an increment of a standard Wiener process [18].

The presence of wildlife creates benefits and costs for the local economy. Possible benefits include hunting, eco-tourism, and the control of species that could damage agriculture or forestry if left unchecked. Conversely, costs result from wildlife attacks on livestock, pets, or even humans. We denote by $f(x)$ the flow of net wildlife benefits when $X = x$. If $f(x)$ is sufficiently negative, the wildlife density may be reduced by $h$ at a cost $Q(h,x)$, where $0 \leq h \leq x$. 
\( Q(h, x) \) is assumed to be increasing in \( h \) and decreasing in \( x \). For simplicity, we suppose that culling is instantaneous, which is not unreasonable because it usually takes only a few days to find and remove radio-collared wolves.

Let us now consider the problem of a resource manager whose goal is to maximize the wildlife value function by choosing the sequences of wildlife densities at which culling should take place and the number of animals that should be culled each time. The wildlife value function is the sum of the discounted stream of net wildlife benefits plus the discounted culling costs and the possible loss of existence value or the discounted reintroduction costs, if reintroduction is possible and optimum. This is a-priori a difficult task but it can be greatly simplified, as shown below. Let us first consider the case without reintroduction.

Managing the wildlife population without reintroduction

To motivate our approach, it is fruitful to emphasize the parallel between culling and investing. First, the decision to cull is made under uncertainty. Second, it is clearly irreversible because culling entails sunk costs that cannot be recovered if it turns out that too many animals were removed. More importantly, excessive culling may lead to extinction. Finally, the decision to cull is flexible. These characteristics call for using real options.

If we see the wildlife population as an asset whose productivity we can affect by culling (or by reintroduction, when it is possible), the manager of this resource is holding a perpetual compound option. This option gives the right but not the obligation to cull, and it never expires if the resource manager’s time horizon is infinite, which we assume.

Moreover, just as in the Faustmann problem in forestry, the resource manager faces the same problem after each cull, albeit possibly with different starting points. As in [38], we rely on
stochastic dynamic programming and invoke Bellman’s optimality principle to infer that, if there is a unique solution, the optimal strategy is to cull by \( h^* \) as soon as \( X \) reaches \( x^* > K_0 \). \( x^* \) and \( h^* \) are chosen to maximize the present value of net benefits between culls, plus the present value of culling costs. Culling takes place at randomly spaced intervals \( (T_i, i=1\ldots+\infty) \), but since \( X \) is Markovian, the \( T_i \)s are independent and identically distributed.\(^7\) In addition, it is necessary to account for the risk of resource extinction.

Therefore, the wildlife value function \( V_N(.) \) has three components: 1) the flow of net benefits until either culling or extinction; 2) the cost of culling plus the value function after culling if \( X \) reaches \( x^* \) before \( K_0 \); and 3) the loss of existence value if instead \( X \) reaches \( K_0 \) before \( x^* \) and extinction occurs. Hence, for \( x_0 \in (K_0, x^*) \),

\[
V_N(x_0) = F(x_0; x^*) + D_{x^*|x_0; K_0} - Q(x^*, h^*) + V_N(x^* - h^*) - D_{K_0|x_0; x^*} L, \tag{2}
\]

where:

- \( x_0 \) is the current wildlife density;
- \( F(x_0; x^*) = E \left\{ \frac{T_{K_0\vee x^*|x_0}}{\rho} \int_0^{T_{K_0\vee x^*|x_0}} f(X_t) e^{-\rho t} \, dt \right\} \) is the present value of the flow of expected net benefits between the moment where \( X \) equals \( x_0 \) and the first time it hits either \( K_0 \) or \( x^* \);
- \( E \) is the expectation operator with respect to \( X \);
- \( T_{K_0\vee x^*|x_0} \) is the random duration between \( X=x_0 \) and the first time \( X \) hits either \( K_0 \) or \( x^* \);
- \( f(.) \) is the instantaneous flow of net wildlife benefits, including monitoring costs;
- \( \rho \) is the resource manager’s discount rate.
- \( D_{x^*|x_0; K_0} = E e^{-\rho T_{x^*|x_0; K_0}} \) and \( D_{K_0|x_0; x^*} = E e^{-\rho T_{K_0|x_0; x^*}} \) are expected discount factors;
- \( T_{x^*|x_0; K_0} \) is the first time \( X \) hits \( x^* \) starting from \( x_0 \), conditional on hitting \( x^* \) before \( K_0 \);
conversely, \( T_{K_0|x_0;x^*} \) is the first time \( X \) hits \( K_0 \) starting from \( x_0 \), conditional on hitting \( K_0 \) before \( x^* \); 

- \( Q(x^*, h^*) \) is the cost of reducing the wildlife density by \( h^* \) when \( X=x^* \); and

- \( L \) is the existence value of the wildlife population [21].

The method for deriving \( D_{x^*|x_0;K_0} \), \( D_{K_0|x_0;x^*} \), and \( F(x_0;x^*) \) is outlined in the appendix.

Likewise, the value of the wildlife asset immediately after culling is

\[
V_N^*(x^* - h^*) = F(x^* - h^*; x^*) + D_{x^*|x^*-h^*;K_0} - Q(x^*, h^*) + V_N(x^* - h^*) - D_{K_0|x^*-h^*;x^*} L. \tag{3}
\]

Isolating \( V(x^* - h^*) \) in (3) and plugging it into (2) allows us to reformulate the resource manager’s problem as follows:

\[
V_N^*(x_0) = \max_{(h, x, \ 0 \leq h \leq x - K_0)} \left\{ FA_N(x_0, x, h) + \frac{D_{x^*|x_0;K_0}}{1 - D_{x^*-h;K_0}} FA_N(x - h, x, h) \right\}, \tag{4}
\]

where

\[
FA_N(x_0, x, h) = F(x_0; x) - D_{x^*|x_0;K_0} Q(x, h) - D_{K_0|x_0;x^*} L \tag{5}
\]

is the present value of the expected flow of net benefits from wildlife, including control costs, monitoring costs, and the expected loss of existence value, between \( X=x_0 \) and the moment where the wildlife population is either culled or becomes extinct. Likewise, \( FA_N(x - h, x, h) \) is the present value of the expected flow of net wildlife benefits during a management cycle. \(^8\)

Let us now derive necessary first-order conditions for an interior solution. As noted in [38], both first order conditions need to be written for \( x=x_0=x^* \) and \( h=h^* \). First order necessary conditions for \( x_0 \neq x^* \) merely indicate whether or not the maximum of (4) has been attained at \( x_0 \).
Both necessary first order conditions for (4) have the same structure, so let \( \zeta \) be a dummy variable representing either \( h \) or \( x \). Taking the derivative of Equation (4) with respect to \( \zeta \), equating it to zero, and rearranging terms gives

\[
\frac{\zeta \frac{\partial F_{A_N}(x_0, x, h)}{\partial \zeta}}{F_{A_N}(x - h, x, h) - 1 - D_{x|x-h;K_0}} = \hat{\varepsilon}^{F_{A_N}}_{\zeta} + \hat{\varepsilon}^{D_N}_{\zeta}, \text{ at } x_0 = x^*, h = h^*.
\]

where \( \hat{\varepsilon}^{F_{A_N}}_{\zeta} = -\frac{\partial \ln F_{A_N}(x - h, x, h)}{\partial \ln \zeta} \) is the elasticity with respect to \( \zeta \) of the present value of the expected flow of net wildlife benefits during a management cycle and

\[
\hat{\varepsilon}^{D_N}_{\zeta} = -\frac{\partial \ln \left[ D_{x|x_0;K_0} \frac{1 - D_{x|x-h;K_0}}{1 - D_{x|x-h;K_0}} \right]}{\partial \ln \zeta}
\]

is the elasticity with respect to \( \zeta \) of \( D_{x|x_0;K_0} \frac{1 - D_{x|x-h;K_0}}{1 - D_{x|x-h;K_0}} \). On the left side of (6), the numerator is the average present value of the flow of net wildlife benefits during a management cycle, and the denominator is the sum of the expected flow of net wildlife benefits during all management cycles.

To gain some insight into the timing of culling, let us look at (6) for \( \zeta = x \). First, consider the elasticity of the modified discount factor. Waiting to cull delays incurring the culling costs and increases the flow of net wildlife benefits; culling is justified only if this flow is negative at \( x^* \), which we now assume. If a small increase in \( x^* \) (i.e., waiting longer) causes the discount factor to increase a lot, future culling costs are heavily discounted. The resource manager is then more likely to cull now because the present value of future culling costs is small compared to the value of the flow of wildlife damages. Conversely, if a small increase in \( x^* \) causes the discount factor to increase very little, discounted future culling costs are now only slightly lower so the manager is more likely to wait. Now consider the elasticity of the present value of the expected
flow of net wildlife benefits. If the flow of net benefits increases a lot while waiting, the
manager culls now. Conversely, if this flow changes little with a small increase in $x^*$, culling is
delayed. The same logic holds for the first order condition with respect to $h$.

Formulating the optimum management strategy in terms of elasticities provides wildlife
managers with a convenient rule of thumb if estimates of elasticity measures are available.

Managing the wildlife population with reintroduction

When reintroduction is feasible and optimum, key problem features are unchanged. If there is a
unique solution, it is therefore still optimum to reduce the wildlife density by $h^*$ as soon as it
reaches $x^*$, although the value of $x^*$ and $h^*$ here usually differ from the case without
reintroduction. For simplicity, we assume that reintroduction increases the wildlife density from
$K_0$ up to $x^*-h^*$, and we denote reintroduction costs by $R(x^*-h^*)$. Given $x_0 \in (K_0, x^*)$, the value
function of the wildlife asset, now denoted by $V_R(\cdot)$, becomes

$$V_R(x_0) = F(x_0; x^*) - D_{x^*|x_0; K_0} Q(x^*, h^*) - D_{K_0|x_0; x^*} R(x^*-h^*) + D_{x^*\lor K_0|x_0} V_R(x^*-h^*). \tag{7}$$

In (7), $D_{x^*\lor K_0|x_0} = E e^{-\rho T_{x^*\lor K_0|x_0}}$ is the expected discount factor for $T_{x^*\lor K_0|x_0}$, the first time $X$
hits either $x^*$ or $K_0$ starting from $x_0$ (see appendix); other terms are defined as above.

If we compare (7) to the value function for the no-reintroduction case (Equation (2)), the
resource manager never loses the wildlife asset, although reintroduction costs may be incurred
periodically. Writing (7) for $x_0 = x^*-h^*$, isolating $V(x^*-h^*)$, and plugging it back into (7) allows
us to reformulate the resource manager’s problem as follows:

$$V_R(x_0) = Max_{(h,x, \ 0 \leq h \leq x-K_0)} \left\{ FA_R(x_0, x, h) + \frac{D_{x\lor K_0|x_0}}{1-D_{x\lor K_0|x-h}} FA_R(x-h, x, h) \right\}, \tag{8}$$
where

\[ FA_R(x_0, x, h) = F(x_0; x) - D_{x|x_0; K_0} Q(x, h) - D_{K_0|x_0; x} R(x - h) \]  

(9)

is the present value of the expected flow of net wildlife benefits, including control, monitoring and reintroduction costs, between \( X = x_0 \) and the moment where wildlife is either culled or reintroduced. Likewise, \( FA_R(x - h, x, h) \) is the present value of the expected flow of net wildlife benefits, including control and reintroduction costs, during a management cycle. Comparing (5) and (9), we see that \( FA_M(.) \) and \( FA_R(.) \) are similar except that \( L \) in the former replaces \( R(x-h) \) in the latter.

Proceeding as above, if \( \zeta \) represents either \( h \) or \( x \), the two first order necessary conditions for an interior solution, which hold for \( x = x^* \) and \( h = h^* \), imposing \( x_0 = x^* \) can be written

\[
\zeta \frac{\partial FA_R(x_0, x, h)}{\partial \zeta} = \zeta^{FA_R} + \zeta^{DR} \cdot \text{at } x = x_0 = x^*, h = h^*, \]  

(10)

where \( \zeta^{FA_R} = -\frac{\partial \ln FA_R(x - h, x, h)}{\partial \ln(\zeta)} \) is the elasticity with respect to \( \zeta \) of \( FA_R(x - h, x, h) \) and

\[
\zeta^{DR} = -\frac{\partial \ln \left[ D_{x \vee K_0|x_0} 1 - D_{x \vee K_0|x - h}^{-1} \right]}{\partial \ln(\zeta)} \]  

is the elasticity with respect to \( \zeta \) of \( D_{x \vee K_0|x_0} 1 - D_{x \vee K_0|x - h}^{-1} \). The right sides of Equations (6) and (10) have the same interpretation.

The difference between \( V_R(x_0) \) and \( V_N(x_0) \) gives the value of the option to reintroduce the wildlife population.
Comparing management alternatives

Reintroduction may be possible but it may be too expensive, or a species may cause very high damages and have a very low existence value. In this case, eradication may be optimum. With the notation defined above, the corresponding value function is then

\[ V_E(x_0) = -L - Q(x_0, x_0), \]  

(11)

where \( x_0 \) is the current wildlife density.

To choose between management with or without reintroduction and instant eradication, we simply select the management strategy corresponding to the highest value functions at the current density. From the Bellman optimality principle, if we follow now a management strategy, then it should remain optimum in the future unless the nature of our problem changes over time, which we ruled out by assumption.

4. An Application to the Management of Wolves

Let us now apply our framework to the management of wolves. For simplicity, we assume that \( X \) follows the geometric Brownian motion (GBM)

\[ dX = \mu X dt + \sigma X dz, \]  

(12)

where \( \mu \) and \( \sigma > 0 \) are respectively the infinitesimal mean and volatility parameters, and \( dz \) is an increment of a standard Wiener process [18]. Equation (12) implies that the wildlife density tends to increase \((\mu > 0)\) around an exponential trend, which is reasonable for fast growing wildlife populations not limited by food availability for example [10] and [37]. It appears adequate for wolves in many situations since the growth rate of the wolf populations in Michigan (1990-2001) and Yellowstone (1995-2002) exceeds 30% per year [http://www.wolf.org/wolves/learn/basic/populations/uspop.asp: International wolf Center].
Assuming that $X$ follows a GBM ignores the natural carrying capacity $K_i$ of the territory where the wolf population lives, so it leads to overestimating future wolf damages since they increase with wolf density; $x^*$ with the GBM is thus likely to be lower than its optimum value.

The GBM assumption is reasonable, however, if the wildlife population should be controlled significantly below $K_i$, which is often the case for wolves living in close proximity to humans. Assuming a GBM for $X$ also yields much simpler analytical results than for a mean-reverting process (see [38] for some results), which explains its popularity in finance and economics.

When we solve (A.1) with boundary conditions (A.2) and (A.3), we obtain

$$D_{x|0:K_0} = \frac{K_0^{\theta^+} - K_0^{\theta^-}}{\frac{K_0^{\theta^+}}{x} - \frac{K_0^{\theta^-}}{x}}, \quad D_{K_0|x:0} = \frac{x^{\theta^-} - x^{\theta^+} - x^{\theta^+} - x^{\theta^-}}{\frac{K_0^{\theta^+}}{x} - \frac{K_0^{\theta^-}}{x}}. \quad (13)$$

and

$$D_{x^+:K_0|x_0} = \frac{(K_0^{\theta^-} - x^{\theta^-})x_0^{\theta^+} + (x^{\theta^+} - K_0^{\theta^+})x_0^{\theta^-}}{\frac{K_0^{\theta^-}}{x} - \frac{K_0^{\theta^+}}{x}}. \quad (14)$$

$\theta^+ > 0$ and $\theta^- < 0$ are defined by

$$\theta^\pm = 0.5\sigma^2 - \mu \pm \sqrt{(0.5\sigma^2 - \mu)^2 + 2\rho\sigma^2}. \quad (15)$$

For the net flow of amenities resulting from the presence of wolves, we suppose that

$$f(x) = -a_0 - a_1x^2. \quad (16)$$

In (16), $a_0 > 0$ is the flow of monitoring costs for the wolf population and $a_1x^2$ ($a_1 > 0$) is the flow of net damages; there is no flow of benefits because we could not find data linking wolf density to ecological benefits (see Section 2) or to increases in tourism, although there is some evidence that economic growth is stimulated by the conservation of wilderness habitats that sustain wolves.
Our decision rule thus generates a lower $x^n$ than optimum. With (16), the derivation of
$F(x_0; x)$ is straightforward (see appendix):

$$F(x_0; x) = \frac{-a_0}{\rho} \left\{ \frac{(x_{\theta^-} - K_{\theta^-})x_0^{\theta^-} - (x_{\theta^+} - K_{\theta^+})x_0^{\theta^+}}{x_{\theta^+} - x_{\theta^-}} + 1 \right\} +$$

$$\frac{-a_1}{\rho - 2\mu - \sigma^2} \left\{ \frac{(x_{\theta^-} K_0^2 - x_0^2 K_{\theta^-})x_0^{\theta^-} - (x_{\theta^+} K_0^2 - x_0^2 K_{\theta^+})x_0^{\theta^+}}{x_{\theta^+} - x_{\theta^-}} + \lambda_0 \right\}. \quad (17)$$

With reintroduction, monitoring costs are always incurred so their total contribution to the objective function is simply $-a_0/\rho$, case 1 is more complex because of the risk of extinction.

In addition, we suppose that the cost of reducing the wolf density by $h$ when the total wolf density is $x$ can be adequately described by

$$Q(x, h) = q_0 + q_1 \frac{h}{x}, \quad (18)$$

where $q_0 > 0$ and $q_1 > 0$ are respectively the fixed and variable culling costs. $Q(x, h)$ thus increases with the culling level and decreases with the wolf density.

Finally, we model reintroduction costs using the functional form

$$R(\xi) = r_0 + r_1 \xi. \quad (19)$$

Fixed costs ($r_0$), which are not directly dependent on the number of wolves translocated, have several components. The first one is the cost of hiring scientists to craft a reintroduction plan: select wolves, decide where they should be released, and assess their potential impacts on ranchers, hunters, other local residents, and wildlife. In addition, administrative costs are incurred to secure approval for the reintroduction process from local, state, and federal authorities, and in the case of Yellowstone and Idaho, from the Canadian government. Finally, information campaigns need to be organized to educate the public about the reintroduction process, its potential impacts, and how to manage potential conflicts with reintroduced wolves.
Variable costs \((r_1)\) include the costs of capturing wolves, transporting them close to the release site, preparing temporary pens to acclimatize them (for a “soft release,” as in Yellowstone), and releasing them. With a “hard-release,” which was used in Idaho, wolves are released directly without prior acclimatization.

Before we proceed to find \(x^*\) and \(h^*\), let us first analyze the deterministic case to get a benchmark for the impact of uncertainty.

The Deterministic Case

When \(\sigma=0\) in \((12)\), the wolf population is no longer at risk of becoming extinct and we have a one to one relationship between time and the density of the wolf population. If we proceed as above and formulate the problem in terms of \(x^*\) and \(h^*\), the deterministic objective function is

\[
\max_{x^0, h^0, K_0 \geq 0} \left\{ \frac{a_1 x^2}{2 \mu - \rho} \left( \frac{x_0}{x} \right)^2 - \left( \frac{x_0}{x} \right)^\delta + \frac{x_0^\delta}{x^\delta - x - h^\delta} \left[ \left( \frac{1 - h}{x} \right)^2 - \left( \frac{1 - h}{x} \right)^\delta \right] \right\}
\]

\[
- \frac{x_0^\delta}{x^\delta - x - h^\delta} \left[ q_0 + q_1 \frac{h}{x} \right] - \frac{a_0}{\rho} \right\},
\]

where the constraint \(x^\delta - h^\delta \geq K_0\) is added for consistency with the stochastic case. From our assumptions, the flow of net benefits from wolves is always negative. It is not optimum to cull continuously, however, because of fixed costs \((q_0 > 0\) in \((18)\)). When culling takes place, it brings the wolf density all the way down to \(K_0\). We will see that this is not the case for the stochastic case. The relevant objective function is therefore obtained by setting \(h^*\) to \(x^* - K_0\) in \((20)\).

Data

For our data, we searched the economic and the biological literatures, as well as sources in the
U.S. (including the National Prairie Wolf Research Center, the International Wolf Center, the Wisconsin Department of Natural Resources, and the U.S. Fish and Wildlife Service), Canada, Sweden, Norway, and India. We rely mostly on information from the Yellowstone area, which we complement with data from other U.S. states.

Only relatively short annual time series are available to estimate $\mu$ and $\sigma$ in (12). Using maximum likelihood, we find a low value for $\mu$ of 0.09 / year ($\sigma=0.24 / \sqrt{\text{year}}$) for Montana (1985-2001), and a high of 0.34 ($\sigma=0.33$) for Yellowstone (1995-2002), with Wisconsin ($\mu=0.28$, $\sigma=0.20$ for 1985-2001) and Michigan ($\mu=0.30$, $\sigma=0.29$ for 1990-2001) in between. For our base case, we therefore select $\mu=0.34 / \text{year}$ and $\sigma=0.33 / \sqrt{\text{year}}$ [http://www.wolf.org/wolves/learn/basic/populations/uspop.asp: International Wolf Center].

These high growth rates are in line with wolf biology: after a gestation period of 65 days, wolves give birth once a year to 1 to 9 pups, with an average litter size of 5 to 6 pups. Mortality is fairly high, however: only 1 or 2 of every 10 pups born live, survives to leave the pack to find a mate [9].

The survival of a wolf population does not hinge only on a minimum density; it also depends on factors such as the availability of preys and land use (e.g., road density). In addition, a minimum number of wolves are needed for genetic diversity. The Yellowstone area is 25,000 square miles ($\text{mi}^2$) and it offers an abundant food supply, so a starting population as small as 25 wolves is likely to thrive, as shown by recent history. We therefore set $K_0$ to the fairly low value of one wolf per 1000 $\text{mi}^2$.

To estimate reintroduction costs for Yellowstone, we rely mostly on [47]. Fixed costs, which comprise staff/support, public education/law enforcement and administration/support, total approximately $r_0 = $320,000.9 Variable costs, which include monitoring of donor pack,
facility preparation/care, capture/transport, and confine/release, reach $13,000 per wolf. This number is high: for Idaho, variable costs were only $5,300 per wolf released because there was no acclimation period. Since our decision variable is wolf density per 1,000 mi$^2$, our variable cost coefficient is then $r_1 = 13,000 \times 25 = 325,000$, in $\$1,000$ per wolf (recall that the greater Yellowstone area is 25,000 mi$^2$). These costs exclude monitoring, which we estimate at $\$100,000 per year ($a_0 = \$100,000$): $\$50,000 for the salary of the wolf program coordinator, and $\$50,000 for radio telemetry expenses and howl surveys (see also [50]).

Evaluating the stream of damages from wolves is challenging because relatively few data are available. To come up with an estimate for $a_1$ in (16), we use OLS with White heteroskedasticity-consistent standard errors to link wolf density with compensation paid to ranchers for livestock depredation, and we find $a_1 = 524 \$ (1,000$ mi$^2$ per wolf)$^2$. This approach underestimates true damages because carcasses sometimes disappear and cannot be attributed to the misdeeds of wolves. However, it compensates partly for the GBM hypothesis, which overestimates the growth potential of the wolf population and therefore its potential damages.

For control costs, we only found data from Minnesota because until April 1$^{st}$, 2003, it was the only state with a sizeable wolf population where culling was permitted (wolves were listed as threatened instead of endangered). When we use OLS with White heteroskedasticity-consistent standard errors to estimate (18), we find $q_0 = \$108 and $q_1 = \$974 \times 1,000$ mi$^2$ per wolf. Estimations for both damage costs and culling costs are detailed in Tables 1A and 1B.

Existence value, which is measured by applying contingent valuation, is one of the most difficult parameters to estimate. We found only a couple of studies that estimate existence value for wolves [5] [47]. We adopt the existence value ($L = \$8.3$ million) from [47], since it focuses on the reintroduction of wolves in Yellowstone National Park.
Finally, we choose a discount rate $\rho$ of 7% per year, a value commonly used by the federal government for conducting cost-benefit analyses. Table 2 summarizes the meaning and estimated values of our model parameters.

Since uncertainty surrounding some parameters is substantial, we conduct an extensive sensitivity analysis using the following values: $\mu \in [0.10; 0.50]$, $\sigma \in [0.10; 0.60]$, $K_0 \in [1.0; 4.0]$, $a_1 \in [-1,600; -500]$, $L \in \left[4*10^6; 12*10^6\right]$, and $\rho \in [1.25\%, 10.5\%]$. We also examine how lower reintroduction costs ($r_0=220,000$ and $r_1=132,500$) impact $x^*$ and $h^*$. Results were generated using MathCad on a PC. They are discussed below and illustrated on Figures 1 through 4.

**Results**

Let us first start with our base parameters (see Table 2). For case 1 (no reintroduction), we find $x^* = 4.92$ wolves per 1000 mi$^2$ and $h^* = 0.70$ wolf per 1000 mi$^2$, while for case 2 (with reintroduction), $x^* = 3.93$ wolves per 1000 mi$^2$ and $h^* = 0.87$ wolf per 1000 mi$^2$. The corresponding value functions at $X = 1.5$ wolves per 1000 mi$^2$ (an arbitrary value) are -$2,295,000$ and -$1,626,000$ respectively; they are negative because the presence of wolves is assumed to generate a flow of costs. The possibility of reintroduction increases the value of wolves to society and the best policy is to manage them according to case 2. These results compare with $x^* = 1.29$ wolves per 1000 mi$^2$, $h^* = 0.29$ wolves per 1000 mi$^2$, and $V = -$1,444,000 for the deterministic case. Managing the wolf population with the deterministic policy parameters ($x^*$ and $h^*$) would lead to a value function of -$8,470,000$ for case 1, and it simply would not make sense for case 2 since culling would induce the need for reintroduction, leading to very large and unnecessary management costs (due to constant culling and reintroduction). Accounting for volatility in the wolf population is therefore important here.
Contrasting case 2 with case 1, we see that culling takes place at a 20% lower density and it is 24% more severe for case 2: since reintroduction costs are much lower than existence value, the wildlife manager keeps a lower wolf density to reduce damages, even if it requires more frequent reintroductions. The difference between case 1 and case 2 increases with uncertainty, especially for $h^*$ (Figure 1A) and for the value function (Figure 1B).

From Figures 1A and 1B, we also see that $x^*$ and $h^*$ increase monotonically with volatility for both cases: a higher volatility increases the risk of a population crash so the wildlife manager needs to maintain a higher wolf density (and thus tolerate a higher flow of damages) in order to reduce the risk of loss of existence value or of incurring reintroduction costs. When uncertainty goes to 0, however, both $x^*$ and $h^*$ tend towards their deterministic values.

A good estimate of the wolf population growth rate ($\mu$) turns out to be important for $x^*$ and $h^*$. From Figure 2, we observe that as $\mu$ increases, both $x^*$ and $h^*$ decrease monotonically, and the difference between the two cases is sharply reduced. Indeed, when the rate of growth of the wolf population increases, the risks of extinction or of incurring reintroduction expenses decrease, and so does the difference between cases 1 and 2: the value functions at $X=1.5$ for $\mu=0.50$ are respectively -$1.747$ and -$1.517$ million for cases 1 and 2.

The value of $K_0$ also impacts $x^*$ and $h^*$ (Figure 3). As $K_0$ increases, the risks of extinction (case 1) or of incurring reintroduction costs increases, so $x^*$ becomes larger ($x^*=12.82$ (11.74) when $K_0=4$ for case 1 (case 2)), but this also swells wolf damages and lowers the value function ($V_N=-6.920 \times 10^6$ and $V_R=-3.878 \times 10^6$ at $K_0=4$). To partly compensate, culling ($h^*$) increases, especially for case 2: at $K_0=4$, $h^*=0.95$ for case 1 and 2.42 for case 2.

By contrast, the damage coefficient ($a_1$), the existence value ($L$), and especially the discount rate ($\rho$) have a limited impact on $x^*$ and $h^*$. This result is particularly important for $a_1$.
since many ranchers are claiming that only a fraction of the damages caused by wolves is officially acknowledged; they are therefore calling for much more severe culling. Yet if we triple the rate of damages \( (a_1) \) to 1600, \( x^* \) decreases to only 3.25 (from 3.93) while \( h^* \) also decreases to 0.61 (down from 0.87) for case 2 (the relevant situation here; see Figure 4). The underlying reason is that wolf damages are relatively small compared to other costs (e.g., reintroduction or monitoring costs). From these results, we see that the optimal policy when damages increase is to cull more frequently (at a lower wolf density) but also less severely. This also holds for case 1.

Although existence value justifies the reintroduction of wolves in our models, our results indicate that it is not necessary to have a precise measure of it because it is a large cost to avoid. This is good news for the practical implementation of our approach. Indeed, when \( L \) increases from $4 million to $12 million, \( x^* = 4.34 \) and \( h^* = 0.67 \) increase only to \( x^* = 5.20 \) and \( h^* = 0.71 \) in case 1 (it does not intervene directly in case 2). So when existence value increases, culling is less frequent but slightly more severe.

The discount rate \( \rho \) also has a fairly small impact on \( x^* \) and \( h^* \), except for very small values. When \( \rho \) grows from 2% to 10.5%, \( x^* \) increases from 4.48 to 4.91 and \( h^* \) grows from 0.68 to 0.70 for case 1; for case 2, \( x^* \) inches down from 3.99 to 3.89 and \( h^* \) decreases from 0.90 to 0.86. When the value of future damages decreases, it is therefore optimal to cull less frequently but slightly more severely for case 1, whereas the reverse holds for case 2. As expected, the value functions decrease when \( \rho \) increases as future damages are discounted more heavily. When \( \rho \) is below 2%, \( x^* \) and the value function start decreasing sharply; around \( \rho = 1.25\% \), immediate extinction becomes the preferred solution.
Finally, decreasing reintroduction costs (for \( r_0=220,000 \) and \( r_1=132,500 \)) increases the value function to \(-$1,554,000\) by limiting wolf damages: the wildlife manager culls more frequently \((x^*=3.56\) instead of 3.90\) although less severely \((h^*=0.78\) instead of 0.87\).

5. Conclusions

We have presented a continuous time framework that offers simple rules for managing a valuable endangered wildlife population under uncertainty, which also accounts for the risk of extinction and the possibility of reintroduction. These two important features have been neglected so far in the environmental economics literature.

Our application to the management of wolves in the greater Yellowstone area indicates that the possibility of reintroduction adds approximately 30\% to the social value of the wolf population. A comprehensive sensitivity analysis reveals that the wolf population growth rate \((\mu)\), its volatility \((\sigma)\), and the minimum wolf density \((K_0)\) are important determinants of the policy variables \(x^*\) (when to cull) and \(h^*\) (by how much to cull). By contrast, the damage coefficient \((a_I)\), the existence value \((L)\), and especially the discount rate \((\rho)\) have a limited impact on \(x^*\) and \(h^*\). This matters because of the difficulty of measuring existence value or of selecting a discount rate for preserving species. In addition, a number of Western ranchers have been asking for a reduction in wolf density on the basis of underestimated damages (or for a complete removal of wolves). Our results therefore do not support their claims.

Our results also highlight the importance of accounting for existence value in wildlife management models. However, it is critical that a mechanism be in place so that the general public (for whom wolves are a public good) compensates ranchers and hunters (for whom wolves are instead a public bad [6]), who bear the brunt of wolf damages. This compensation
mechanism can be private, as illustrated by the successful creation by Defenders of Wildlife of the Bailey Wildlife Foundation Wolf Compensation Trust, which pays livestock owners for losses from wolf predation.\textsuperscript{12}

Although our illustration assumes that the wolf density follows a GBM, our framework can be easily applied to mean-reverting processes (such as a logistic Brownian motion [38]) in order to account for carrying capacity. Useful extensions could include modeling uncertainty in damages (or benefits) at a given population density; incorporating errors in measuring the size of the wolf population; quantifying ecosystem benefits from wolves; analyzing land use policies that minimize conflicts with endangered species; and modeling interactions with other species when bioeconomic data become available.

References


Table 1A: Culling Costs Estimation

Model: \( Q(h/x) = q_0 + q_1 h/x \)
Method: Least Squares with White Heteroskedasticity-Consistent Standard Errors & Covariance

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>107.51</td>
<td>4.416</td>
<td>0.002</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>973.57</td>
<td>2.282</td>
<td>0.052</td>
</tr>
</tbody>
</table>

\( R^2 \) 0.226 Mean dependent variable 170.26
\( \bar{R}^2 \) 0.130 S.D. dependent variable 41.14
S.E. of regression 38.38 Akaike info criterion 10.31
Sum squared residuals 11784.32 Log likelihood -49.55

Table 1B: Damage Costs Estimation

Model: \( f(x) = a_1 x^2 \)
Method: Least Squares with White Heteroskedasticity-Consistent Standard Errors & Covariance
Sample: 1995-2000; 6 observations.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>523.90</td>
<td>3.334</td>
<td>0.021</td>
</tr>
</tbody>
</table>

\( R^2 \) 0.073 Mean dependent variable -6950.09
\( \bar{R}^2 \) 0.073 S.D. dependent variable 7144.67
S.E. of regression 6877.72 Akaike info criterion 20.66
Sum squared residuals 2.37E+08 Log likelihood -60.98
Table 2: Base Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>Parameter estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Rate of growth</td>
<td>0.34 per year.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Volatility</td>
<td>0.33 per year$^{0.5}$.</td>
</tr>
<tr>
<td>$K_0$</td>
<td>Minimum viable density</td>
<td>1 per 1000 mi$^2$.</td>
</tr>
<tr>
<td>$Q(x,h) = q_0 + q_1 \frac{h}{x}$</td>
<td>Culling costs</td>
<td>$108+974*(h/x)$.</td>
</tr>
<tr>
<td>$L$</td>
<td>Existence value</td>
<td>$8.3$ million.</td>
</tr>
<tr>
<td>$f(x) = -a_0 - a_1 x^2$</td>
<td>Stream of monitoring and damage costs.</td>
<td>-$100,000-$524 x$^2$ per year.</td>
</tr>
<tr>
<td>$R(\xi) = r_0 + \eta \xi$</td>
<td>Reintroduction costs</td>
<td>$320,000+325,000 \xi$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
<td>0.07 per year</td>
</tr>
</tbody>
</table>
Figure 1A: $x^*$ and $h^*$ versus $\sigma$, with and without reintroduction.

Notes: This figure was generated with our base parameters, except for $\sigma$, the volatility coefficient of the wolf population. $x_N^*$ and $h_N^*$ are respectively the density of the wolf population at which culling should take place and by how much the wolf density should be reduced without reintroduction. $x_R^*$ and $h_R^*$ have the same meaning with reintroduction.
Notes: This figure was generated with our base parameters, except for $\sigma$, the volatility coefficient of the wolf population. $V_N$ and $V_R$ are the value functions for case 1 (no reintroduction) and case 2 (reintroduction) respectively. They are calculated at 1.5, an arbitrary value of the wolf density.
Figure 2: $x^*$ and $h^*$ versus $\mu$, with and without reintroduction.

Notes: This figure was generated with our base parameters, except for $\mu$, the growth rate of the wolf population. $x_N^*$ and $h_N^*$ are respectively the density of the wolf population at which culling should take place and by how much the wolf density should be reduced without reintroduction. $x_R^*$ and $h_R^*$ have the same meaning with reintroduction.
Figure 3: Impact of $K_0$ on $x^*$ and $h^*$.

Notes: This figure was generated with our base parameters, except for $K_0$, the minimum density of the wolf population. $x_N^*$ and $h_N^*$ are respectively the density of the wolf population at which culling should take place and by how much the wolf density should be reduced without reintroduction. $x_R^*$ and $h_R^*$ have the same meaning with reintroduction.
Figure 4: Impact of $a_1$ on $x^*$ and $h^*$.

Notes: This figure was generated with our base parameters, except for $a_1$, the damage coefficient. $x_N^*$ and $h_N^*$ are respectively the density of the wolf population at which culling should take place and by how much the wolf density should be reduced without reintroduction. $x_R^*$ and $h_R^*$ have the same meaning with reintroduction.
Appendix

Let $a$, $b$, and $x_0$ be three real numbers such that $0 < a < x_0 < b$.

**Expression of $D_{a|x_0;b}$ and $D_{a\lor b|x_0}$**

We suppose that $b$ is absorbing. Let $W(x_0) = D_{a|x_0;b} = E e^{-\rho T_{a|x_0;b}}$, where $X$ follows (1).

Using a Taylor expansion, the law of total probabilities and the Markov property, Karlin and Taylor (1981) show that $W(.)$ solves

$$
\frac{\nu^2 (\xi)}{2} \frac{d^2 W (\xi)}{d \xi^2} + m(\xi) \frac{d W (\xi)}{d \xi} - \rho W (\xi) = 0.
$$

(A.1)

By construction,

$$W(a) = 1.$$  

(A.2)

In addition, since $T_{a|x_0;b}$ is the first time $X$ hits $a$ starting from $x_0$, conditional on hitting $a$ before $b$ and $b$ is assumed absorbing,

$$W(b) = 0.$$  

(A.3)

$$V(x_0) = D_{a\lor b|x_0} = E e^{-\rho T_{a\lor b|x_0}}$$

also verifies (A.1); based on the definition of $D_{a\lor b|x_0}$, the boundary conditions are simply

$$V(a) = V(b) = 1.$$  

(A.4)
**Expression of** $F(x_0; x^*)$

Let $R(x_0) = E \left[ \frac{\mathcal{L} e^{\mathcal{L} t} - e^{-\rho t}}{\rho} d t \mid X(0) = x_0 \right]$. Using the same approach as for $D_{a|x_0,b}$, Karlin and Taylor [18] show that $R(.)$ solves

$$
\frac{\nu^2(\xi) d^2 R(\xi)}{2} + m(\xi) \frac{dR(\xi)}{d\xi} - \rho R(\xi) + f(\xi) = 0, \quad (A.5)
$$

with boundary conditions

$$R(a) = R(b) = 0. \quad (A.6)$$

If $X$ follows the GBM described by (12), $m(\xi) = \mu \xi$ and $\nu(\xi) = \sigma \xi$. A general solution of (A.5) with boundary conditions $R(K_0) = R(x) = 0$ is then

$$R(x_0) = \frac{x^{\theta^-} R_0(K_0) - R_0(x) K_0^{\theta^+}}{x^{\theta^-} K_0^{\theta^+} - x^{\theta^-} K_0^{\theta^+}} x_0^{\theta^-} - \frac{x^{\theta^+} R_0(K_0) - R_0(x) K_0^{\theta^-}}{x^{\theta^+} K_0^{\theta^-} - x^{\theta^-} K_0^{\theta^-}} x_0^{\theta^-} + R_0(x_0), \quad (A.7)$$

where $R_0(.)$ is a particular solution of (A.5). If $f(\xi) = -a_1 \xi^2$, a particular solution of (A.5) obtained by substitution is $R_0(\xi) = \frac{-a_1 \xi^2}{\rho - 2\mu - \sigma^2}$. To model monitoring costs we simply solve (A.5) again with $f(\xi) = -a_0$, where $a_0 > 0$. A particular solution of (A.5) obtained by substitution is $R_0 = \frac{-a_0}{\rho}$. Since $F(x_0; x) \equiv R(x_0)$, Equation (17) follows.
Umbrella species require habitats that encompass the habitats of many other species [30].

In the Rocky Mountains, habitat requirements exceed 400 km$^2$ for mountain lions, reach 900 km$^2$ for grizzly bears, and range between 250 and 2000 km$^2$ for wolf packs [30].

For a history of the interactions between wolves and humans in Europe and a synthesis of its current status there, see [11].

Variations on the tale of Little Red Riding Hood are heard around the world. Apart from Western Europe, basic plot elements can also be found in tales from Japan, China and Korea (see http://mld.ursinus.edu/Maerchen/rrhintro.html). Such stories probably contribute to the bad image of wolves among young children [17].

To this list, we need to add 9 llamas, and one horse [33]. These are documented attacks, so actual damages may be larger.

Throughout this paper, $X$ denotes a random variable and $x$ is one of its realizations.

Indeed, since $X$ follows a diffusion process, it is Markovian, i.e. all the relevant information about $X$ is contained in its current state [18].

A management cycle goes from $x^*-h^*$ to either $x^*$ or $K_0$. If $X$ reaches $K_0$, the wildlife population becomes extinct if reintroduction is not possible; otherwise, reintroduction costs are incurred.

$50,000 of staff expenses are assumed to be allocated to monitoring (see footnotes in [50]).

This classification is not absolute. According to Ed Bangs (U.S. Fish and Wildlife Service reintroduction coordinator for the Northern Rockies), reintroduction costs could in fact be much lower for the reintroduction of only a few wolves (personal communication, 12/08/03).
